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Luís Melo Gonçalves

Measuring efficiency and frontier shift in Portuguese hospital industry: a non-parametric approach.

WP04/2018

Working Papers

ISSN 2184-3325

Measuring efficiency and frontier shift in Portuguese hospital industry: a non-parametric approach.

Luís Melo Gonçalves
PhD in Management (ISCTE - Lisbon university).
Professor of Finance in ISCAL

Abstract

Hospitals are complex bodies which produce multiple outputs through the utilisation of multiple inputs (Fare et al., 1994). Besides complexity, many relevant issues are raised in regard to its operation, and efficiency is one of them. It is estimated that half of the resources assigned to the health sector are allocated to hospitals operations.

In Portugal, reforms in the public sector have started in the decade of 90's of the last century and great efforts have been made to fight back the excessive expenditure in the health sector. Under this movement, some experiences in hospital management have been carried out, in particular, the privatisation of the management system of 34 hospitals of the Portuguese National Health System by converting them into 31 corporations running the hospital business.

The study we have conducted concerns the economic efficiency of hospitals and our analysis is based on the concept of the efficiency frontier (Farrell, 1957). For achieving this objective we have explored two possible methodologies, the parametric approach which is the econometric one, and the non-parametric approach, the data envelopment analysis.

As main conclusions of our research, the newly established hospital corporations helped to improve the efficiency frontier of the entire hospital industry, notwithstanding public hospitals could show higher technical efficiency scores than those that were transformed into corporations.

Key words: hospital efficiency; *Data Envelopment Analysis (DEA); Stochastic Frontier Analysis (SFA); New Public Management*.

Classificação JEL: I18, C02

1. THE FRAMEWORK

The evolution of Portuguese hospital system was marked by a series of events or milestones that shed light on the existing model in nowadays. In 1968, it was released the hospital status which defines the role and responsibilities of the hospital in providing health care. In 1990, it was approved the Basic Law on Health (Law No. 48/90 of 24 August) that, aligned with the welfare state of ex-EEC, advocated greater participation in the financing of the NHS. In 1993, it was approved the status of the SNS that in Articles 28 and 29 considers the existence of public-private partnerships for the management of hospitals. It was precisely under these articles that in 1995 was granted the management of an NHS hospital (Hospital Fernando Fonseca also known as Amadora - Sintra) to a private group.

One year later on, it was done the split of functions of purchaser (financing) and provider of health care services, and it have begun the process of setting up agencies for contracts within the Regional Health Departments.

In this year, it was given a new step in the “quasi-corporatization” of the NHS hospitals. The 1st experience took place at the Hospital de Sao Sebastiao in Santa Maria da Feira in 1996. Later, in 1999 and 2001, new near-corporatization experiences have taken place in the Local Health Unit of Matosinhos, and in the Hospital of Barlavento Algarvio, respectively.

The most relevant experience was the “corporatization” of 34 traditional NHS hospitals, which gave origin to 31 SA hospitals. This event took place in 2002 under Law No. 27/2002 of 8 November, during the right centre minority government. The transformation of 31 SA hospitals and five SPA hospitals in EPE hospitals occurred in 2005, during the Government of absolute majority of socialist party, under Decree No. 93/2005 of 7 June.

The changes in the hospital industry, in particular, the transformation of a significative part of SPA hospitals in SA hospitals, have created conditions for the introduction of competition in the industry, with impact on the efficiency of hospitals.

2. THE RESEARCH QUESTION

In this paper, it is our aim to investigate whether, with the establishment of SA hospitals, the efficiency frontier of hospital industry has moved or not, towards the improvement of technical efficiency. What we seek to understand with this research question is whether, with the transformation of a part of hospital industry in SA hospitals, there has been greater overall efficiency, as a result of the combination of a number of factors, such as better performance of the group composed by SA hospitals, better performance of SPA hospitals (or better performance in both groups), or mixed situations with the resulting overall effect of improving the efficiency of the hospital industry.

In general terms, we want to assess the impact on efficiency production frontier of the hospital industry, as a consequence of the transformation of part of SPA hospitals into SA ones.

The methodology approach for this research is based on analysis of the production efficiency frontier. As the efficiency frontier is not known, it must be estimated using hospital data sample. For the estimation, there are two broad possible categories of approach: i) non-parametric methodology based on an optimization problem of operational research, that define the frontier by segments, such as the DEA - Data Envelopment Analysis; ii) parametric or stochastic frontier estimated by econometric models such as the SFA - Stochastic Frontier Analysis.

In the next chapter we discuss the methodologies for the production efficiency frontier.

The issue of economic efficiency has been studied in terms of scientific research, over time, especially after the conceptual and theoretical formulation of Farrell, in 1957, in the very famous article "The measurement of productive efficiency".

In the health sector, the field on which is focused our work, varied analytical work on efficiency and efficiency frontier can be found in the literature, such as Burgess et al. (1996), Ferrier et al. (1996), Fare et al. (1994), Grosskopf et al. (1987). However, these just mentioned papers use the non-parametric approach, such data envelopment analysis methodology. Others have made the analysis using the stochastic or parametric approach for calculating the efficiency frontier, such as Eakin et al. (1988), Zuckerman et. al. (1994) and Wagstaff et al. (1992).

This paper is divided into five chapters including chapters 1 and 2 about Framework and the Research Question, respectively. In chapter 2, an introduction is made about the problem we propose to investigate. Chapter 3 presents the methodology for the production efficiency frontier and discusses the advantages of both approaches. The main findings of the investigation are presented in chapter 4 and the final conclusions of the research are discussed in chapter 5.

3. THE METHODOLOGY

The research methodology was the efficiency frontier of production. The following figures help to understand the basics behind this approach.

Figure 1 shows the production function of an industry, depicting the placement of a unit A, also known as DMU (Decision Making Unity). If the DMU was on the frontier, that is, if it was technically efficient, for the same level of resource X, it would produce the same as B. Or, for the same level of production, the DMU would use fewer resources and would be located in C.

All the inner area to the curve that defines the efficiency frontier represents the set of production possibilities.

In figure 2, it is also represented the rays from the origin to point A, B and C. In the case of DMU A, the productivity is given by the slope of the ray passing through point A (Y / X). If the DMU operating in section B (technical efficiency) instead of A, the slope or measure of technical efficiency is now higher than in A. In turn, in C, the measure of efficiency will be higher, and the slope now greater. In this case, we are in the optimal point where the scale and technique efficiencies are the highest.

Figure 1 – Technical efficiency

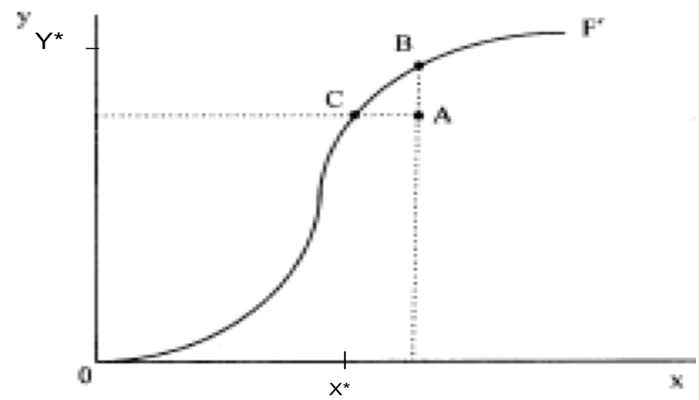
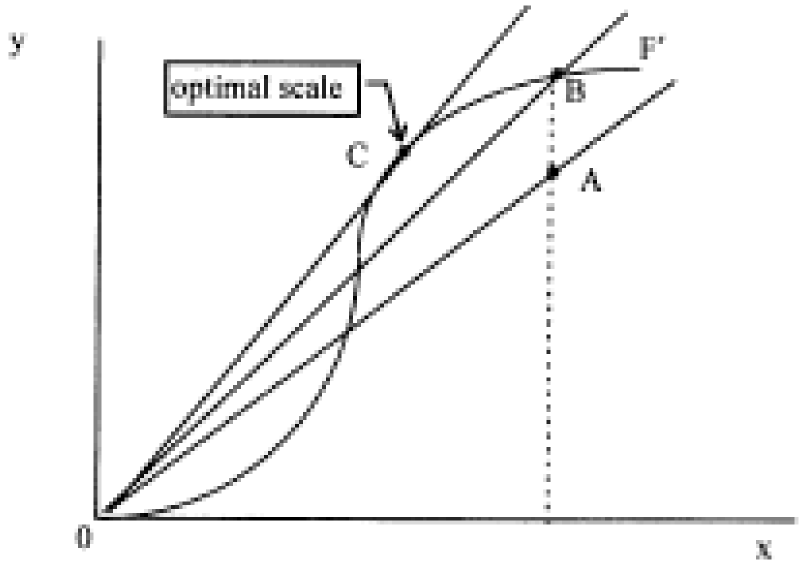


Figure 2 – Scale efficiency



There are two broad approaches to estimate the efficiency frontier and the scores of the DMUs: DEA (Data Envelopment Analysis) and SFA (Stochastic Frontier Analysis).
In very brief terms, the difference between the two approaches lies in the fact that in econometric approach (SFA) we determine the efficiency of the organization comparing to a theoretical frontier, while in the DEA approach we seek to determine the efficiency of the organization in relation to other units (peers) in the same industry.

Data Envelopment Analysis

The calculation of efficiency scores in DEA approach is the result of the ratio of the weighted sum of outputs in relation to the weighted sum of inputs, where the weights (multipliers) for both the outputs and for inputs follows the Pareto optimal for each homogeneous (DMU), with the constraint that the efficiency scores of DMUs can not exceed 100% which is the maximum relative efficiency of the sample (Charnes et al., 1995). Stated in an alternative way, Data Envelopment Analysis is a nonparametric approach that uses mathematical programming to define the efficiency frontier. In this approach, the efficiency can be measured as the ratio of the weighted sum of outputs in relation to the weighted sum of inputs (Hollingsworth et al., 1999).
An important issue is the choice of orientation of the optimization problem, which may have two variants: i) input-orientated, where the aim is to produce quantities set with minimal resources, ii) output-orientated when it wants to maximize production for a given level of resources.

On the other hand, concerning the technology underlying the production process, we have basically, two options: a) the production process is constant returns to scale (CRS), this is, the production process is such that optimality is independent of the scale of production, or b) variable returns to scale (VRS)[1]. The VRS option generates, in principle, higher scores than the one of CRS and it is considered the most flexible option in defining the production frontier.

The DEA model, input orientated, and constant returns to scale (CRS) was the 1st model to be widely applied, and was developed by Charnes, Cooper and Rhodes (1978). In the most intuitive, can be presented as a ratio problem where the measure of the efficiency is the ratio of all outputs on all inputs: $u'y_i / v'x_i$

The optimal weights are obtained by solving a problem of mathematical programming (DEA CRS) of this type:

$$Max_{u,v}(u'y_i / v'x_i)$$

subject to:

$$u'y_j / v'x_j \leq 1 \qquad j=1,2,.., N$$

$$u, v \geq 0$$

where

u- x1 vector of output weights

v –Kx1 vector of the inputs weights

y – M x N array of outputs

x –K x N array of inputs

i – Index for ith hospital

j – Index for hospitals of the sample

N- Number of hospitals

According to Coelli et al.(1998), duality in linear programming is preferred to resolve because it involves fewer restrictions:

Min Φ, λ Φ

subject to:

[1] Proportional consumption of resources is lower (IRS-increased returns to scale) or higher (DRS-decreased returns to scale) than output obtained.

$-y_i + Y\lambda \geq 0$

$\Phi x_i - X\lambda \geq 0$

$\lambda \geq 0$

where

Φ - scalar (efficiency score)

λ - Nx1 vector of constants

y_i - vector of outputs to the hospital i

Y – array of outputs of all hospitals

x_i - vector of inputs of i^{th} hospital

The calculation of efficiency scores is based on the resolution of a linear programming problem which minimizes the weighted sum of inputs, under the constraint that the weighted sum of outputs equal to 1 . The efficiency scores represent the proportion of use of resources (inputs) in order to achieve the efficiency frontier.

A deep knowledge of the DEA methodology can be found, for example, in the following publications: i) Fried, H. and S. Schmidt (1993), The measurement of productive efficiency: Techniques and Applications, Oxford University Press, New York; ii) Fare, R. and S. Grosskopf (1994), *Production Frontiers*, Cambridge University Press, Cambridge; iii) Charnes, A. and W. Cooper (1995), *Data Envelopment Analysis: Theory, Methodology, and Application*, Kluwer Academic Publishers; iv) W. Cooper and L. Seiford (2000), *Data Envelopment Analysis: A Comprehensive Text with Models, Applications, References and DEA-Solver Software*, Kluwer Academic Publishers; v) Cooper, W. et al. (2011), Handbook on Data Envelopment Analysis, International series in operations research management science, Springer; vi) Zhu j.(2014), Quantitative Models for Performance Evaluation and Benchmarking, Springer; vii) Paradi, j. et al. (2017), Data Envelopment Analysis in the Financial Services Industry, Springer

Stochastic Frontier Analysis SFA

In SFA econometric models, the random variable of the estimation function, usually referred as error or random disturbance, is decomposed into two terms: one of inefficiency and another of stochastic noise. This one is the error with normal distribution with mean 0 and variance equal to 1, which comprises the set of explanatory variables not explicitly present in the model.

In its more traditional representation, the error term must meet a set of assumptions for econometric estimation in order it can be considered appropriate. In studies of efficiency, the most interesting component of the formulation is the stochastic residue that distinguishes SFA models from COLS ones. Thus, while in COLS models all the residue is considered inefficiency, in SFA models the residue is divided into terms: stochastic error (v_i) and inefficiency (u_i).

Several functional forms can be used to specify the production function or cost one. Transcendental logarithmic function (translog) is one of the most used due to its flexibility. But in contrast, requires a high number of parameters to be estimated, reducing, therefore, the degrees of freedom.

The model parameters are specified taking into account “a priori” knowledge and econometric evidence on the characteristics of production process.

The stochastic frontier function of production was independently proposed by Aigner, Lovell and Schmidt (1977) and Meeusen and van den Broeck (1977).

The original specification of the production function contains an error term with two components: a term for the random component, and another for technical inefficiency. (Coelli, 1996).

$$Y_i = x_i\beta + (V_i - U_i) \qquad ,i=1,...,N,$$

For more details on the methodology of stochastic frontier production (Stochastic Frontier Analysis) see, for example, Aigner, D. et al. (1977); Christensen, L. et al. (1973); Coelli et al. (1998); Jacobs et al. (2006); Ray, S. et al. (2015).

Discussion on methods of efficiency frontier

As we saw before, different methods to evaluate the efficiency can be grouped into two broad categories: parametric and nonparametric ones. Parametric methods are associated with a functional form, while the non-parametric ones do not require a functional specification. Another possible classification is based on a statistical (stochastic data) or non- statistical methods.

The non-statistical method (DEA) is non-parametric or deterministic, while statistical methods are parametric (or stochastic), assuming, therefore, a statistical noise. Under

SFA we estimate a stochastic production frontier with a stochastic error and it requires restrictive assumptions about the production technology and the inefficiency term, such as truncated normal distribution.

In the DEA approach, efficiency frontier is built by sections or segments, with minimal assumptions about the technology which makes it less susceptible to specification errors, but no ability to adjust random error.

Having into attention that in DEA approach "noise" is considered inefficiency, it is natural that the scores of the SFA approach are higher than those of DEA.

Being DEA a nonparametric approach, it has the advantage of being able to deal with complex production technologies, involving multiple inputs and multiple outputs. As disadvantage, we point out the non-statistical noise tackling, which means that statistical tests cannot, in principle, be done.

The estimates of efficiency scores in DEA approach are very sensitive to data set considered, while in the SFA approach, estimates of efficiency are based on average parameters estimated, so are not as sensitive to changes in hospital data at the individual level. But it has the disadvantage of not clearly distinguish the random component of the inefficiency component.

Notwithstanding the potential difficulties in applying these two methods, there has been, in fact, a growing interest in applying these techniques in the measurement of hospital efficiency.

5. ESTIMATIONS AND RESULTS

The main source of data concerning the different input and output variables of all hospitals was the Portuguese DG HEALTH publications, covering the period 2002-2004.

The models estimations were done using the DEAP program for non-parametric (or deterministic) approach, and Frontier for the parametric approach.

The data collection involved the following output variables: *i)* inpatient discharges *ii)* outpatient visits, *iii)* emergency cases and, as a variable quality of service: *iv)* readmission rate within 30 days after discharge from hospital.

As input variables, we considered: *i)* number of doctors, *ii)* number of nurses *iii)* number of "other hospital staff" and *iv)* beds capacity. Therefore, four output variables and four input variables.

In order to understand how the efficiency frontier evolved in the hospital industry during the period of the management experience of SA hospitals, three estimates were done in a phase 1: DEA (VRS or variables returns to scale), DEA (CRS or constant returns to scale) and DEA Malmquist.

Relating DEA estimates, CRS and VRS were tested. As VRS results are considered more robust than CRS, we decided to present only the estimation results for VRS.

Figure 3 -Technical Efficiency for all hospital
(DEA VRS)

2002	2003	2004
0,861	0,917	0,903

Regardless of the technology option chosen (CRS or VRS), the average efficiency increases from 2002 to 2003, and decreases slightly between 2003 and 2004.

Considering only the model runs DEA VRS (see table above), technical efficiency rises 5.6 pp between 2002 and 2003, and decreases 1.4 pp in 2004.

Trying to interpret the results, the management change in hospital industry in early 2003, gave rise to a climate of competition in hospital industry where SA hospitals, being the subject of special attention, have been monitored consistently by the Office of Mission SA hospitals which has been producing regular monitoring reports of SA hospitals. In terms of media, they were at the focus of what was going on.

It is therefore natural that the 1st year of operation of SA hospitals had resulted in efficiency gains in hospital industry. The managers of SPA hospitals, in turn, have felt the responsibility of the competition with the comparison group (SA hospitals) and are likely to have implemented a more dynamic and efficient management. For these reasons, it is thought that the efficiency frontier hospital have improved during the 1st year of implementation of this experience.

The 2004 results are apparently of process consolidation contrarily to 1st year of novelty. In fact, in2004, there has been a slight regression of more than 1 percentage point.

Results of the Malmquist index

DEA estimates involving distance functions of Malmquist index are particularly adequate to examine the overall evolution of the factors over time. As research question is focused on the efficiency evolution of hospital industry, Malmquist Index can greatly help to understand how the efficiency frontier evolved over 2002-2004.

Figure 4 - All Hospitals (SA+SPA hospitals)

MALMQUIST INDEX SUMMARY OF ANNUAL MEANS

	1=3×4	2	3	4	5=1×2
Year	effch	techch	pech	sech	tfpch
2	1.029	1.186	1.007	1.022	1.221
3	1.024	0.932	1.015	1.009	0.954
mean	1.026	1.051	1.011	1.016	1.079

Note: year 2 = 2003; year 3 = 2004

The above table shows, for each year, the change in total factor productivity (tfpch – total factor productivity change) and its decomposition. The total factor productivity decomposition is as follows:

1) technical efficiency change (Effch). In the biennium (2003, 2004), the growth is, on average, 2.6% per year

2) technological efficiency change (techch). This will grow 18.6% in 2003 and decreases 6.8% in the following year. For the period, the average growth is 5.1%;

Technical efficiency change breaks down in:

3) pure efficiency change (pech). Practically, in the period, there have been no evolution worthy of mention. In terms of the biennium, the average increase is 1.1% per year.

4) scale efficiency change (sech). The growth is 2.2% in 2003 and 0.9% in next year. The average growth is 1.6% per year

5) total factor productivity change (tfpch) grows, on average, about 7.9% per year, with strong growth in 2003 (+22.1%) and 4.6% decrease in 2004. The 2003 growth of 22.1%, is mainly due to a favourable evolution of the efficiency frontier (frontier shift). This may be explained by the effect of competition that has emerged in hospital industry, due to the transformation of the SPA hospitals in corporate hospitals (SA). The contribution of improved technical efficiency in 2003 was only +2.9% and resulted primarily from improving efficiency of scale that grows 2.2%. That is, hospitals started operating in a more favourable cost curve as consequence of more appropriate economies of scale.

In 2004, total factor productivity decreased almost 5%, again owing to the behaviour of the technological frontier that after significant growth in 2003 (+18.6%), decreased 6.8% in 2004. The technical efficiency improved 2.4% which means that hospitals continued to move up closer to the frontier. The improvement of technical efficiency was due mainly to pure efficiency (+1.5%) than to scale efficiency (+0.9%).

The following figure (Figure 3) depicts the comparison of different indices of efficiency for the three groups of hospitals: SA, SPA and SA + SPA.

Figure 4 shows the trend over 2002-2004 of the different efficiency components of total factor productivity (overall efficiency) change for the all hospitals of our sample (SA+SPA hospitals).

Figure 5 – Comparison of average values of the period

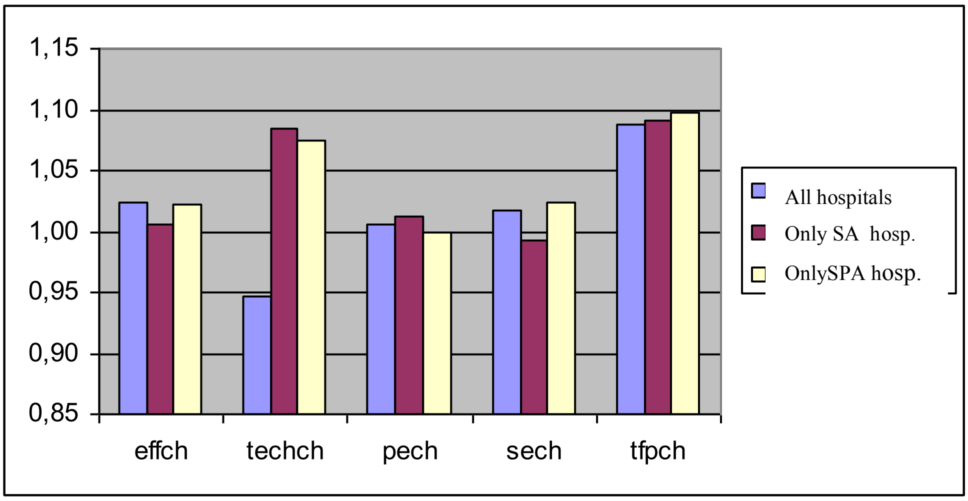
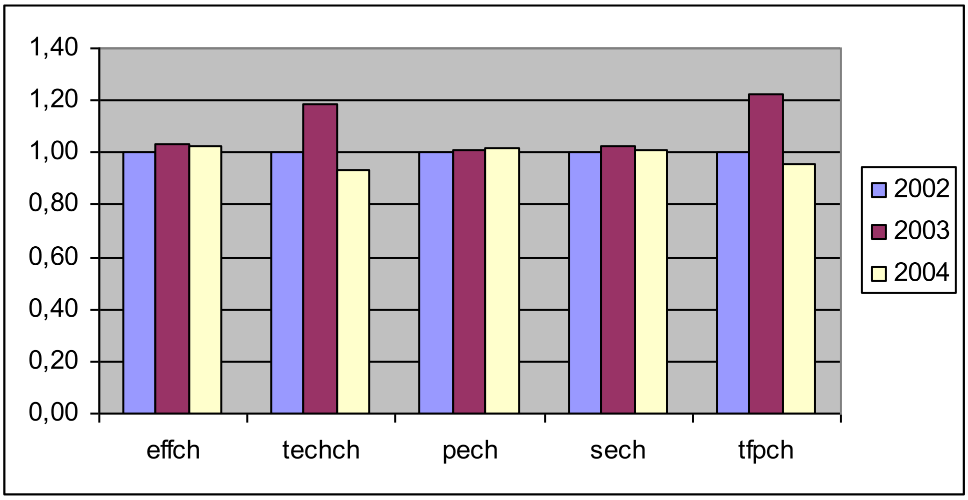
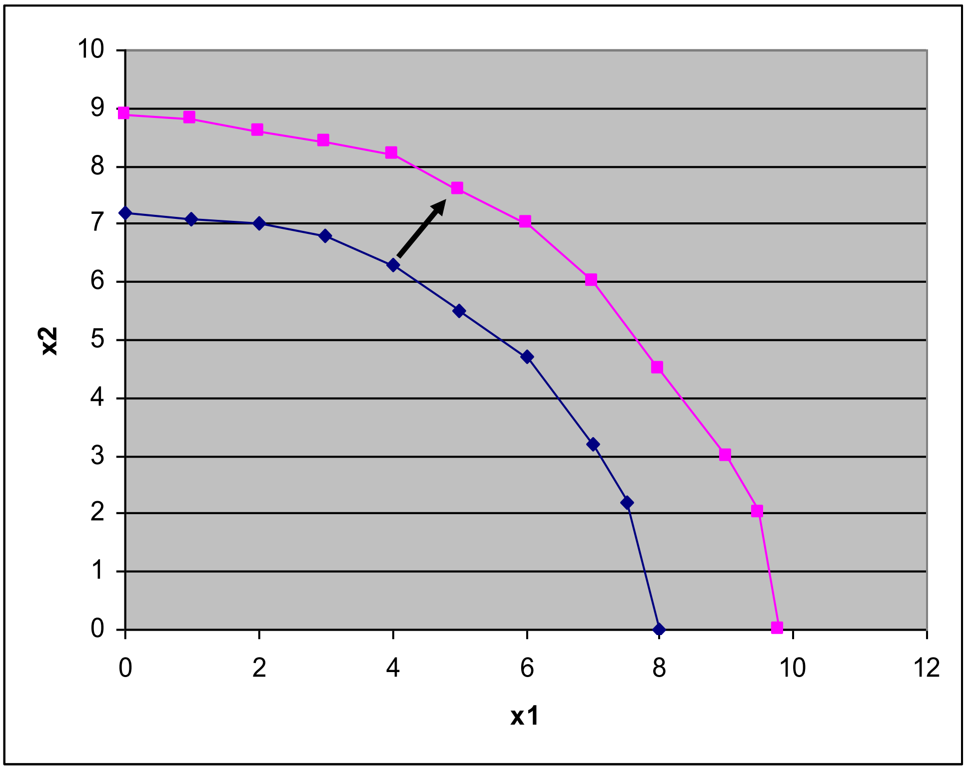


Figure 6 – Malmquist index – all hospitals



The interpretation of the results for all hospitals becomes evident that the technical efficiency began improved 2.9% between 2002 and 2003, and increased 2.4% between 2003 and 2004. However, it is not clear that changes referred are due to transformation of part of SPA hospital into SA hospitals.

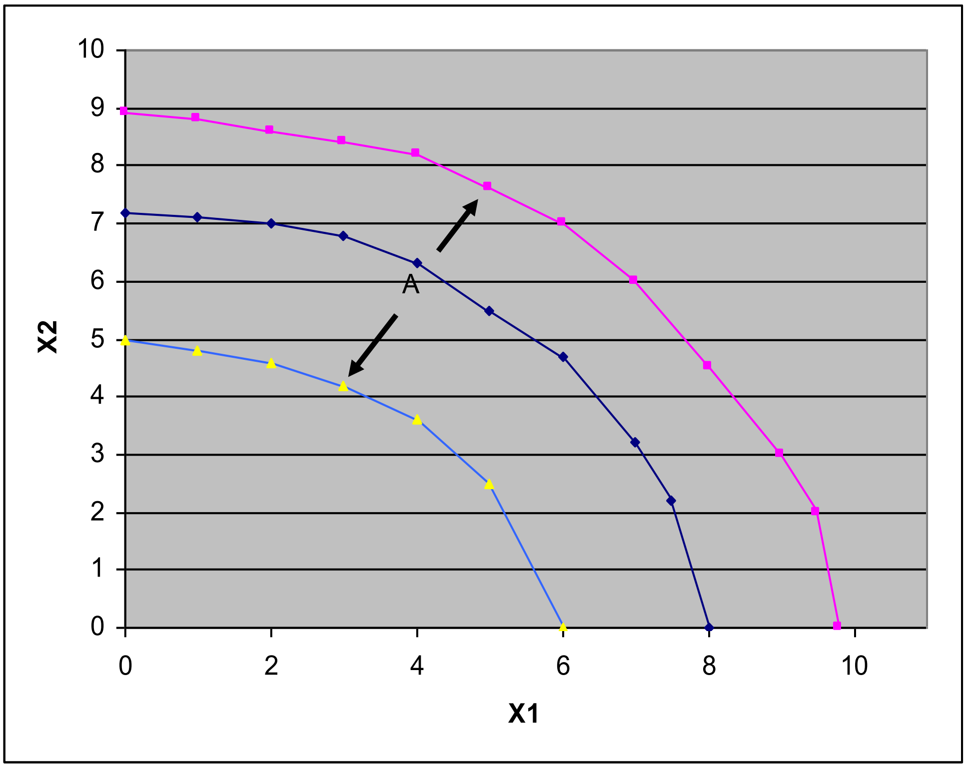
Figure 7 – Evolution of technical efficiency



The interpretation of the results for all hospitals shows that the technical efficiency improved by 2.9% between 2002 and 2003 and 2.4% between 2003 and 2004. However, it is not clear whether changes were due, or not due, to the transformation of some SPA into SA hospitals.

The frontier shift between 2002 and 2003 can have many explanations and interpretations. For example, if there was no transformation into SA hospitals, hypothetically the frontier could, instead of having improved from A to B, could have decreased to C (see figure 8). In this case, the shift frontier (improvement) was due to the superior performance of SA hospitals since the SPA hospitals would have had negative development.

Figure 8 – Decomposition of the evolution of technical efficiency



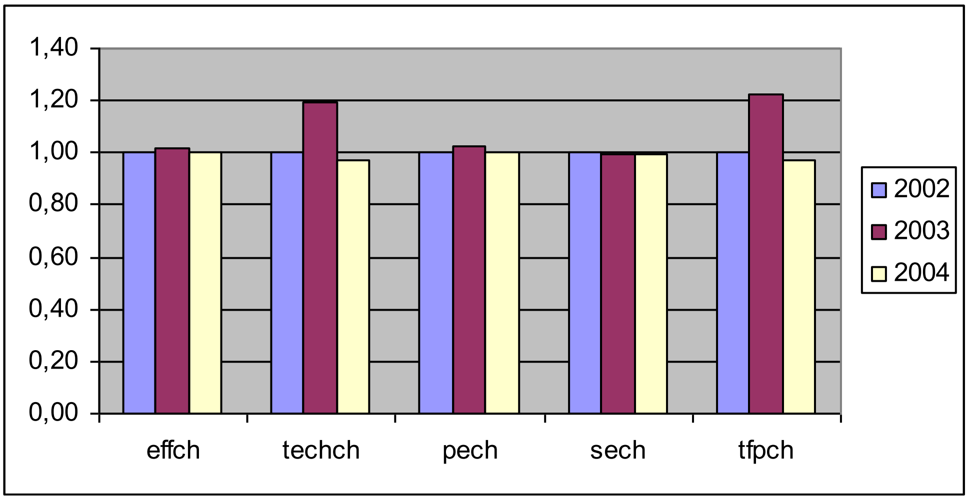
To help shed light on these questions, two Malmquist DEA estimates, one for each group of hospitals (SA and SPA, separately), were carried over. The aim, now, is to try to understand how the efficiency for each of these groups has evolved. The results are shown below. In this estimation, the efficiency frontier was defined only on the basis of hospitals that were transformed into SA hospitals

Figure 9 – SA Hospitals only

MALMQUIST INDEX SUMMARY OF ANNUAL MEANS

	1=3×4	2	3	4	5=1×2
Year	effch	techch	pech	sech	tfpch
2	1.015	1.192	1.024	0.992	1.211
3	0.999	0.969	1.004	0.994	0.968
Mean	1.007	1.075	1.014	0.993	1.082

Figure 10 – Malmquist Index – SA Hospitals only



The results show (figure 9) strong growth of total factor productivity change in 2003 (+21.1%), having fallen 3.2% in 2004. Again the main cause is the change in technology (frontier shift or technological change) with positive evolution (+19.2%) in 2003 and moving in the opposite direction (-3.1%) in 2004.

The technical efficiency grew in 2003, 1.5% due to favourable evolution in pure efficiency which grew 2.4%. In the following year, the technical efficiency stabilized (-0.1%). On average, an increase of 0.7% per year between 2002 and 2004 has been recorded. The technological efficiency will have increased by around 19.2% in 2003 and decreased 3.1% in 2004. The average change between 2002-2004 has been 7.5% per year.

Concerning technical efficiency, while the pure efficiency showed some progression, scale efficiency stabilized in both years of the biennium (2003-2004).

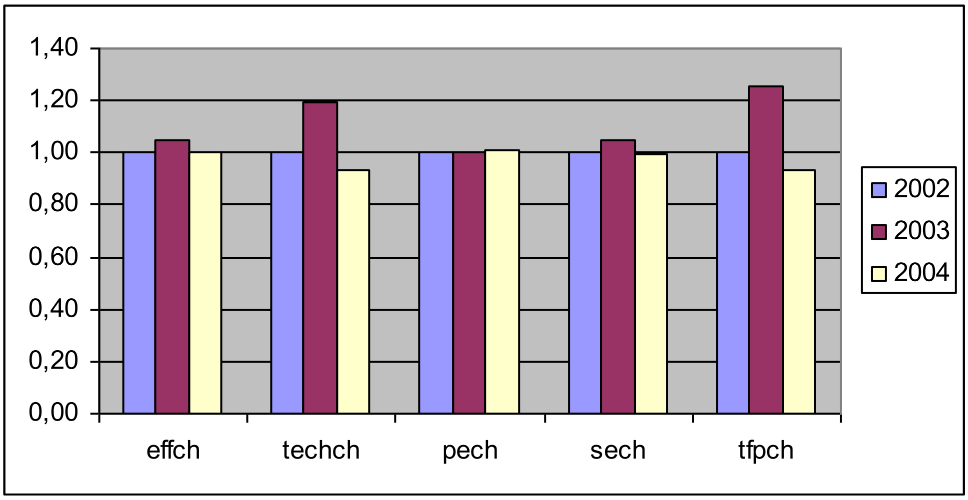
The estimation results for the annual average, considering only those hospitals that were not transformed in SA, this is SPA hospitals, are those listed in table below.

Figure 11 – Efficiency evolution and decomposition. SPA hospitals only

MALMQUIST INDEX SUMMARY OF ANNUAL MEANS

	1=3×4	2	3	4	5=1×2
year	**effch**	**techch**	**pech**	**sech**	**tfpch**
2	1.050	1.197	0.999	1.051	1.256
3	1.000	0.937	1.009	0.991	0.937
mean	1.024	1.059	1.004	1.020	1.085

Figure 12 – Malmquist Index– SPA hospitals only



In figure 11, it can be seen that technical efficiency of SPA hospitals increased 5.0% between 2002 and 2003, which is a higher rate than the one of SA hospitals (+1.5%), but in 2004, we verify the stabilization of technical efficiency, which is similar to what happened for SA hospitals.

Also the technological efficiency increases significantly in 2003 (19.7%), decreasing thereafter (-6.3%). The total factor productivity recorded average annual growth of 8.5%, with significant growth in 2003 (+25.6%) and decreased in 2004 (-6.3%). It should be referred that technical efficiency growth in the biennium is mainly due to the scale efficiency, which means that the SPA hospitals were operating in a more interesting position of the curve of average costs.

The combination of results of DEA estimations with the results of Malmquist index show that in the 1st year (2003), the favourable shift frontier was due to improvement of technical efficiency in both groups of hospitals (SA and SPA), but mostly in the SPA. In 2004, as seen before, the efficiency frontier for both hospital groups has somewhat stabilized. In the biennium, while the technical efficiency of the SA group of hospitals remained virtually unchanged (+0.7% per year), in the SPA group, the average annual growth was 2.4%. For all hospitals (SPA and SA), the average annual growth of technical efficiency was 2.6%.

5. CONCLUSIONS

The main aim of the research was to determine to what extent the transformation of SPA hospitals into SA ones in late 2002, contributed to the improvement of technical

efficiency frontier in hospital industry. In addition, it was also investigated which hospitals group is responsible for technical efficiency frontier shift of hospital industry.
For this purpose, we used both DEA (VRS) and Malmquist index.
The outputs from DEA runs allow the following findings:
- In 2003, technical efficiency of hospital industry improved, and in 2004 declined compared with 2003;
- In two years (2003 and 2004), the improvement of technical efficiency frontier of hospital industry was 4.9%, or 2.4% on geometric average (per year).

Analysis with Malmquist index shows that,
- In both years, technical efficiency has improved. In the 1st, the improvement was 2.9%, and in 2nd year, 2.4% (geometric average of 2.6%).
- During the biennium, frontier shift has improved by 5.1%.
- Detailing the analysis by hospital groups (SPA and SA), it appears that technical efficiency growth in 2003 is 1.5% for SA hospitals and 5.0% for SPA. In 2004 the technical efficiency of both groups of hospitals remained practically unchanged. On average, SA hospitals grew 0.7% per year while the SPA grew 2.4% per year.

We can check that, in fact, results of DEA (VRS) and Malmquist index approaches are relatively consistent.
In this context, one can conclude that the technical efficiency for hospital industry has improved in biennium 2002-2004, with a most significant contribution from the SPA hospitals.

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