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Children's understanding of length measurement using a ruler in preschool and primary education: A cross-national longitudinal study

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ABSTRACT

Measuring length is a practice of everyday life and a mathematical principle shared across cultures. A cross-national and longitudinal study was designed to analyse the evolution of the strategies used by Spanish and Portuguese preschoolers and 1st graders when measuring length. The instruments were rulers designed to discriminate between the strategies demonstrated by young children in previous research, taking as variables the number of correct measurements and the strategy used to measure, and country as a covariate. Neither country nor age influenced correct measurements while the ruler used affected the strategy developed by the children. No significant differences were found in the strategies used from preschool to 1st grade, except for the ruler with numbered units and marks. Considering the sequence of sophistication of the strategies, our results reinforce the scaffolding role of discrete units for the understanding of length measurement towards the use of the standard ruler.

1. Introduction

Knowing how to measure correctly is a fundamental aspect of knowing the world around us accurately. The development of length measurement is an important topic in school curricula from the earliest years because of its relevance both in everyday life and in geometry, which links number and space. At school, the measurement of magnitudes is present in both the mathematics and experimental science curricula; in fact, it is one of the fundamental building blocks recognised by the National Council of Teachers of Mathematics (NCTM, 2000). One of the first ideas about magnitude that children manifest from an early age is the notion of length, which they apply in situations of comparing objects. This is shown in MacDonald's (2012) study in which she asked children aged 4–6 years to draw themselves measuring, and found that most related measurement to length. Length is a magnitude recognised as fundamental in physics and mathematics, but it is also associated with a basic cognitive ability, as other magnitudes such as area or volume, which are based on it. Because of its fundamental nature, this article is dedicated to discovering how children measure the length of objects at an early age.

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1.1. Length measurement

The teaching of length measurement usually begins in early childhood education with direct comparison of lengths and measurement with discrete units. It is later, in the first years of primary school, that measuring with rulers is taught. Despite this, there are studies with children younger than the beginning of formal instruction in ruler measurement that obtain evidence of the correct use of this measuring instrument by some of them (Bragg and Outhred, 2000a, 2000b; Hiebert, 1981; McDonough and Sullivan, 2011). Therefore, it can be interpreted that some children have some intuitive notion of how to use this instrument.

However, different authors (Congdon et al., 2018; Gómezescobar et al., 2020; Ho & Lowrie, 2013; Irwin et al., 2004; Kamii & Clark, 1997; Levine et al., 2009; Sisman & Aksu, 2016; Solomon et al., 2015); that delved into students' understanding of ruler length measurement found that students have difficulties in interpreting the component elements of the ruler: numbers, marks (or ticks) and units. Regarding the numbers on the ruler, Gómezescobar (2020) concludes that when children measure lengths with a ruler, they tend to read the number that coincides with the end of the object, regardless of the starting point of the measurement. It seems, therefore, that children do not make an adequate interpretation of what the numbers on the ruler represent: the accumulation of units from the zero mark. As for the marks, children tend to count them instead of the units, thus assigning them the entity of unit of measurement and not what they really represent: unit delimiters. As far as units are concerned, only some children seem to conceptualise them. These children tend to perform an action on the interval by counting the units that make up the length of the object and making a correct measurement (Gómezescobar et al., 2018).

Regarding the use of instruments to measure real objects, studies are scarce. We found studies in which the authors propose paper-based tests that do not allow direct interaction with the measuring instrument and the object to be measured (Bragg & Outhred, 2000a, 2000b; Cullen & Barrett, 2010; Ho & Lowrie, 2013; Irwin et al., 2004; Solomon et al., 2015). In these works, the authors provide tools to tackle the tasks, for example, the broken ruler or the displaced object (from the origin of the measurement), which would require using concepts such as the starting point of the measurements. However, the objects to be measured are pictorial representations, which does not allow for physical interaction with the material. Aiming at deepening the knowledge in this regard, in line with other studies that provide manipulative material or rulers (Boulton-Lewis et al., 1996; Nunes et al., 1993), the present research offers different types of rulers so that children can face real measurement situations.

Another aspect to consider in order to understand children's interpretation of the measurement is the strategy they use. In this regard, to the best of our knowledge, the literature reviewed does not promote children's feedback through verbalisation of the strategy used. Several studies (Bragg & Outhred, 2004; Cogndon et al., 2018; Cullen & Barrett, 2010; Irwin et al., 2004; Kamii & Clark, 1997; Levine et al., 2009; Solomon et al., 2015) infer the measurement strategy employed by the participant through the numerical outcome of the measurements. For example, if the result is one unit more than the length of the object, the authors interpret that the learner has counted marks. As in other studies (Boulton-Lewis et al., 1996; Nunes et al., 1993), the present research explicitly asks about the measurement strategy used to encourage the child's reflection on his or her performance. However, it happens that in some cases children verbalise that they are counting marks, but when asked to demonstrate this, they read the sequence of numbers under the marks, which would mean that they are actually reading the number that coincides with the right end of the object. Therefore, in order to obtain reliable information in this research, the children's answers about the strategy used were cross-checked with the interviewer's observation since children's self-reported strategies may not be a reliable source of data about their actual strategy. This is one of the contributions of this research work, having used recorded interviews to obtain information that cannot be obtained when the child is simply asked to circle the correct answer.

1.1.1. Elements of the ruler, strategies and their evolution in the measurement of length

According to several studies (Cogndom et al., 2018; Cullen, 2009; Cullen & Barrett, 2010; Gómezescobar et al., 2018; Gómezescobar et al., 2020; Kamii, 1995; Levine et al., 2009; Sisman & Aksu, 2016; Solomon et al., 2015), the main strategies used by young children to measure lengths are related to the elements of the ruler indicated above: numbers, marks (or ticks) and units. The first element is related to reading the number on the end of the object (EP, end point), the second to counting marks (CM) and the third to acting on the interval (INT). These three strategies are considered to be hierarchical, in a sense, with respect to the level of conceptualisation of measurement, so that conceptualisation would be more complete when the unit is conceived. Therefore, among these strategies, the last of the three is considered to be the most conceptually elaborated. It seems that pupils' use of these strategies is not static over time, but evolves as their knowledge of length measurement deepens (Congdon et al., 2018; Gómezescobar et al., 2020; Levine et al., 2009; Solomon et al., 2015). It seems that children focus their attention on the numbers on the ruler, and that the strategy they use most often at the beginning is EP. However, this strategy does not always lead to a correct measurement, as it will only be valid if the starting point of the measurement is the zero mark, whereas if the object is displaced on the ruler, simply reading the number at the end without the relative interpretation of the position, will not lead to a correct measurement. As for those who focus their attention on the marks, the strategy they employ is their counting, CM. In this case, the measurement will be correct when a mark is biunivocally assigned to a unit. On the contrary, if the marks are simply counted starting with the first one, the measurement will be incorrect. Finally, children who visualise the unit tend to point out on the ruler with an action, using the INT strategy. Moreover, they establish a two-way correspondence between the action and the units, resulting in a correct measurement in all cases.

The elements of the ruler seem to determine the strategy used, but this does not remain invariant over time, but evolves with the age of the children. In this line, the study by Solomon et al. (2015), working with preschoolers and 2nd graders, indicates that younger children tend to read the number at the end of the object (EP), while older children use the strategy of counting marks (CM). We assume that the CM strategy is more evolved than the EP strategy, as it involves finding an element that is repeated to compose a length, and can be counted. Barrett et al. (2012) consider the use of this strategy as a necessary transitional phase until the identification of the unit. In fact, several studies (Congdon et al., 2018; Gómezescobar, 2020; Levine et al., 2009; Solomon et al., 2015) indicate that pupils who count marks on their measurements are more prepared to identify units or intervals within the ruler than pupils who simply read the number on the end of the object.

Since quantifying a length measurement, or measuring, involves counting the number of units that make up the length of the object being measured, the strategy of counting marks conceptually assumes some identification of the unit. It is therefore closer to the conceptualisation of the interval than the EP strategy. The study by Congdon ' et al. (2018) with 1st graders revealed that, after instruction with actions or gestures to identify units, several pupils switched from having previously used the EP strategy to using CM, but never the other way around. Moreover, those who used CM in the pretest benefited more from the instruction.

Finally, the INT strategy can be considered to have a higher conceptualisation of length measurement, as it fully identifies units and closely reflects the idea of measuring: dividing a magnitude into equal parts and counting these parts (Clements & Sarama, 2009). Also Akdeniz and Argün (2021) highlight the INT strategy as the most conceptually advanced and associated with the understanding of the concept of unit in the learning trajectory of a student with learning difficulties undergoing a teaching experiment. Therefore, the evolution in terms of conceptual sophistication of the length measurement strategies would be as follows: EP, CM and INT.

1.1.2. Measuring length in the transition from Preschool to primary education

In the context of this research, the transition from Preschool to Primary Education is associated with certain methodological changes that have been noted in different studies (González, Muñoz, & Zubizarreta, 2011; Castro, 2016; Tamayo, 2014). Teaching in Preschool is conceived as more globalised than in Primary Education. In the case of materials and tasks, in Preschool Education they tend to be more manipulative and focused on everyday life, while in Primary Education manipulation decreases considerably comparatively to the Preschool stage, using printed materials to a greater extent.

At the change of stage, children obviously get older. But the role of age in the correctness of the measurements has not been conclusively evidenced. Kotsopoulos, Makosz, Zambrzycka, and McCarthy (2015) find this association, while Nunes et al. (1993) and Gómezescobar et al., (2018, 2020) rule out that measuring correctly is associated with age for children up to 8 years old. Therefore, this question remains unanswered, and this study will try to shed more light on it.

1.2. Cultural issues in measurement

According to Bishop (1991) measurement is one of the six constituent mathematical principles of the cross-cultural axis, so it seems that there should be no differences between countries. However, other studies (Irwin et al., 2004; Delgado, 2013) do report some differences in length measurement in this respect. The work of Irwin et al. (2004) compares concepts of length measurement in Philippine and New Zealand children of similar ages. This study is mainly descriptive and does not use statistical inference to show statistically significant differences, but it does report differences between countries in the percentages of correct measurements in measurement tasks using the ruler. However, in none of the proposed tasks is the conventional commercial ruler used: in one task the ruler is altered (broken at the beginning) and in another the ruler is drawn on paper, so it does not allow any manipulation.

In the study by Delgado (2013) developed in the English-speaking world, the author concludes that those pupils who use the International System of Measurement in their daily lives obtain better results in conceptual tasks on scale and measurement than those who use the traditional British system of units (US customary system). Another consequence of using different measurement systems is to mix units from both systems (centimetres and inches) in measurements, as noted by Irwin et al. (2004) in their study with New Zealand and Philippine children. They report that New Zealand children mix these units, despite the fact that inches have not been included in the New Zealand curriculum for more than 30 years. They justify this by the influence of the everyday context at school, as inches are still used in everyday life in New Zealand.

Following on from the measurement experiences of other countries, in Australia the emphasis is on informal measurement, e.g. using sticks or paper clips, for at least two years before moving on to measurement with standard units. However, in the list of competencies to be achieved by Philippine pupils, informal measurement appears only once in the first grade and, in later grades, measurement is dealt with in science and only with standard units.

To our knowledge, studies with international samples on length measurements with children are scarce. Therefore, this article tries to fill this gap by working with two groups of children from different countries, Spain and Portugal, where the transition from Preschool to Primary Education officially takes place at the age of 6 years.

1.2.1. Measuring with the conventional ruler in the Spanish and Portuguese curricula

Economic investments in education in Spain and Portugal are very similar, which translates into similar ratios of pupils in the classroom. Moreover, teaching is in the mother tongue of the pupils, Spanish and Portuguese respectively. Both the ratio and the language of instruction are reported as influential factors in the study by Irwin et al. (2004). In the same study, it is emphasised that differences in curriculum may affect outcomes.

In both the Spanish and Portuguese curricula, length measurement with rulers is included as a learning objective around the 2nd grade of Primary Education (7–8 years old). Previously, measurement is worked on through direct and indirect comparisons and measurements with non-standard units. Moreover, the end of Preschool Education and the beginning of Primary Education is set at the age of 6, so that the transition from Preschool to Primary Education occurs at the same age. Hence the importance of the age range of the sample chosen in the design of this study: the last year of Preschool and 1st grade of Primary Education, when they have not yet received instruction in ruler measurement at school.

1.3. Objective of the research

The aim of this study is to explore through a longitudinal study the conceptual knowledge that boys and girls have about measuring length with a ruler, and to explore possible differences due to the country (Spain/Portugal) and the transition from Preschool to Primary Education (5–6 years). We will analyse whether there are differences in this conceptual knowledge between countries or differences due to age. More specifically, we will explore whether the strategy used is related to the type of ruler used to measure.

The research questions would be as follows:

1. Is there any difference between the correct measurements of Spanish and Portuguese pupils in Preschool and Primary Education?
2. What measurement strategies do they use?
3. Are the strategies related to the measuring instrument and to the age?

2. Materials and methods

This research is an exploratory longitudinal study covering the last year of Preschool and the first year of Primary Education.

2.1. Participants

The sample consisted of 80 pupils, 44 Spanish and 36 Portuguese, who participated in two consecutive academic years. The two countries share the age at which the transition from Preschool to Primary Education takes place, which determined our choice. The participation of the pupils was made possible thanks to the authorisation of their families. The children belonged to two Portuguese and two Spanish public schools, chosen for their proximity to the university, so it was a non-probabilistic sample of convenience.

The first data collection corresponded to the time when they were preschoolers and the second when they were pupils in the 1st[st] year of Primary Education. In the second data collection, 2 Spanish pupils and 7 Portuguese pupils changed schools, so the sample at this time was composed of 71 participants, 42 Spanish (22 girls) and 29 Portuguese (16 girls). The average age was 69.21 months.

During the time the study took place, no participants were instructed in the school environment on measuring length with rulers.

2.2. Instrument

The instrument consisted of a set of four rulers, used in the study by Gómezescobar, Guerrero and Fernández-Cézar (2018), which brought into play the most relevant elements of the conventional ruler: numbers, marks and units. These elements were combined to create measurement tools (Lehrer et al., 1998). Moreover, precisely these three elements are considered essential for the understanding of linear measurement by Barrett et al. (2012) in their review of the learning trajectory of length.

The first ruler, 1NUM (Fig. 1), is the most similar to the conventional ruler, as it has numbers below the marks. The second, 2MAR (Fig. 2), has marks delimiting the beginning and end of each unit. The third ruler, 3DUNUM (Fig. 3), has numbered units; and the fourth, 4DU (Fig. 4), has unnumbered units.

2.3. Procedure

Each pupil was interviewed and recorded individually at two points in time: the last year of Preschool and the first year of Primary Education. In order to be able to use the recordings from the first year and compare them with those from the second year of the same participant, they were pseudonymised by coding, guaranteeing non-personal identification, in compliance with the data protection law in force in Spain and Portugal.

Four measurements were taken, one with each ruler. The interviewer (the first author) placed a strip of 4 units of length at the 2 mark (the third mark, corresponding to two units) and asked: "how long is it?", and then "how did you know?". The first question revealed information about whether the measurement was correct. The second question was key, as it provided information about the student's strategy for measuring.

These strategies were classified as follows:

- OTH, non-mathematical arguments (e.g. "It (the strip) measures 5 because I am 5 years old"), does not answer or verbalises that he/she does not know.
- EP, reading the number at the end (left or right) or reading the sequence of numbers under the marks, although the child verbalises that he/she counts marks.

Fig. 1. 1NUM.

Fig. 2. 2MAR.

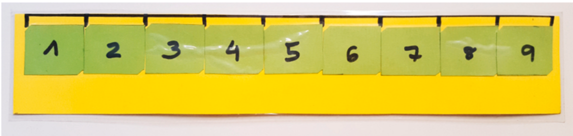

Fig. 3. 3DUNUM.

Fig. 4. 4DU.

- CM, counting marks
- INT, action on the interval

In some measurements participants did not answer the first question and some interviews were incomplete. In order not to lose the rest of the information about these participants for this study, the rest of the task responses were considered valid. Therefore, in the results' tables the absolute frequencies may not correspond to the relative frequencies, given the variation of the totals according to the task.

To avoid trial-and-error learning, the task was counterbalanced by setting up 4 different sequences (Table 1) that were randomly assigned to participants.

2.4. Analysis

The data obtained were analysed with Statistical Package for Social Sciences, SPSS, v. 24. The conceptual knowledge of the measurement was measured through the number of correct answers as a numerical variable, and the strategies used, which were counted as a percentage. The age was considered a covariate and was studied using tests for related samples, and the association between the number of correct answers, the strategy and the country was analysed using χ^2 tests.

3. Results and discussion

The aim of this study was to explore children's understanding of length measurement with ruler, and to explore possible differences due to country (Spain/Portugal) and age in the transition from Preschool to Primary Education (5–6 years). Results are presented on the correctness of the measurements, firstly, and on the strategies used, secondly.

3.1. Correction of measurements

In the interview, the question "How long is it?" allows us to know the correctness or error in the measurements made by each child with each ruler. The correct measurements made by participants in each country, as well as by participants in the two time points, the last year of Preschool and the first year of Primary Education, are shown in Table 2. Also included in this table is the value of the

Table 1
Sequence of rulers.

Sequence	1st measurement	2nd measurement	3rd measurement	4rd measurement
A	1NUM	2MAR	3DUNUM	4DU
B	2MAR	1NUM	4DU	3DUNUM
C	3DUNUM	4DU	1NUM	4DU
D	4DU	3DUNUM	2MAR	1NUM

Table 2
Absolute frequency and percentage of correct measurements for each ruler, country and stage, and statistical differences by country.

Ruler	Preschool Spain	Preschool Portugal	Primary Education Spain	Primary Education Portugal	Preschool (Spain vs Portugal)	Primary Education (Spain vs. Portugal)
1NUM	2 (5%)	4 (14.3%)	3 (7.1%)	2 (6.9%)	χ^2 (1, N = 68) = 1.77, p = . 18	χ^2 (1, N = 71) = 0.00, p = . 97
2MAR	3 (8.1%)	3 (11.1%)	4 (9.5%)	3 (10.3%)	χ^2 (1, N = 64) = 0.17, p = . 68	χ^2 (1, N = 71) = 0.01, p = . 91
3DUNUM	5 (13.2%)	4 (13.8%)	9 (21.4%)	4 (13.8%)	χ^2 (1, N = 67) = 0.01, p = . 94	χ^2 (1, N = 71) = 0.67, p = . 41
4DU	14 (36.8%)	14 (50%)	16 (38.1%)	14 (48.3%)	χ^2 (1, N = 66) = 1.14, p = . 29	χ^2 (1, N = 71) = 0.73, p = . 39

coefficient used to assess the possible influence of country (χ^2).

At none of the stages where participants measure with each of the four rulers are there significant differences with respect to country. It seems that country does not condition the correctness of the length measurements in this study. Pupil/classroom ratio and mother tongue instruction generated differences in measurements between pupils in the Philippines and New Zealand in the study by Irwin et al. (2004). However, both the ratio and the development of mother tongue instruction are largely the same in the countries considered, Spain and Portugal. In parallel, another differentiating aspect in the Philippines and New Zealand was the curriculum, but not in our case, since in both countries the curriculum places the teaching of ruler measurement in later grades, so that all the pupils in the sample have not yet received specific instruction in ruler length measurement at school.

Other authors, such as Delgado (2013), find differences between countries due to family-school differences, as a discrepancy is detected between the units used in everyday life (US family-customary system) and those used at school (decimal units). In our case, the non-difference obtained by country could be justified by the absence of this discrepancy, since both in Spain and Portugal the units in everyday life and at school are those of the metric system, and the same units are used both at home and at school.

Since our results are in line with Bishop's (1991) assertion that there are no differences in measurement across cultures/countries, the rest of the results are presented considering only one sample.

Table 3 shows the absolute values and the percentages of correct measurements for each ruler in the two years covered by the study, when the participants were in the last year of Preschool and in the following year, in the first year of Primary Education, as well as the statistical difference between the two years in which the data were taken.

In view of the results, it is worth noting that, except in the case of the ruler most similar to the conventional ruler, the 1NUM ruler, the participants make more correct measurements when they move on to Primary Education. However, the longitudinal nature of this study allows us to explore whether there are significant differences associated with the pupils ' change of stage, and no such differences were found in any of the four rulers used. In comparison with the literature, the work by Gómezescobar, Guerrero and Fernández-Cézar (2018) uses the same set of rulers with children of similar ages, pupils in the last two years of Preschool and the first year of Primary Education, although in a cross-sectional study. Despite in this case the research is a longitudinal study, our results coincide with those obtained in the aforementioned work, as they do not show differences in the correct measurements with any of the rulers.

In other longitudinal studies, such as McDonough and Sullivan (2011), the authors proposed actual measurements with the same participants for 3 years, starting in the last year of Preschool and receiving no instruction in the use of the ruler during all these years. This is, in terms of design, comparable to the present work, which would also allow us to compare results. However, although they detect an increase in correct answers with the age of the participants, they do not indicate whether this increase is statistically significant as they do not use statistical inference. It can be said that our results are in line with theirs in that we also obtain an increase, but the statistical contrast used in our case reveals that this increase is not statistically significant.

Among the studies of cross-sectional designs, the results regarding the evolution of measurement correctness with age are controversial. For example, Gómezescobar et al. (2020) and Nunes et al. (1993) do not find such a relationship, while Kotsopoulos et al. (2015) do. These latter studies, being cross-sectional, do not work with the same subjects at different ages, something that does occur in longitudinal studies as is the present research. Therefore, it seems to us that the line of research that remains open is to continue carrying out longitudinal studies to confirm or disprove this plausible relationship between the evolution of the correctness of the measurements with the age of the subjects.

Regarding the rulers considered individually, in both grades, correct measurements with the ruler that has discrete units (4DU) are more abundant than with the other rulers (χ^2 (3, N = 66) = 33.97, p < .00 for Preschool and χ^2 (3, N = 70) = 34.88, p < .00 for Primary Education). This result is consistent with the findings of the cross-sectional study by Gómezescobar et al. (2018), who also

Table 3
Absolute frequency and percentage of correct measurements for each ruler in Preschool and Primary Education and the result of the statistical contrast.

Ruler	Correct measurements (%) Preschool	Correct measurements (%) Primary education	Wilcoxon test for related samples
1NUM	6 (8.8%)	5 (7%)	Z = −0.38, p = .71
2MAR	6 (9.4%)	7 (9.9%)	Z = 0.00, p = 1
3DUNUM	9 (13.4%)	13 (18.3%)	Z = −1, p = .32
4DU	28 (42.4%)	30 (42.3%)	Z = −43, p = .67

obtained a higher percentage of correct answers with the discrete unit ruler. It seems that the inclusion of discrete units scaffolds the conceptualisation of measurement.

3.2. Measurement strategies

The question "How did you know?" allowed information to be collected about the strategy each child used to state his/her measurement result. Asking specifically about the measurement strategy is one of the strengths of this work, as this question forces the participants to verbalise a reasoned response, rather than the researchers inferring the strategy from the numerical result of the measurements. Table 4 shows the absolute frequency and percentage of each strategy for each ruler in the two years in which this study was conducted.

Below are some transcripts of interviews showing examples of each of the strategies:

Strategy EP (Fig. 5), participant 23, 1NUM ruler, the interviewer placed a strip 4 units long on the 2 mark and asked:

- *How long is it?*
- *[Pointing to each number and speaking softly, 2, 3, 4, 5 and 6] Six.*
- *Six? And how do you know that?*
- *Counting.*
- *And how did you count?*
- *[Pointing to each number] Two, three, four, five and six.*

CM strategy (Fig. 6), participant 80, 2MAR ruler, interviewer placed a strip 4 units long on the 2 mark and asked:

- *How long is it?*
- *[Counting and nodding forward at each number] Five.*
- *Five? And how do you know that?*
- *[Starting at mark 2 and pointing to each mark] One, two, three, four and five.*

Strategy INT (Fig. 7), participant 37, 3DUNUM ruler, the interviewer placed a strip 4 units long on the 2 mark and asked:

- *How long is it?*
- *[Counting and nodding forward] Four.*
- *Four? And how do you know that?*
- *[Pointing to the numbered units with his little finger] Counting the numbers.*

Some of the pupils who measured correctly used different strategies from those employed by the pupils who failed. Thus, strategies were associated with correct and incorrect measurement (see Table 5). In this sense, EP and CM are associated with incorrect results and INT with correct results. Therefore, in the rulers containing numbers, 1NUM and 3DUNUM, the most used strategy is reading the number at the end of the object (EP); in the ruler with marks, 2MAR, the most used strategy is mark counting (CM); and in the ruler with unnumbered discrete units, 4DU, the most used strategy is action on the interval (INT). These results occur regardless of whether the participants are preschoolers or 1st graders. It appears that when the rulers have numbers, pupils tend to use them using EP, and when numbers are not present on the measuring instrument, pupils count the highlighted items: marks in the 2MAR ruler, and discrete units in the 4DU ruler.

The longitudinal nature of the study allows us to explore whether there are differences in the strategies used by the pupils as a consequence of the change of stage. Such differences are found for the 3DUNUM ruler ($Z = -2.31$, $p = .02$), but not for the other three rulers in the set ($Z = -1.08$, $p = .28$ for 1NUM; $Z = -1.55$, $p = .12$ for 2MAR; $Z = -1.37$, $p = .17$ for 4DU). The 3DUNUM ruler is the one that specifically contains the three elements that are combined in the ruler set: numbers, marks and units. It is observed that, from one stage to the next, i.e. moving on to Primary Education, non-mathematical responses (OTH) and the use of the end point reading strategy (EP) decrease, giving way to two other more sophisticated and conceptually deeper strategies: mark counting and action on the interval (CM and INT).

These results resemble those obtained by Solomon et al. (2015) in experiment 2 of their study, in which they proposed to participants to measure an object displaced from zero (as in the present study) with a ruler with numbers and with an instrument conceptually similar to the 3DUNUM ruler, featuring numbered concatenated circles. Second graders had a statistically higher rate of correct answers than preschoolers s, as the latter tended to use the EP strategy in this task. As reported by Solomon et al. (2015), we also think that, in the shifted object task, participants often do not make use of the fact that the number is representing the accumulation of units up to that point, as they do not take into account the starting point for measurement. Therefore, the lack of understanding of the role that numbers play in the measuring instrument hinders the understanding of length measurement at these ages. Therefore, it is crucial to promote reflection with pupils on the facilitating role of numbers in the measuring instrument, to emphasise

Table 4
Absolute frequency and percentage of use of each strategy for each measurement in Preschool and Primary Education.

	OTH		EP		CM		INT	
	Preschool	Primary Education	Preschool	Primary Education	Preschool	Primary Education	Preschool	Primary Education
1NUM	16 (23.5%)	12 (16.9%)	39 (57.4%)	43 (60.6%)	13 (19.1%)	15 (21.1%)	0	1 (1.4%)
2MAR	22 (34.4%)	16 (22.5%)	0	0	38 (59.4%)	52 (73.2%)	4 (6.3%)	3 (4.2%)
3DUNUM	17 (25.4%)	13 (18.3%)	39 (58.2%)	37 (52.1%)	5 (7.5%)	8 (11.3%)	6 (9%)	13 (18.3%)
4DU	24 (36.4%)	19 (26.8%)	0	0	9 (13.6%)	13 (18.3%)	33 (50%)	39 (54.9%)

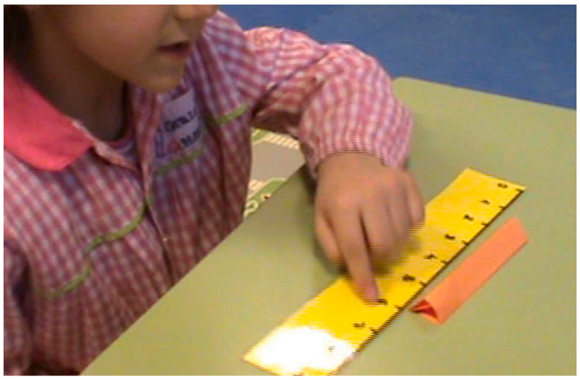

Fig. 5. EP strategy.

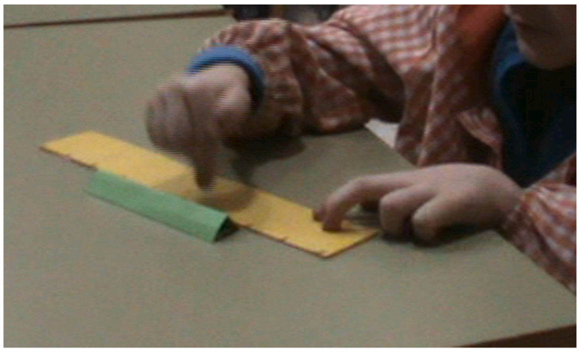

Fig. 6. CM strategy.

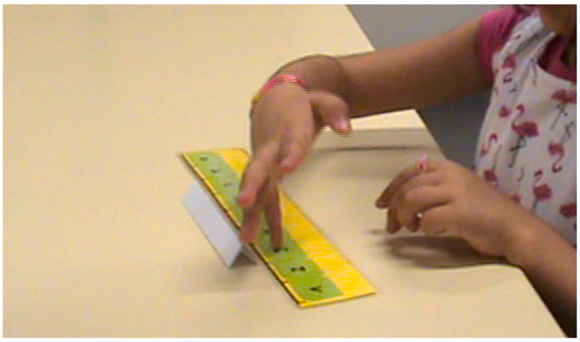

Fig. 7. INT Strategy.

Table 5
Statistic for the association between strategy and correctness in the measurement with each ruler at each stage.

	Preschool	Primary Education
1NUM	χ^2 (2, N = 68) = 9.02, p = .01	χ^2 (3, N = 71) = 19.06, p < .00
2MAR	χ^2 (2, N = 64) = 10,27, p < .00	χ^2 (2, N = 71) = . 12, p < .00
3DUNUM	χ^2 (3, N = 67) = 44,95, p < .00	χ^2 (3, N = 71) = 0.71, p < .00
4DU	χ^2 (2, N = 66) = 25.55, p < .00	χ^2 (2, N = 71) = 22.54, p < .00

that they prevent us from having to count units every time we want to measure an object.

With regard to the increase in the use of the CM strategy (5–8 with the 3DUNUM ruler and 9–13 with 4DU) and in the use of INT strategy (6–13 with the 3DUNUM ruler and 33–39 with 4DU), it is worth noting that in Primary Education the increase in the use of the

INT strategy, which is more than double that in Preschool, takes place when children use the ruler with numbered units. Therefore, age seems to influence the use of the INT strategy, although there is no parallel increase in the use of the CM strategy, nor in the correctness of the measurement. This result does not coincide with that provided by the study by Solomon et al. (2015), who report an increase in correct answers in pupils in the 2nd grade (we infer that through the INT strategy, although they do not explicitly indicate this). If we take into account the possible evolution of the strategies, understanding marks counting, CM, as an intermediate strategy between reading the endpoint (EP, more rudimentary) and the action on the interval (INT, more sophisticated), it seems reasonable to argue that these differences in the use of these two strategies that were observed in our sample, and which were not associated with an increase in the correctness of the measurements, are due to the fact that the Primary Education pupils in this study are still in the first grade, not in the 2nd grade as they were in the study by Solomon et al. (2015). Therefore, although they have changed the strategy towards CM, they would still need to evolve in a greater extent to the use of the action on the interval, INT, which is the strategy that would be associated with providing a correct measurement.

As mentioned above, the 3DUNUM ruler is the one that we found statistically significant differences in the strategies used by the pupils as a result of the change of stage. In the three remaining rulers used in the present study, 1NUM, 2MAR and 4DU, the pupils in the present study do not show statistically significant differences in measurement strategies when changing stages. Thus, as reported by Gómezescobar (2020), when subjects measure with the ruler with numbers under the marks (1NUM), as well as with the other ruler containing numbers (3DUNUM), they mostly use the endpoint reading, EP, in both stages. The 1NUM ruler is the most similar to the conventional ruler, and corresponds to the one used in experiments 1 and 2 of Solomon et al. (2015) study. However, they found that their Preschool pupils tended to use the EP strategy but their Primary Education pupils mostly used the CM strategy, which was not observed in our study, as we did not find an evolution in the use of the EP strategy. The use of the CM strategy to measure a displaced object using a ruler with numbers is also used by the participants in the pretest carried out by Levine et al. (2009) with pupils from 2nd grade. As with the 3DUNUM ruler, it seems that our Primary Education pupils are conceptually at a stage prior to the transition to this intermediate strategy, CM, as they still mostly use endpoint reading. This fact could be due to the fact that our participants are 6 years old and were not instructed in measuring with a ruler, unlike those in the other studies, who are 7 years old, and could have been instructed in the use of the ruler in different educational systems, although this is an assumption because this information is not explicitly provided by the authors.

The 2MAR ruler is crucial for understanding the pupils ' interpretation of the units it contains. The fact that this ruler does not contain numbers forces pupils to use other elements of the ruler in their measurements, in this case, the marks. The majority use of the strategy of counting marks, CM, throughout the two years of this study shows that these pupils interpret the marks as if they were units in themselves, just as occurs with the pupils who use the 2MAR ruler in the study by Gómezescobar (2020). That is, they do not perceive these marks as unit delimiters. The 2MAR ruler is similar to the one used in experiment 3 of Solomon et al. (2015), where the use of the CM strategy by Primary Education 2nd pupils is more frequent than by preschoolers, who also made use of EP. We attribute this to the fact that in this ruler there are no numbers that can be read, which makes it difficult to use EP. In our interviews, when the pupil refers to a number that is not in the ruler, he/she is asked why he/she indicates such a number as an answer, which often leads him/her to give us a result and an explanation based on elements present in the rulers (in this ruler, for example, based on the marks).

In any case, once again, we believe that these results reinforce the conceptual position of the CM strategy as an intermediate strategy between EP and INT. The ruler with marks (2MAR) can be interesting for pupils to count these marks and then to propose the counterexample referred to by Barrett et al. (2003) where pupils are asked to draw a figure of one unit of length. The pupils will be faced with the choice of not being able to draw anything at all if they really rely on the mark as an unit of measurement, or drawing a figure that actually measures one unit, but lies between two marks.

Finally, in the ruler with discrete units, 4DU, the action on the interval (INT) strategy predominates in the two consecutive stages, which coincides with the results obtained by Gómezescobar (2020) with the same ruler. The 4DU ruler could be considered conceptually similar to the concatenated coins and unnumbered circles used in experiments 1 and 3, respectively, by Solomon et al. (2015). The authors report rates of correct answers of over 95% for the 2nd grade group, and for the Preschool group the correct answers are close to 100% with the coins and around 80% for the unnumbered circles. In the latter case, there are again some cases of use of the EP strategy despite the absence of numbers.

It should be noted that the study by Solomon et al. (2015) uses objects (coins or circles) that are not embedded in a ruler. In contrast to these authors, Levine et al. (2009) propose the use of units on top of the ruler in their intervention with pupils at 2nd grade, which improves the results when measuring displaced objects. Levine et al. (2009) argue this by indicating that 7 of the 8 children who had counted marks started to get correct results, although they did not argue why. We think that this improvement could be due to a shift in strategy towards action on the interval, INT. This result, together with the predominant use of the INT strategy in the 4DU ruler, emphasises the importance of the inclusion of discrete units in the ruler to support unit visualisation. The scaffolding role of this inclusion leads to a full conceptual understanding of length measurement with a ruler.

4. Conclusions

The aim of this study was to explore through a longitudinal study the conceptual knowledge that children, boys and girls, have about measuring length with a ruler, and to explore possible differences due to the country (Spain/Portugal) and the transition from Preschool to Primary Education (5–6 years). For this purpose, we worked with a sample of 42 Spanish and 29 Portuguese pupils during two consecutive years, the last year of Preschool and the first year of Primary Education, to whom we applied the test described in the Procedure section and analysed the results. The following main conclusions can be drawn:

No statistically significant differences were found in correct measurements between pupils in the countries involved, in line with

Bishop's (1991) considerations that measurement is one of the six mathematical principles that bridge cultures. This absence can be attributed to the cultural similarity between Spain and Portugal, both in everyday life and at school. In everyday life, similar units of length, the Decimal Metric System, are used, and in school, the curricula also mark the beginning of length measurement with rulers in the same grade, 2nd grade, around the age of 7–8 years.

With respect to the transition from Preschool to Primary Education, no significant differences were found, so it can be concluded that the participants do not improve the correctness of their measurements when they move to the higher educational stage. This lack of improvement in measurement, despite the fact that the children were older at the higher stage, may be related to the fact that the children had not yet been instructed in the use of ruler measurement. It seems that the improvement detected in other studies may be more due to the instruction received at the upper age than to the cognitive maturity strictly associated with age. No significant differences are found in terms of the strategies used by pupils when moving from one stage to another, except for the 3DUNUM ruler. In this ruler, participants decrease the use of the EP strategy and non-mathematical responses (OTH) to use other more sophisticated and conceptually deeper strategies in Primary Education: counting marks and action on the interval (CM and INT). It would be useful to know whether in the following year, 2nd grade of Primary Education, this evolution towards more sophisticated strategies also occurs in the rest of the rulers. This finding could have some implications for Primary School teachers, who may assume that their pupils have more elaborate strategies for ruler use than preschoolers, when this study provides evidence that this is not the case. This evidence would imply for teachers the need to go deeper with preschoolers in the understanding of iteration of congruent units, and that in Primary Education there should be a teaching focused on the understanding of the use of the ruler as a conventional measuring instrument.

As for the possible association between the ruler and the strategy used, it seems that when the rulers have numbers, pupils tend to use them using EP, and when they do not, they count the highlighted elements, marks (CM) in the 2MAR ruler and units (INT) in the 4DU ruler. The latter ruler, 4DU, is the one with which pupils, regardless of their age, provided more correct measurements, which leads us to think that it is the one that helps the most to conceptualise length measurement properly, as it provides a scaffolding for the visualisation of the unit.

It is important that pupils work with the four types of rulers used in this study. Seeing their activity and their successes or difficulties will help the teacher to identify their ways of thinking and offer guidance using whichever tool seems best in sequence. In light of the results of our study, we recommend starting by using the discrete unit ruler without numerical labels, 4DU, when pupils begin to work with length measurement. The other rulers should follow in the following order: 3DUNUM, 2MAR and 1NUM, as proposed by Gómezescobar (2020). If the teacher detects that the children have difficulties with any of the rulers, he/she can guide them, for example, by going back to the previous ruler they had success with in the sequence, to reinforce their learning.

Data Availability

Data will be made available on request.

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Declarations of interest

None.

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