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One-loop pseudoscalar mass in a 2HDM with a Z_3 symmetry

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Abstract: A two-Higgs doublet model with a discrete Z_3 symmetry acquires, in its scalar and gauge sectors, an accidental continuous U(1) symmetry. One therefore finds, after spontaneous symmetry breaking of those symmetries, that a massless pseudoscalar arises, as expected by Goldstone's theorem. In the fermion sector, however, it is possible to obtain Yukawa matrix textures which distinguish between the Z_3 and U(1) symmetries, so one expects loop corrections to originate a non-zero pseudoscalar mass for the Z_3 case. We perform an explicit calculation that shows that at one-loop the pseudoscalar remains massless for all but one tested Z_3 Yukawa matrices. The pseudoscalar mass thus found is highly suppressed by the hierarchy of fermion masses.

Keywords: Multi-Higgs Models, Discrete Symmetries, Global Symmetries

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1 Introduction

The Higgs boson was discovered by the LHC collaborations in 2012 [1, 2] and precision measurements of its properties have shown it behaves very much as one would expect in the context of the Standard Model (SM) [3, 4]. This was the last particle predicted for the SM that remained to be discovered, and that theory is now "complete" — except that the SM leaves many unanswered questions, such as any explanation for the hierarchy of fermion masses; a satisfactory description of the matter-antimatter asymmetry in the Universe; and a valid candidate for Dark Matter, among other issues. Beyond the Standard Model (BSM) theories have been proposed since the SM itself was proposed, and extensions of the scalar sector, wherein more scalars than just the SM Higgs doublet are considered, have long been a popular and fertile field of study. Considering an extra scalar gauge singlet has been proposed to allow for an explanation of electroweak baryogenesis [5]; adding a SU(2) triplet to the SM [6] originates the see-saw mechanism, which provides a natural explanation for the smallness of the neutrino masses; and one of the most simple SM extensions is the two Higgs doublet model (2HDM), proposed by T. D. Lee in 1973 [7] so that CP violation could be the result of spontaneous symmetry breaking (for a review, see [8]). The model has a rich phenomenology, predicting the existence of 3 neutral spin-0 particles (not just one) and a charged one. Some versions of the 2HDM also include natural candidates for dark matter [9–12]. However, while the SM scalar potential is characterized by 2 independent parameters, the most general 2HDM has, seemingly, 14 parameters, though the freedom to redefine both doublets reduces that number to 11 [13]. As such, it is quite useful to impose discrete or continuous global symmetries on the 2HDM, thereby reducing the number of parameters of the model and increasing its predictive power. Not only that but those symmetries can provide natural solutions for phenomenological problems: for instance, a Z_2 symmetry was proposed by

Glashow, Weinberg and Paschos [14, 15] so that flavour changing neutral currents (FCNCs) mediated by neutral scalars become forbidden — and since such interactions are present in the most general 2HDM lagrangian, and are highly constrained by experimental data, the Z_2 symmetry provides a natural explanation for their absence, all the while reducing the number of free parameters in the scalar potential to 7. Likewise, a continuous U(1) symmetry was proposed by Peccei and Quinn as a possible explanation for the strong QCD problem [16], from which a 2HDM scalar potential with only 6 parameters emerges.

All in all, six different global symmetries may be imposed on the $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ scalar potential [17, 18]. Due to the freedom to redefine doublets by choosing different basis for those fields, two *a priori* different scalar potentials may well correspond to the same symmetry. For instance, the Z_2 symmetry, resulting from the field transformations $\Phi_1 \to \Phi_1$ and $\Phi_2 \to -\Phi_2$, yields a scalar potential with the same physics than a potential invariant under a permutation symmetry, $\Phi_1 \leftrightarrow \Phi_2$, even though both potentials have very different relations imposed on its parameters. Another issue is that of *accidental symmetries*: since we are considering a renormalizable theory the scalar potential will have at most quartic terms in the fields, different field transformations yield physically equivalent potentials. For instance, a scalar potential invariant under a Z_3 symmetry, where doublets transform as $\Phi_1 \to \Phi_1$ and $\Phi_2 \to e^{2i\pi/3}\Phi_2$, is physically indistinguishable from one invariant under a U(1) Peccei-Quinn symmetry, for which $\Phi_1 \to \Phi_1$ and $\Phi_2 \to e^{i\theta}\Phi_2$, with arbitrary values for θ. Indeed, while we consider only the scalar and gauge sectors, there is no distinction between these two possibilities. The same can be said for a 2HDM scalar sector invariant under a Z_N symmetry, with $N \geq 3$, and as mentioned this is caused by the fact that the scalar potential only has terms up to fourth power in the doublet fields. However, this picture changes when one considers the Yukawa sector. Though most of the six symmetries of the scalar potential can be extended to the full lagrangian yielding acceptable fermion phenomenology, there are different possibilities of achieving that. We can therefore have physically different theories which are identical in the scalar and gauge sectors but differ substantially in their Yukawa interactions. For instance, it is possible to have a U(1)-invariant 2HDM where its Yukawa sector has no FCNCs, and a Z_3 symmetric 2HDM, which shares the same scalar potential as the U(1) theory, but with FCNCs in its scalar-fermion interactions.

This then raises an interesting question, which we will study in this paper: since the U(1) 2HDM is invariant under a continuous symmetry, any vacuum which spontaneously breaks that symmetry (i.e. one for which both doublets acquire a vacuum expectation value (vev)) yields a massless scalar (other than the 3 Goldstone bosons one expects from electroweak symmetry breaking)- Indeed, minimising the scalar potential and computing the scalar masses one finds that the pseudoscalar of the model, A, is massless in this theory.[1] This is to be expected from Goldstone's theorem [22, 23]. But with a Z_3 2HDM we are left with a lagrangian which is invariant under a *discrete* symmetry but whose scalar potential is identical to the Peccei-Quinn one. One then finds at tree-level that upon spontaneous symmetry breaking the pseudoscalar A is also massless in this case. Is this a possible exception of

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[1] This is usually avoided by adding a soft-breaking quadratic term in the potential. In this work we will only deal with potentials without such soft breakings. It should be noted, however, that the Peccei-Quinn 2HDM symmetry is anomalous, and a non-zero but very small axion mass is expected to appear [19] from, in principle, 3-loop contributions. For an alternate way to generate axion masses in 2HDMs, see [20, 21].

Goldstone's theorem, which explicitly applies to models with continuous symmetries? We will show in the present work that is not the case and Goldstone's theorem remains valid. Since both models, the U(1) and Z_3 invariant 2HDMs, differ only in their Yukawa sectors, we should compute the contributions from fermions to scalar masses, and hopefully see that those mass corrections to the pseudoscalar mass are non-zero for the Z_3 model, remaining zero for the U(1) case. But fermions have no contribution to scalar masses at tree-level, which leads us to compute one-loop corrections to the scalar sector. We will discover that indeed a Z_3 symmetry yields a massive pseudoscalar at the one-loop level, while all U(1)-invariant Yukawa sectors still originate a massless A. However, we will also show that, out of all possible Z_3-invariant Yukawa matrices (the seemingly complete list of such textures was presented in [24]), only *one* gives a non-zero mass to A.

This paper is organised as follows: in section 2 we present the 2HDM and discuss in detail the U(1) and Z_3 symmetries imposed on it. We show that at tree-level both symmetries yield a massless pseudoscalar. We review how these symmetries are extended to the fermion sector, and how they generate different Yukawa textures. In section 3 we present the one-loop potential and discuss its minimisation and the formalism needed to compute one-loop contributions to the masses of the scalars of the model. We present the contributions to the pseudoscalar and neutral Goldstone masses from the scalar and gauge sectors in section 4 and show how both of those masses remain zero, even at one-loop. In section 5 we then compute the one-loop corrections from fermions, showing that a specific Z_3 Yukawa texture yields a non-zero pseudoscalar mass, all the while preserving the masslessness of the neutral Goldstone boson. We will also show that the other possible Z_3 Yukawa textures still yield a massless pseudsocalar, as do all U(1)-invariant Yukawas. We will discuss these results and present our conclusions in section 6.

2 The two Higgs doublet model: U(1) and Z_3 symmetries

The 2HDM extends the scalar sector of the SM by considering two hypercharge $Y = 1$ SU(2)$_L$ doublets. The most general renormalizable scalar potential that one can then write which is invariant under the SU(2)$_L$ × U(1)$_Y$ gauge symmetries is given by, at tree-level,

$$
\begin{aligned}
V_0 = {} & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - [m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}] + \tfrac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \tfrac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) \\
& + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \left\{ \tfrac{1}{2}\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + [\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)] \Phi_1^\dagger \Phi_2 + \text{h.c.} \right\}. \qquad (2.1)
\end{aligned}
$$

Other than the m_{12}^2 and $\lambda_{5,6,7}$ coefficients, all parameters are real. Since the two doublets are not physical fields (those will be the scalar mass eigenstates found after spontaneous symmetry breaking), any unitary transformation of Φ_1 and Φ_2 which leaves the theory's kinetic terms invariant leads to a physically equivalent model. These *basis transformations*, defined as

$$
\Phi_i' = U_{ij}\, \Phi_j\,, \qquad (2.2)
$$

for a generic 2 × 2 unitary matrix U, allow one to simplify the model in many circumstances. For instance, such basis transformations can show that the most general 2HDM scalar potential has only 11 independent real parameters (and not 14 as a naïve parameter counting of eq. (2.1) would lead to conclude [13]).

The most general 2HDM, when extended to the fermion sector, would induce tree-level flavour-changing neutral currents (FCNC) mediated by scalar particles. These interactions may have sizeable contributions to several meson physics observables (such as B and K meson oscillation parameters, for instance) and as such are constrained by experiments to be quite small. One way to naturally ensure the absence of these FCNC is to impose a discrete symmetry on the lagrangian — a simple Z_2 symmetry, as proposed by Glashow, Weinberg and Paschos [14, 15], wherein invariance of the lagrangian is required for the transformation $\Phi_1 \to \Phi_1$, $\Phi_2 \to -\Phi_2$, forces a single scalar doublet to couple to fermions of the same electric charge, thereby eliminating tree-level FCNC in scalar-fermion interactions. The impact of this symmetry on the scalar potential is to set to zero the parameters m_{12}^2, λ_6 and λ_7.[2]

Another global symmetry that one can impose on the 2HDM is a continuous U(1) symmetry, as proposed by Peccei and Quinn [16] to solve the strong QCD problem. To wit, one requires invariance of the lagrangian under the transformations

$$\Phi_1 \to \Phi_1' = \Phi_1\,, \quad \Phi_2 \to \Phi_2' = e^{i\,\theta}\Phi_2\,, \tag{2.3}$$

with θ an arbitrary real number. This induces another restriction on the 2HDM scalar parameters, rendering λ_5 equal to zero, so that the potential becomes

$$V_0 = m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 + \tfrac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \tfrac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)\,. \tag{2.4}$$

Let us now consider electroweak symmetry breaking, assuming both doublets acquire real and neutral vevs,[3] $\langle\Phi_1\rangle = (0\,,\, v_1/\sqrt{2})^T$ and $\langle\Phi_2\rangle = (0\,,\, v_2/\sqrt{2})^T$ such that $v_1^2 + v_2^2 = v^2$, with $v = 246\,\mathrm{GeV}$ for correct electroweak symmetry breaking. This vacuum clearly breaks the global U(1) symmetry imposed on the model.[4] We then parameterize the doublets' real components φ_i such that

$$\Phi_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}\varphi_1 + i\varphi_3\\ v_1 + \varphi_5 + i\varphi_7\end{pmatrix}, \qquad \Phi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}\varphi_2 + i\varphi_4\\ v_2 + \varphi_6 + i\varphi_8\end{pmatrix}, \tag{2.5}$$

so that the tree-level minimization conditions are found by setting all φ_i to zero and computing the derivatives of the potential in order of the vevs,

$$\begin{aligned}\frac{\partial V_0}{\partial v_1} = 0 \iff \left(m_{11}^2 + \frac{1}{2}\lambda_1 v_1^2 + \frac{1}{2}\lambda_{34}v_2^2\right)v_1 = 0\\ \frac{\partial V_0}{\partial v_2} = 0 \iff \left(m_{22}^2 + \frac{1}{2}\lambda_2 v_2^2 + \frac{1}{2}\lambda_{34}v_1^2\right)v_2 = 0\,,\end{aligned} \tag{2.6}$$

---

[2] In the basis where the Z_2 symmetry has the form mentioned. In other basis there might appear different relations between parameters but the physical consequences of said symmetry are the same.

[3] It is actually impossible that charge breaking vevs occur for this model. Likewise, complex vevs which would spontaneously break CP cannot occur in this model as well [25].

[4] For some regions of parameter space, the model also allows for a vacuum in which one of the doublets remains vevless and therefore preserves the U(1) symmetry. That yields a Inert 2HDM [9–12] and would have a non-zero pseudoscalar mass already at tree-level.

where for convenience we defined $\lambda_{34} = \lambda_3 + \lambda_4$. The tree-level scalar squared mass matrix is then given by

$$\left[M_S^2\right]_{ij} = \frac{\partial^2 V_0}{\partial\varphi_i \partial\varphi_j}\bigg|_{\text{min}} \tag{2.7}$$

where by "*min*" we indicate that these second derivatives are computed with all $\varphi_i = 0$. With the vevs considered and our ordering of the real components φ_i, the 8×8 mass matrix above breaks into four 2×2 blocks. Two of those blocks (corresponding to $\{\varphi_1, \varphi_2\}$ and $\{\varphi_3, \varphi_4\}$) are identical and give the charged mass matrix,

$$\left[M_{H^\pm}^2\right] = \begin{pmatrix} m_{11}^2 + \frac{1}{2}\lambda_1 v_1^2 + \frac{1}{2}\lambda_3 v_2^2 & \frac{1}{2}\lambda_4 v_1 v_2 \\ \frac{1}{2}\lambda_4 v_1 v_2 & m_{22}^2 + \frac{1}{2}\lambda_2 v_2^2 + \frac{1}{2}\lambda_3 v_1^2 \end{pmatrix} . \tag{2.8}$$

If evaluated at the tree-level solution of the minimization equations (2.6), this matrix has a zero eigenvalue, corresponding to the mass of the charged Goldstone $G^\pm$, and a non-zero one giving the tree-level charged scalar mass. The block corresponding to fields $\{\varphi_5, \varphi_6\}$ is the CP-even mass matrix,

$$\left[M_H^2\right] = \begin{pmatrix} m_{11}^2 + \frac{3}{2}\lambda_1 v_1^2 + \frac{1}{2}\lambda_{34} v_2^2 & \lambda_{34} v_1 v_2 \\ \lambda_{34} v_1 v_2 & m_{22}^2 + \frac{3}{2}\lambda_2 v_2^2 + \frac{1}{2}\lambda_{34} v_1^2 \end{pmatrix} . \tag{2.9}$$

When the eigenvalues of this matrix are evaluated with vevs satisfying the conditions of eqs. (2.6) we find two non-zero values, the squared masses of the lighter CP-even scalar, h, and the heavier one, H.

Finally, the block corresponding to the component fields $\{\varphi_7, \varphi_8\}$, which are the imaginary parts of the neutral components of both doublets, will give us the masses of the pseudoscalar A and the neutral Goldstone boson G^0. This block is found to be diagonal, and the tree-level pseudoscalar mass matrix is then given by

$$\left[M_A^2\right] = \begin{pmatrix} m_{11}^2 + \frac{1}{2}\lambda_1 v_1^2 + \frac{1}{2}\lambda_{34} v_2^2 & 0 \\ 0 & m_{22}^2 + \frac{1}{2}\lambda_2 v_2^2 + \frac{1}{2}\lambda_{34} v_1^2 \end{pmatrix} . \tag{2.10}$$

At the tree-level minimum which satisfies eqs. (2.6), we find that all four entries of this matrix vanish — which means that, in accordance with Goldstone's theorem, the spontaneous breaking of the continuous Peccei-Quinn U(1) symmetry produces a massless pseudoscalar, along with the massless neutral Goldstone boson one would expect since electroweak symmetry breaking is also occurring. As for the gauge boson masses, they arise as usual from the kinetic terms in the lagrangian,

$$\mathcal{L}_K = (D_\mu \Phi_1)^\dagger (D^\mu \Phi_1) + (D_\mu \Phi_2)^\dagger (D^\mu \Phi_2) \tag{2.11}$$

where the covariant derivatives are written as

$$D_\mu = \partial_\mu - \frac{i}{2} g' B_\mu - \frac{i}{2} g\, \sigma^a W_\mu^a , \tag{2.12}$$

with $a = \{1, 2, 3\}$ and σ^a are the Pauli matrices. The usual 2HDM gauge boson mass matrices are then found, and the gauge boson masses given by

$$m_W^2 = \frac{g^2}{4}\left(v_1^2 + v_2^2\right), \quad m_Z^2 = \frac{g^2 + g'^2}{4}\left(v_1^2 + v_2^2\right). \tag{2.13}$$

At this stage, let us consider the 2HDM with a different symmetry — a Z_3 symmetry, whereas instead of a generic angle θ as was considered in the U(1) symmetry in transformation of eq. (2.3), we restrict ourselves to a single complex phase of $2\pi/3$ and require invariance of the lagrangian under

$$\Phi_1 \to \Phi_1' = \Phi_1\,, \quad \Phi_2 \to \Phi_2' = e^{2i\pi/3}\Phi_2\,. \tag{2.14}$$

Now, it is easy to verify that requiring invariance under Z_3 or U(1) yields exactly the same scalar potential, eq. (2.4). This is because all terms of the form $\Phi_1^\dagger\Phi_2$ (or their hermitian conjugates) transform non-trivially under both symmetries[5] and as such all coefficients which multiply such a term, with the exception of λ_4, are set to zero by both symmetries. As a consequence, imposing the discrete Z_3 symmetry on the 2HDM scalar potential results in an *accidental* continuous U(1) symmetry, and as such it is not surprising that spontaneous symmetry breaking yields a massless pseudoscalar in both cases. The gauge sector also does not distinguish between both symmetries so a scalar + gauge 2HDM with a Z_3 symmetry is indeed, by accident, a Peccei-Quinn U(1) 2HDM. Indeed, the same occurs for any other Z_N symmetry, with $N \geq 3$.

However, this picture changes when we introduce fermions in the theory. We will only look at the quark sector, but leptons would be handled in a similar manner. The interactions between quarks and scalars are given by the Yukawa lagrangian, written as

$$-\mathcal{L}_\mathcal{Y} = \bar{Q}_L\Gamma_1\Phi_1 n_R + \bar{Q}_L\Gamma_2\Phi_2 n_R + \bar{Q}_L\Delta_1\tilde{\Phi}_1 p_R + \bar{Q}_L\Delta_2\tilde{\Phi}_2 p_R + \text{h.c.}\,, \tag{2.15}$$

where the Γ_i, Δ_i are 3×3 complex matrices of Yukawa couplings, and n_R (p_R) are 3-vectors in flavour space containing the 3 right handed negative (positive) quark fields who, upon rotation to the basis of quark mass eigenstates, will describe the down (up) quarks. Finally, Q_L stands for a 3-vector in flavour space containing the 3 left quark doublets, i.e. $(Q_L)_i = (p_i\,,\, n_i)_L^T$. The quark mass matrices are then given by

$$M_n = \frac{1}{\sqrt{2}}\left(\Gamma_1 v_1 + \Gamma_2 v_2\right), \quad M_p = \frac{1}{\sqrt{2}}\left(\Delta_1 v_1 + \Delta_2 v_2\right), \tag{2.16}$$

which are diagonalised by 3×3 unitary matrices U_{qL}, U_{qR} such that the physical down and up masses appear as

$$\begin{aligned} M_d &= \text{diag}(m_d\,,\, m_s\,,\, m_b) = U_{dL}^\dagger\, M_n\, U_{dR}\,, \\ M_u &= \text{diag}(m_u\,,\, m_c\,,\, m_t) = U_{uL}^\dagger\, M_p\, U_{uR}\,. \end{aligned} \tag{2.17}$$

Extending a U(1) or Z_3 symmetry to the Yukawa sector is achieved by requiring invariance of the lagrangian (2.15) under simultaneous transformations of the scalars under eqs. (2.3)

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[5] Albeit with different complex phases, $e^{i\theta}$ for U(1) and $e^{2i\pi/3}$ for Z_3.

or (2.14), respectively, and of the fermion fields under

$$\begin{aligned} Q_L &\to Q'_L = \mathrm{diag}(e^{i\alpha_1}\,,\, e^{i\alpha_2}\,,\, e^{i\alpha_3})\, Q_L\,, \\ n_R &\to n'_R = \mathrm{diag}(e^{i\beta_1}\,,\, e^{i\beta_2}\,,\, e^{i\beta_3})\, n_R\,, \\ p_R &\to p'_R = \mathrm{diag}(e^{i\gamma_1}\,,\, e^{i\gamma_2}\,,\, e^{i\gamma_3})\, p_R\,, \end{aligned} \tag{2.18}$$

with generic real phases α_i, β_i and γ_i. Under these transformations the Yukawa matrices transform as

$$\left(\Gamma_a\right)_{ij} = e^{i(\alpha_i - \beta_j - \theta_a)}\left(\Gamma_a\right)_{ij}\,, \tag{2.19}$$

$$\left(\Delta_a\right)_{ij} = e^{i(\alpha_i - \gamma_j + \theta_a)}\left(\Delta_a\right)_{ij}\,, \tag{2.20}$$

where $\theta_1 = 0$ and $\theta_2 = \theta$. A given entry of a Γ or Δ matrix will be non-zero if and only if the combinations of phases in the exponentials above are equal to 0 (mod(2π)). The complete[6] list of possible Yukawa matrices that are (a) invariant under these transformations, (b) generate 3 massive up and down quarks and (c) can produce a realistic Cabbibo-Kobayashi-Maskawa (CKM) quark mixing matrix were obtained in ref. [24], where it was shown that there are many different, and physically non-equivalent, ways of extending symmetries of the scalar sector to the Yukawa one. For instance, there are many Yukawa matrix textures which are themselves U(1) invariant. One example out of many is to choose $\alpha_i = 0$ and $\beta_i = -\gamma_i = -\theta$ for all $i = 1, 2, 3$, which forces all fermions to couple only to Φ_2, the Yukawa matrices resulting thereof given by

$$\begin{aligned} \Gamma_1 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \Gamma_2 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}, \\ \Delta_1 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \Delta_2 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}, \end{aligned} \tag{2.21}$$

where "×" denotes a generic complex number. This is a well-known realisation of a "Type I" 2HDM, and the Yukawa matrices above have no scalar-mediated FCNCs. Like many other U(1)-invariant textures, they are obtained for completely generic values of the phase θ.

But there are Yukawa textures which can only be obtained if $\theta = 2\pi/3$ and are thus specifically Z_3-symmetric.[7] All such cases are listed in section III.C.5 of ref. [24].[8] One such example are the textures

$$\begin{aligned} \Gamma_1 &= \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & 0 \\ 0 & 0 & \times \end{pmatrix}, \qquad \Gamma_2 = \begin{pmatrix} 0 & \times & 0 \\ 0 & 0 & \times \\ \times & 0 & 0 \end{pmatrix}, \\ \Delta_1 &= \begin{pmatrix} 0 & \times & 0 \\ 0 & 0 & \times \\ \times & 0 & 0 \end{pmatrix}, \qquad \Delta_2 = \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & 0 \\ 0 & 0 & \times \end{pmatrix}. \end{aligned} \tag{2.22}$$

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[6] It should be emphasised that in ref. [24] only textures resultant from abelian symmetries were obtained. Other textures are therefore possible. It also does not seem that the Z_3 textures considered in [26] are included in the list provided in [24].

[7] There are also Yukawa matrix textures which can only be obtained if $\theta = \pi$ and are therefore Z_2-symmetric. They are listed in section III.C.4 of ref. [24] but we will not consider them in this paper.

[8] We detected a typo in eq. (89) of ref. [24]: the (1,3) entry of matrix Γ_1 should be zero.

Looking at the entries of the Γ matrices we obtain the following conditions on the α and β phases (we already chose $\alpha_1 = 0$, which can always be made without loss of generality),[9]

$$\begin{aligned} \Gamma_1 :\ & -\beta_1 = 0\,; \quad \alpha_2 - \beta_2 = 0\,; \quad \alpha_3 - \beta_3 = 0 \\ \Gamma_2 :\ & -\beta_2 - \theta = 0\,; \quad \alpha_2 - \beta_3 - \theta = 0\,; \quad \alpha_3 - \beta_1 - \theta = 2n\pi\,, \end{aligned} \tag{2.23}$$

with n a generic integer number. It is then simple to verify that for these equations to have a solution one must have $\alpha_2 = \beta_2 = -\theta$ and $\alpha_3 = \beta_3 = -2\theta$. The final equation then implies $-3\theta = 2n\pi$, for which $\theta = 2\pi/3$ is a solution (as is $\theta = -2\pi/3$, but both solutions correspond to exactly the same physics).

The models corresponding to the U(1) textures of (2.21) and the Z_3 ones of (2.22) are not the same: the former has neutral scalars which preserve flavour in their interactions with quarks, whereas the latter has FCNCs in those same interactions. There is therefore a physical distinction between the U(1) and Z_3 lagrangians, even though their scalar sectors are seemingly identical. But then we are left with a question: if the Z_3 lagrangian has a discrete symmetry, then Goldstone's theorem does not apply, and there should be a massive pseudoscalar after spontaneous symmetry breaking. That, however, is not what we have already seen in eq. (2.10), where the tree-level minimisation conditions yield a massless pseudoscalar. Before considering that Godlstone's theorem might be incorrect, however, we should consider that both lagrangians, the U(1) and Z_3-symmetric ones, differ only in their Yukawa sector, and as such we may anticipate that contributions to the pseudoscalar mass from the fermionic sector will distinguish between both models, and give rise to a non-zero mass for the Z_3 case. Such contributions only appear at the one-loop level, which motivates us to undertake the computation of radiative corrections to the pseudoscalar mass in both models.

3 One-loop potential and one-loop scalar masses

In this paper we will closely follow the formalism developed by Stephen P. Martin in a series of papers [27–29]. In this section we will briefly review the formalism required for our purposes and refer the reader to those references for further details. The one-loop effective potential V_1 is given by

$$V_1 = V_0 + \Delta V_1\,, \tag{3.1}$$

where we have already written the tree-level potential V_0 in eq. (2.4) and the one-loop contribution is

$$\Delta V_1 = \frac{1}{64\pi^2} \sum_\alpha n_\alpha\, m_\alpha^4 \left[\log\left(\frac{m_\alpha^2}{\mu^2}\right) - \frac{3}{2}\right]\,, \tag{3.2}$$

where we are working in the Landau gauge and considering a Dimensional Reduction regularization scheme.[10] In this expression μ is an arbitrary renormalization scale (typically

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[9] In these equations obviously all terms equal to "0" should be interpreted as "0 (mod(2π))", but for our purposes it is enough to only consider a factor of "$2n\pi$" in the last equation.

[10] Had we used Dimensional Regularization, the factors of 3/2 in ΔV_1 would be replaced by 5/6 for the gauge boson contributions.

considered of the order of the largest masses of the theory, to numerically minimize the size of the logarithms present in this expression) and the α index runs through all particles in the model. The m_α are the tree-level masses for each particle, and the factors n_α are, for a particle of spin s_α, given by

$$n_\alpha = (-1)^{2s_\alpha}\,(2s_\alpha + 1)\,C_\alpha\,Q_\alpha\,, \tag{3.3}$$

with $C_\alpha = 3$ for particles with colour and 1 for all others; and $Q_\alpha = 2$ for particles with electric charge and 1 for neutral ones. The one-loop minimization equations are therefore

$$\frac{\partial V_1}{\partial v_i} = \frac{\partial V_0}{\partial v_i} + \frac{\partial \Delta V_1}{\partial v_i} = 0 \tag{3.4}$$

and a simple calculation gives

$$\begin{aligned}
\left(m^2_{11} + \frac{1}{2}\lambda_1 v_1^2 + \frac{1}{2}\lambda_{34} v_2^2\right) v_1 + \frac{1}{32\pi^2}\sum_\alpha n_\alpha\, m^2_\alpha\, \frac{\partial m^2_\alpha}{\partial v_1}\left[\log\left(\frac{m^2_\alpha}{\mu^2}\right) - 1\right] &= 0\,,\\
\left(m^2_{22} + \frac{1}{2}\lambda_2 v_2^2 + \frac{1}{2}\lambda_{34} v_1^2\right) v_2 + \frac{1}{32\pi^2}\sum_\alpha n_\alpha\, m^2_\alpha\, \frac{\partial m^2_\alpha}{\partial v_2}\left[\log\left(\frac{m^2_\alpha}{\mu^2}\right) - 1\right] &= 0\,.
\end{aligned} \tag{3.5}$$

Numerically, these equations may be solved for the values of m^2_{11} and m^2_{22} once all other couplings and vevs are chosen.[11] The tree-level masses depend on the vevs, so their derivatives can be obtained easily from eqs. (2.8), (2.9), (2.10) and (2.13) for the scalars and gauge bosons. In section 5 we will show how to handle the fermionic masses. In all of these expressions, the tree-level expressions for each particle mass are evaluated with vevs which satisfy the one-loop minimization equations. As mentioned earlier, the tree-level masses of eqs. (2.8), (2.9) and (2.10) evaluated at the tree-level minimum given by eqs. (2.6) yield three massless Goldstone bosons and a massless pseudoscalar — the former due to spontaneous electroweak symmetry breaking, the latter from the spontaneous breaking of the U(1) global symmetry. In the evaluation of the one-loop effective potential, or its derivatives, tree-level masses are evaluated at the one-loop minimum and therefore no massless eigenvalues are expected. But when the one-loop scalar mass matrix is evaluated at the one-loop minimum we expect to again find three massless eigenvalues corresponding to the Goldstone bosons, and again a massless pseudoscalar from spontaneous breaking of U(1).

In [28, 29] a method of computing the one-loop scalar self-energies was presented. The procedure uses a mass-independent renormalization scheme, in which observables — such as masses or cross sections — are outputs, but the inputs of the model are renormalized running couplings and mass parameters. This method is ideal for situations where tree-level predictions for the mass of a given particle are expected to be substantially altered through radiative corrections, albeit in a perturbative manner. For instance, in SUSY models the Higgs mass is predicted to be less than the mass of the Z boson at tree-level — but one-loop contributions to that mass can substantially enhance it, and make the SUSY value compatible with the current experimental value [30, 31]. Likewise, this method is appropriate for the question we are considering in this paper — to investigate whether the tree-level prediction,

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[11] Notice, however, that those two parameters enter into the definitions of the squared scalar masses present in the one-loop terms above.

$m_A = 0$, is preserved by loop corrections both for the case of a U(1)-invariant lagrangian and a Z_3-invariant one.

The physical masses are independent of both gauge and external momentum, and are defined as the poles of the propagator of the particle in question. Following [28, 29], for a theory with n scalars with tree-level masses $m(0)_i$, the one-loop mass $m(1)_k$ is the value of s which solves

$$\text{Det}\left[\left(m(0)_i^2 - s\right)\delta_{ij} + \frac{1}{16\pi^2}\,\Pi_{ij}\left(m(0)_k^2\right)\right] = 0\,, \tag{3.6}$$

where $s = m(1)_k^2 = -p^2$, p being the external momentum, and Π_{ij} are the scalar self-energies. Since the self-energies Π_{ij} have a complicated dependence on the variable s, this equation will typically be solved in an iterative way: start with $s = 0$ and compute the eigenvalues of the one-loop mass matrices, take those for the new values of s and repeat the process until s converges to a single value for each mass. This formalism is valid for a general theory containing an arbitrary number of scalars (S) R_i, spin-1/2 Weyl fermions (F) ψ_I and gauge bosons (G) A_a^μ,[12] interacting according to the partial Lagrangians

$$\mathcal{L}_\text{S} = -\frac{1}{6}\lambda^{ijk}R_iR_jR_k - \frac{1}{24}\lambda^{ijkl}R_iR_jR_kR_l\,, \tag{3.7}$$

$$\mathcal{L}_\text{SF} = -\frac{1}{2}y^{IJk}\psi_I\psi_JR_k + \text{h.c.}\,, \tag{3.8}$$

$$\mathcal{L}_\text{SG} = -g^{aij}A_a^\mu R_i\partial_\mu R_j - \frac{1}{4}g^{abij}A_a^\mu A_{b\mu}R_iR_j - \frac{1}{2}g^{abi}A_a^\mu A_{b\mu}R_i\,. \tag{3.9}$$

The conventions are clear: $\mathcal{L}_\text{S}$ describes the self interactions between the scalars R_i, and the symmetric tensors λ^{ijk} and λ^{ijkl} will be deduced from the scalar potential of eq. (2.4) for the U(1) and Z_3 models. $\mathcal{L}_\text{SF}$ describes the interactions between scalars and fermions and is therefore a different way of expressing the Yukawa lagrangian of eq. (2.15), and the y^{IJk} couplings will be related to the Γ and Δ matrices of (2.15). The y^{IJk} tensors are symmetric on the indices I, J, and raising/lowering their indices is tantamount to complex conjugation, *i.e* $y^{IJk} = y^*_{IJk}$.

Interactions between gauge bosons and scalars are contained in $\mathcal{L}_\text{SG}$, and the g^{abi} and g^{abij} couplings will be deduced from the scalar kinetic terms of eq. (2.11). With these definitions the one-loop scalar self-energies are found to be [28, 29]

$$\begin{aligned}\Pi_{ij} = {} & \frac{1}{2}\lambda^{ijkk}A(m_k^2) - \frac{1}{2}\lambda^{ikl}\lambda^{jkl}B(m_k^2, m_l^2) \\ & + g^{aik}g^{ajk}B_{SG}(m_k^2, m_a^2) + \frac{1}{2}g^{aaij}A_G(m_a^2) + \frac{1}{2}g^{abi}g^{abj}B_{GG}(m_a^2, m_b^2) \\ & + \text{Re}\left[y^{KLi}y_{KLj}\right]B_{FF}(m_K^2, m_L^2) + \text{Re}\left[y^{KLi}y^{K'L'j}M_{KK'}M_{LL'}\right]B_{\bar{F}\bar{F}}(m_K^2, m_L^2)\,,\end{aligned} \tag{3.10}$$

where the A, B are functions of s and masses, and their explicit expressions are shown in appendix A. The first line of the equation above includes all scalar-only contributions to the self-energies, Π_{ij}^S; the second line all contributions involving gauge bosons, Π_{ij}^G; and

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[12] These fields correspond to the eigenstates of the tree-level mass matrices for scalars, fermions and gauge bosons, and each will be described by, respectively, lowercase indices i, j, k, l, uppercase indices I, J, K, L and lowercase indices a, b.

all fermionic contributions are gathered in the third line, Π^F_{ij}, which also involves M_{IJ}, off-diagonal elements figuring in fermionic propagators in the Weyl formalism (see [27, 28] for further details). Since in this work we are interested in the loop corrections to the pseudoscalar mass, we use the tree-level mass matrix from eq. (2.10) and obtain, for the one-loop mass matrix evaluated at the desired minimum, the following expression

$$\begin{aligned}\left[M^2_A\right] &= \begin{pmatrix} m^2_{11} + \frac{1}{2}\lambda_1 v_1^2 + \frac{1}{2}\lambda_{34} v_2^2 + \frac{1}{16\pi^2}\,\Pi_{G^0G^0} & \frac{1}{16\pi^2}\,\Pi_{G^0A} \\ \frac{1}{16\pi^2}\,\Pi_{AG^0} & m^2_{22} + \frac{1}{2}\lambda_2 v_2^2 + \frac{1}{2}\lambda_{34} v_1^2 + \frac{1}{16\pi^2}\,\Pi_{AA} \end{pmatrix} \\ &= \frac{1}{16\pi^2}\begin{pmatrix} X_{G^0G^0} & \Pi_{G^0A} \\ \Pi_{AG^0} & X_{AA} \end{pmatrix} \end{aligned} \tag{3.11}$$

with

$$\begin{aligned} X_{G^0G^0} &= \Pi_{G^0G^0} - \sum_\alpha n_\alpha \frac{m^2_\alpha}{2v_1}\frac{\partial m^2_\alpha}{\partial v_1}\left[\log\left(\frac{m^2_\alpha}{\mu^2}\right) - 1\right], \\ X_{AA} &= \Pi_{AA} - \sum_\alpha n_\alpha \frac{m^2_\alpha}{2v_2}\frac{\partial m^2_\alpha}{\partial v_2}\left[\log\left(\frac{m^2_\alpha}{\mu^2}\right) - 1\right], \end{aligned} \tag{3.12}$$

where we have already used the one-loop minimization conditions from eq. (3.5) and made the choice $G^0 \equiv \varphi_7$ and $A \equiv \varphi_8$ (see eq. (2.5); this choice is simplified by the fact that the tree-level pseudoscalar mass matrix (2.10) is diagonal).

We now present detailed calculations of each of the contributions to the loop corrections — scalar, gauge and fermionic — with the expectation that:

- Since the U(1) and Z_3 models are identical in all regards in the scalar and gauge sectors, the respective contributions to $[M^2_A]$ must keep the pseudoscalar massless.
- Using U(1)-symmetric Yukawa matrices should yield fermionic contributions to the self-energies such that both the neutral Goldstone and the pseudoscalar remain massless. From Goldstone's theorem a massless pseudoscalar is to be expected since a continuous symmetry of the lagrangian has been spontaneously broken.
- If Z_3-symmetric Yukawa matrices are considered then the fermionic contributions to the self-energies should be such that while the neutral Goldstone remains massless the pseudoscalar mass receives a finite, non-zero contribution. This will be a verification of Goldstone's theorem since the lagrangian symmetry being spontaneously broken is discrete.

One important detail to consider: since we are interested in loop corrections to the pseudoscalar mass matrix, which includes a massless Goldstone boson, the calculation will in principle yield two different values of s.

4 One-loop masses: scalar and gauge sectors

We will first deal with the contributions to the one-loop minimization conditions of eq. (3.5). These are obvious for the gauge bosons: their masses are given in eqs. (2.13), so their derivatives are trivial to obtain and yield

$$\frac{\partial \Delta V_1^G}{\partial v_i} = \frac{1}{32\pi^2}\,\frac{v_i}{v^2}\,\left[6\,m_Z^2\,A(m_Z^2) + 12\,m_W^2\,A(m_W^2)\right] \tag{4.1}$$

with all spin and charge degrees of freedom already taken into account and the $A(x)$ function is defined as

$$A(x) = x\,\left[\log\left(\frac{x}{\mu^2}\right) - 1\right]\,. \tag{4.2}$$

The scalar contributions are equally simple for the neutral Goldstone and pseudoscalar — using eqs. (2.10) we obtain

$$\begin{aligned} \frac{\partial m_{G^0}^2}{\partial v_1} &= \lambda_1\, v_1\,, & \frac{\partial m_{G^0}^2}{\partial v_2} &= \lambda_{34}\, v_2 \\ \frac{\partial m_A^2}{\partial v_1} &= \lambda_{34}\, v_1\,, & \frac{\partial m_A^2}{\partial v_2} &= \lambda_2\, v_2\,. \end{aligned} \tag{4.3}$$

As for the contributions from the charged and CP-evens scalars, we see from eqs. (2.8) and (2.9) that their squared masses are the eigenvalues of real symmetric 2×2 matrices of the form

$$\left[M_S^2\right] = \begin{pmatrix} a & b \\ b & c \end{pmatrix} \tag{4.4}$$

with vev-dependent coefficients a, b and c. The corresponding eigenvalues are therefore the solutions of the characteristic equation

$$F(x, v_i) = x^2 - (a + c)\,x + ac - b^2 = 0 \tag{4.5}$$

which gives us

$$m_{1,2}^2 = \frac{1}{2}\,\left(a + c \pm \sqrt{(a-c)^2 + 4b^2}\right)\,. \tag{4.6}$$

The lowest of the eigenvalues (associated with the "$-$" sign in this equation) correspond to the lightest CP-even scalar h and to the tree-level charged Goldstone $G^\pm$ squared mass. Analytical expressions for the derivatives of these eigenvalues may be obtained directly from the expression above or, applying the implicit function theorem to eq. (4.5), from

$$\frac{\partial m_{1,2}^2}{\partial v_i} = \frac{(a_i + c_i) m_{1,2}^2 + 2 b b_i - c a_i - a c_i}{m_{1,2}^2 - m_{2,1}^2} \tag{4.7}$$

where a_i, b_i and c_i represent the derivatives of the respective coefficients with respect to v_i, and they are very simple expressions of the λ couplings and vevs. This expression for the derivatives of the squared masses is quite useful if one wants to perform analytical checks but it is also helpful when performing numerical calculations. Putting everything together we have

$$\begin{aligned}(32\pi^2)\frac{\partial\Delta V_1^S}{\partial v_1} &= \Bigg[2\,\frac{(\lambda_1+\lambda_3)v_1 m_{H^\pm}^2+\lambda_4 v_2 b_\pm-(\lambda_1 c_\pm+\lambda_3 a_\pm)v_1}{m_{H^\pm}^2-m_{G^\pm}^2}\,A(m_{H^\pm}^2)\\ &\quad +2\,\frac{(\lambda_1+\lambda_3)v_1 m_{G^\pm}^2+\lambda_4 v_2 b_\pm-(\lambda_1 c_\pm+\lambda_3 a_\pm)v_1}{m_{G^\pm}^2-m_{H^\pm}^2}\,A(m_{G^\pm}^2)\\ &\quad +\frac{(3\lambda_1+\lambda_{34})v_1 m_h^2+2\lambda_{34}v_2 b_H-(3\lambda_1 c_H+\lambda_{34}a_H)v_1}{m_h^2-m_H^2}\,A(m_h^2)\\ &\quad +\frac{(3\lambda_1+\lambda_{34})v_1 m_H^2+2\lambda_{34}v_2 b_H-(3\lambda_1 c_H+\lambda_{34}a_H)v_1}{m_H^2-m_h^2}\,A(m_H^2)\\ &\quad +\ \lambda_1 v_1\,A(m_{G^0}^2)+\lambda_{34}v_1\,A(m_A^2)\Bigg]\,,\end{aligned} \tag{4.8}$$

$$\begin{aligned}(32\pi^2)\frac{\partial\Delta V_1^S}{\partial v_2} &= \Bigg[2\,\frac{(\lambda_2+\lambda_3)v_2 m_{H^\pm}^2+\lambda_4 v_1 b_\pm-(\lambda_3 c_\pm+\lambda_2 a_\pm)v_2}{m_{H^\pm}^2-m_{G^\pm}^2}\,A(m_{H^\pm}^2)\\ &\quad +2\,\frac{(\lambda_2+\lambda_3)v_2 m_{G^\pm}^2+\lambda_4 v_1 b_\pm-(\lambda_3 c_\pm+\lambda_2 a_\pm)v_2}{m_{G^\pm}^2-m_{H^\pm}^2}\,A(m_{G^\pm}^2)\\ &\quad +\frac{(3\lambda_2+\lambda_{34})v_2 m_h^2+2\lambda_{34}v_1 b_H-(\lambda_{34} c_H+3\lambda_2 a_H)v_2}{m_h^2-m_H^2}\,A(m_h^2)\\ &\quad +\frac{(3\lambda_2+\lambda_{34})v_2 m_H^2+2\lambda_{34}v_1 b_H-(\lambda_{34} c_H+3\lambda_2 a_H)v_2}{m_H^2-m_h^2}\,A(m_H^2)\\ &\quad +\ \lambda_{34} v_2\,A(m_{G^0}^2)+\lambda_2 v_2\,A(m_A^2)\Bigg]\,,\end{aligned} \tag{4.9}$$

where $\{a_\pm, b_\pm, c_\pm\}$ ($\{a_H, b_H, c_H\}$) are the entries of matrix (2.8) ((2.9)) following the convention of eq. (4.4).

The self-energies require that we obtain the tensors λ^{ijk}, λ^{ijkl}, g^{aij}, g^{abi} and g^{abij}. To do so we must express the tree-level potential (2.4) and kinetic terms (2.11) in terms of mass eigenstates, both of scalars and gauge bosons. As we already discussed, the neutral Goldstone and pseudoscalar are identified with the scalar components φ_7 and φ_8. The charged component fields $\varphi_{1\ldots4}$ give rise to two charged scalars and Goldstone bosons through the diagonalization angle β of matrix (2.8),[13]

$$\begin{aligned} H^+ &= c_\beta\varphi_1 - s_\beta\varphi_2\,, & G^+ &= s_\beta\varphi_1 + c_\beta\varphi_2\\ H^- &= c_\beta\varphi_3 - s_\beta\varphi_4\,, & G^- &= s_\beta\varphi_3 + c_\beta\varphi_4\end{aligned} \tag{4.10}$$

where we use the notation $c_x = \cos x$ and $s_x = \sin x$. Since we are considering a one-loop minimization of the potential the angle β is *not* the one usually considered in 2HDM calculations, meaning that here $\tan\beta \neq v_2/v_1$. The two CP-even tree-level eigenstates are likewise defined through the diagonalization angle α of (2.9),

$$h = c_\alpha\varphi_5 - s_\alpha\varphi_6 \quad , \quad H = s_\alpha\varphi_5 + c_\alpha\varphi_6\,. \tag{4.11}$$

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[13] Readers will notice that though we use the notation $H^\pm$, $G^\pm$, these are real fields, not the complex ones usually used to represent charged scalars. The formalism of [27–29], however, requires that we indeed use real fields for all eigenstates.

With these definitions and the conventions of eq. (3.7)–(3.9), the tensors required are simply the derivatives of the lagrangian with respect to the tree-level mass eigenstates, with those fields set to zero. So for instance,

$$\lambda^{AAhh} = \frac{\partial^4 V_0}{\partial^2 A \partial^2 h} = s_\alpha^2\, \lambda_2 + c_\alpha^2\, \lambda_{34} \tag{4.12}$$

and

$$\lambda^{AG^+H^-} = \frac{\partial^3 V_0}{\partial A \partial G^+ \partial H^-} = -\frac{\lambda_4\, v_1}{2}\,. \tag{4.13}$$

All non-zero coefficients λ relevant for the current calculation are listed in appendix A. We now show the final expressions for the scalars' contributions to the self-energies, leaving intermediate calculations to the reader. For the off-diagonal term, we have

$$\Pi^S_{AG^0} = \Pi^S_{G^0A} = \frac{\lambda_4}{4}\left\{4 s_\beta c_\beta \left[A(m^2_{G^\pm}) - A(m^2_{H^\pm})\right] + \lambda_4 v_1 v_2 \left[B(m^2_{H^\pm}, m^2_{G^\pm}) + B(m^2_{G^\pm}, m^2_{H^\pm})\right]\right\}, \tag{4.14}$$

where the $B(x, y)$ function is defined in eq. (A.3). We can now use the definition of the diagonalization angle β, eq. (4.10), of the charged mass matrix, eq. (2.8), to obtain the following relation:

$$\lambda_4 = \frac{2 s_\beta c_\beta}{v_1 v_2}\left(m^2_{G^\pm} - m^2_{H^\pm}\right)\,. \tag{4.15}$$

For $s = 0$,[14] the B function is simply $B(x, y) = (A(x) - A(y))/(y - x)$ and therefore it follows that

$$\Pi^S_{AG^0} = \lambda_4\, s_\beta c_\beta \left[A(m^2_{G^\pm}) - A(m^2_{H^\pm}) + \left(m^2_{G^\pm} - m^2_{H^\pm}\right) \frac{A(m^2_{H^\pm}) - A(m^2_{G^\pm})}{m^2_{G^\pm} - m^2_{H^\pm}}\right] = 0\,. \tag{4.16}$$

For the diagonal terms we obtain

$$\begin{aligned}\Pi^S_{G^0G^0} = \frac{1}{2}\Big[&\lambda_{34} A(m^2_A) + 3\lambda_1 A(m^2_{G^0}) + \left(\lambda_{34} c^2_\alpha + \lambda_1 s^2_\alpha\right) A(m^2_H) + \left(\lambda_1 c^2_\alpha + \lambda_{34} s^2_\alpha\right) A(m^2_h)\Big]\\ &+ \left(\lambda_3 c^2_\beta + \lambda_1 s^2_\beta\right) A(m^2_{G^\pm}) + \left(\lambda_1 c^2_\beta + \lambda_3 s^2_\beta\right) A(m^2_{H^\pm}) - \frac{v_2^2 \lambda_4^2}{2} B(m^2_{G^\pm}, m^2_{H^\pm})\\ &- (\lambda_1 v_1 s_\alpha + \lambda_{34} v_2 c_\alpha)^2\, B(m^2_{G^0}, m^2_H) - (\lambda_1 v_1 c_\alpha - \lambda_{34} v_2 s_\alpha)^2\, B(m^2_{G^0}, m^2_h)\,, \end{aligned} \tag{4.17}$$

$$\begin{aligned}\Pi^S_{AA} = \frac{1}{2}\Big[&3\lambda_2 A(m^2_A) + \lambda_{34} A(m^2_{G^0}) + \left(\lambda_2 c^2_\alpha + \lambda_{34} s^2_\alpha\right) A(m^2_H) + \left(\lambda_{34} c^2_\alpha + \lambda_2 s^2_\alpha\right) A(m^2_h)\Big]\\ &+ \left(\lambda_2 c^2_\beta + \lambda_3 s^2_\beta\right) A(m^2_{G^\pm}) + \left(\lambda_3 c^2_\beta + \lambda_2 s^2_\beta\right) A(m^2_{H^\pm}) - \frac{v_1^2 \lambda_4^2}{2} B(m^2_{G^\pm}, m_{H^\pm})\\ &- (\lambda_{34} v_1 s_\alpha + \lambda_2 v_2 c_\alpha)^2\, B(m^2_A, m^2_H) - (\lambda_{34} v_1 c_\alpha - \lambda_2 v_2 s_\alpha)^2\, B(m^2_A, m^2_h)\,. \end{aligned} \tag{4.18}$$

These expressions must now be plugged into eq. (3.11) and (3.12) along with the explicit one-loop potential derivatives of eq. (4.8) and (4.9). With $s = 0$, and expressing all B

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[14] We should definitely perform this computation with $s = 0$ to obtain the mass of the neutral Goldstone boson at one-loop. As we will see, however, when only the scalar and gauge contributions to the self-energies are considered, the choice $s = 0$ yields two zero eigenvalues, which proves that the pseudoscalar is also found to be massless.

functions in terms of A functions, a careful calculation will see a series of cancelations occur — these arise from the fact that the diagonalization angles β and α are not independent of the masses, vevs and couplings of the model, as we saw in eq. (4.15). The following relations were found to be useful for this calculation:

$$\begin{aligned}\lambda_1 &= \frac{1}{v_1^2}\left(c_\alpha^2 m_h^2 + s_\alpha^2 m_H^2 - m_{G^0}^2\right),\\ \lambda_2 &= \frac{1}{v_2^2}\left(s_\alpha^2 m_h^2 + c_\alpha^2 m_H^2 - m_A^2\right),\\ \lambda_{34} &= \frac{m_H^2 - m_h^2}{v_1 v_2}\, s_\alpha c_\alpha\,. \end{aligned} \tag{4.19}$$

The end result is that all entries of the 2×2 matrix of (3.11) become equal to *zero*. The readers are invited to verify this analytic result in full. To illustrate the calculation we show explicitly the vanishing of terms involving the contributions from A and G^0 in $\Pi_{G^0G^0}$ — keeping only those terms in eqs. (3.12), (4.8) and (4.17), we obtain

$$\begin{aligned}X_{G^0G^0} = &\frac{1}{2}\left[\lambda_{34}A(m_A^2)+3\lambda_1 A(m_{G^0}^2)\right]-\left(\lambda_1 v_1 s_\alpha+\lambda_{34}v_2 c_\alpha\right)^2 B(m_{G^0}^2,m_H^2)\\ &-\left(\lambda_1 v_1 c_\alpha-\lambda_{34}v_2 s_\alpha\right)^2 B(m_{G^0}^2,m_h^2)-\frac{1}{2v_1}\left\{\lambda_1 v_1\, A(m_{G^0}^2)+\lambda_{34}v_1\, A(m_A^2)\right\},\end{aligned} \tag{4.20}$$

where the term in curly brackets comes from the derivatives of the one-loop potential. We see that the terms in $A(m_A^2)$ cancel out and, using the relations (4.19), the factors multiplying the B functions above are given by

$$\begin{aligned}\lambda_1 v_1 s_\alpha + \lambda_{34} v_2 c_\alpha &= \frac{m_H^2 - m_{G^0}^2}{v_1}\, s_\alpha\,,\\ \lambda_1 v_1 c_\alpha - \lambda_{34} v_2 s_\alpha &= \frac{m_h^2 - m_{G^0}^2}{v_1}\, c_\alpha\,. \end{aligned} \tag{4.21}$$

Thus, using the explicit $s = 0$ form of the B functions and keeping only the $A(m_{G^0}^2)$ terms, this becomes

$$\begin{aligned}X_{G^0G^0} &= \lambda_1 A(m_{G^0}^2) - \frac{s_\alpha^2}{v_1^2}\left(m_H^2 - m_{G^0}^2\right)^2 \frac{A(m_{G^0}^2)}{m_H^2 - m_{G^0}^2} - \frac{c_\alpha^2}{v_1^2}\left(m_h^2 - m_{G^0}^2\right)^2 \frac{A(m_{G^0}^2)}{m_h^2 - m_{G^0}^2}\\ &= \lambda_1 A(m_{G^0}^2) - \frac{1}{v_1^2}\left(c_\alpha^2 m_h^2 + s_\alpha^2 m_H^2 - m_{G^0}^2\right) A(m_{G^0}^2) = 0\,, \end{aligned} \tag{4.22}$$

where in the final step we once again used the first of eqs. (4.19). Similar calculations are performed for the rest of the terms and show that all one-loop scalar contributions to the pseudoscalar mass matrix vanish at the minimum.

The same in fact holds for the contributions stemming from the gauge sector. The g tensors detailing the scalar and gauge interactions are, as before, obtained through derivatives of the kinetic terms with respect to scalar and gauge mass eigenstate fields, setting those fields to zero. For instance,

$$g^{AAW^+W^-} = -\frac{\partial^4 \mathcal{L}_K}{\partial A^2 \partial W^+ \partial W^-} = \frac{g^2}{2} = \frac{2m_W^2}{v^2}\,. \tag{4.23}$$

We also obtain $g^{abA} = g^{abG^0} = 0$, for all gauge bosons a, b, and another example is

$$g^{ZG^0h} = -\frac{\partial^3 \mathcal{L}_K}{\partial Z_\mu \partial G^0 \partial(\partial^\mu h)} = -\frac{m_Z}{v}\, c_\alpha\,. \tag{4.24}$$

All non-zero coefficients g relevant for the current calculation are listed in appendix A. With these coefficients we can proceed to compute the contributions of the gauge sector to the self-energies in eq. (3.10). For the off-diagonal elements of the pseudoscalar mass matrix we obtain

$$\Pi^G_{AG^0} = \frac{m_Z^2}{v^2} s_\alpha c_\alpha \left[B_{SG}(m_H^2, m_Z^2) - B_{SG}(m_h^2, m_Z^2)\right]\,, \tag{4.25}$$

where the B_{SG} function is defined in eq. (A.4). It is then easy to see that in the Landau gauge and for the limit of zero external momentum, $s = 0$, one obtains $B_{SG}(x, y) = 0$, and therefore $\Pi^G_{AG^0} = 0$. Once again we find that the one-loop contribution to the off-diagonal element of the pseudoscalar mass matrix is zero. As for the diagonal elements, we obtain

$$\Pi^G_{AA} = \frac{m_Z^2}{v^2}\left[s_\alpha^2 B_{SG}(m_h^2, m_Z^2) + c_\alpha^2 B_{SG}(m_H^2, m_Z^2) + A_G(m_Z^2) + 2c_W^2 A_G(m_W^2)\right], \tag{4.26}$$

$$\Pi^G_{G^0G^0} = \frac{m_Z^2}{v^2}\left[c_\alpha^2 B_{SG}(m_h^2, m_Z^2) + s_\alpha^2 B_{SG}(m_H^2, m_Z^2) + A_G(m_Z^2) + 2c_W^2 A_G(m_W^2)\right], \tag{4.27}$$

where $c_W = m_W/m_Z$. As mentioned before, the B_{SG} function is zero for $s = 0$ and in the Landau gauge, and the function $A_G(x)$ is equal (in the same circumstances and for a DRED renormalization scheme) to $3A(x)$ (see eq. (A.2)).[15] Thus we obtain

$$\Pi^G_{AA} = \Pi^G_{G^0G^0} = \frac{3m_Z^2}{v^2}\left[A(m_Z^2) + 2c_W^2 A(m_W^2)\right]\,. \tag{4.28}$$

Once this expression is inserted into eq. (3.12) along with the contributions to the minimization conditions involving gauge bosons, from eq. (4.1), all terms in $A(x)$ cancel out.

Thus we conclude that, when considering the one-loop contributions from scalars and gauge bosons to the pseudoscalar mass matrix, said matrix remains identically equal to zero, as it was at tree-level. Therefore we obtain a massless neutral Goldstone boson, as required by electroweak gauge symmetry breaking, and a massless pseudoscalar — as is to be expected since the lagrangian of the model has a continuous U(1) symmetry, the spontaneous breaking of which must needs produce a massless particle. And since in the scalar and gauge sectors the imposition of a U(1) or Z_3 symmetries is indistinguishable, these results confirm the validity of Goldstone's theorem — the imposition of a discrete Z_3 symmetry in a scalar + gauge 2HDM leads to a theory with an accidental continuous symmetry, therefore a massless pseudoscalar follows after spontaneous symmetry breaking. This now changes when we consider, finally, the contributions from fermions to the self-energies.

5 One-loop masses: fermion sector

The fermion sector calculations are made more difficult by the fact that we wish to study generic textures of Yukawa matrices. Models with flavour conservation, such as 2HDMs of

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[15] This factor of "3" counts the spin degrees of freedom of the gauge bosons, see eq. (3.3).

Type I (or II, or X, or Y), allow, as we shall see, for a full analytic calculation of one-loop pseudoscalar mass corrections. But in generic models where FCNCs are present that will not be possible. In fact, since there are three generations of quarks, the eigenvalues of the quark mass matrices cannot in general be obtained analytically, unlike the scalar and gauge cases. Our investigation of the fermion sector will therefore perforce be (mostly) numerical — we will choose random values for both vevs v_1 and v_2, obviously respecting $v_1^2 + v_2^2 = v^2$, and random values for the entries of the Yukawa matrices so that six massive quarks are obtained; we will then compute the value of the one-loop contributions to the self-energies and minimization conditions and determine numerically whether the pseudoscalar acquires a mass or not.

There is however a semi-analytical way to proceed in dealing with the minimization conditions. Considering the definitions of the quark mass matrices, eq. (2.16), and their diagonalization matrices, eq. (2.17), we see that the matrices

$$H_d = M_n\, M_n^\dagger\,, \quad H_u = M_p\, M_p^\dagger \tag{5.1}$$

are diagonalized by the unitary left matrices of eq. (2.17), and their eigenvalues are the squared down and up quark masses, respectively — since these matrices are not bi-diagonalized as the matrices M_n, M_p are, their eigenvalues are guaranteed to be real and positive, and may be obtained numerically in a trivial manner. Notice that for each chosen texture for the matrices Γ_1 and Γ_2 (Δ_1 and Δ_2) we can write simple analytical expressions for each entry of H_d (H_u). For the up sector, the squared masses are the solutions m^2 of a cubic characteristic equation of the form

$$\mathrm{Det}\Big(H_u - m^2\, \mathbb{1}_{3\times 3}\Big) = 0\,. \tag{5.2}$$

Defining the 3×3 matrix $F_u = H_u - m^2\, \mathbb{1}_{3\times 3}$, this equation may now be written as

$$\mathcal{F}_u(m^2, v_a) = \sum_{i,j,k=1}^{3} \epsilon_{ijk}\, F_{1i}\, F_{2j}\, F_{3k} = 0 \tag{5.3}$$

and therefore, using the implicit function theorem, we may write

$$\frac{\partial m^2}{\partial v_a} = -\frac{\dfrac{\partial \mathcal{F}_u}{\partial v_a}}{\dfrac{\partial \mathcal{F}_u}{\partial m^2}}\,. \tag{5.4}$$

The derivatives of $\mathcal{F}_u$ are given by

$$\frac{\partial \mathcal{F}_u}{\partial x} = \sum_{i,j,k=1}^{3} \left[\epsilon_{ijk}\, \frac{\partial F_{1i}}{\partial x}\, F_{2j}\, F_{3k} + \epsilon_{ijk}\, F_{1i}\, \frac{\partial F_{2j}}{\partial x}\, F_{3k} + \epsilon_{ijk}\, F_{1i}\, F_{2j}\, \frac{\partial F_{3k}}{\partial x}\right]\,, \tag{5.5}$$

that is, they are given by the sum of the determinants of the three matrices obtained from F_u by replacing one of their lines by its derivative with respect to the variable x. For a given choice of textures for Δ_1 and Δ_2, all of these determinants are analytical expressions of the entries of those matrices, the vevs and the mass of each quark. Therefore, once the eigenvalues of H_u have been obtained numerically, we may use them in eqs. (5.4) and (5.5) and obtain the numerical values of the derivatives of the squared up quark masses with respect to the vevs. The procedure is analogous for the down sector. The quark contributions to the one-loop minimization conditions will therefore be

$$\frac{\partial \Delta V_1^F}{\partial v_i} = -\frac{12}{32\pi^2} \sum_{\text{quarks}} \frac{\partial m_q^2}{\partial v_i} A(m_q^2) \,, \tag{5.6}$$

where the factor "−12" stems from eq. (3.3).

The self-energies of eq. (3.10) use Yukawa couplings from the scalar-fermion interaction lagrangian, eq. (3.8), written in terms of Weyl spinors in the basis of quark mass eigenstates. Since the 2HDM Yukawa lagrangian we wrote, eq. (2.15), or the textures from eqs. (2.21) and (2.22), are written in terms of Dirac fermion notation, a "dictionary" between both notations is needed. First we need to rotate the Γ and Δ matrices to the mass basis — this is accomplished through the rotation matrices in eq. (2.17), so that

$$\bar{\Gamma}_a = U_{dL}^\dagger \, \Gamma_a \, U_{dR} \,, \quad \bar{\Delta}_a = U_{uL}^\dagger \, \Delta_a \, U_{uR} \,, \tag{5.7}$$

where the bars indicate Yukawa matrices in the quark mass basis. In this paper we are interested in loop corrections to the pseudoscalar sector, so we can restrict ourselves to the interactions of G^0 and A in the Yukawa lagrangian of eq. (2.15). Since these two scalar eigenstates coincide (at tree-level) with the imaginary neutral components of, respectively, Φ_1 and Φ_2 we obtain, for the three up-type quarks u_i and three down-type quarks d_j, the following lagrangian:

$$\begin{aligned} -\mathcal{L}_{\mathcal{Y}} = & \left[-\frac{i}{\sqrt{2}} \left(\bar{\Gamma}_1^\dagger\right)_{ij} \bar{d}_i \, P_L \, d_j + \frac{i}{\sqrt{2}} \left(\bar{\Delta}_1^\dagger\right)_{ij} \bar{u}_i \, P_L \, u_j + \text{h.c.} \right] G^0 \\ & + \left[-\frac{i}{\sqrt{2}} \left(\bar{\Gamma}_2^\dagger\right)_{ij} \bar{d}_i \, P_L \, d_j + \frac{i}{\sqrt{2}} \left(\bar{\Delta}_2^\dagger\right)_{ij} \bar{u}_i \, P_L \, u_j + \text{h.c.} \right] A + \ldots \end{aligned} \tag{5.8}$$

where we have introduced the usual left and right projectors P_L and P_R. Let us now follow the notation of [27] and recall that any Dirac spinor Ψ may be represented by two Weyl spinors, ξ and χ — in this language "ξ" will embody the left-handed component of the Dirac spinor Ψ, and "$\chi^\dagger$" its right-handed one, so that we will have, for two different fermions,

$$\overline{\Psi}_i \, P_L \, \Psi_j = \chi_i \, \xi_j \,, \quad \overline{\Psi}_i \, P_R \, \Psi_j = \xi_i^\dagger \, \chi_j^\dagger \,. \tag{5.9}$$

To make the connection with the lagrangian from eq. (3.8), then, we introduce 12 Weyl spinors, numbered such that odd/even indices correspond to χ/ξ fields, to wit

$$\begin{aligned} &\psi_1 \equiv \chi_u \,, \quad \psi_3 \equiv \chi_c \,, \quad \psi_5 \equiv \chi_t \,, \quad \psi_7 \equiv \chi_d \,, \quad \psi_9 \equiv \chi_s \,, \quad \psi_{11} \equiv \chi_b \,, \\ &\psi_2 \equiv \xi_u \,, \quad \psi_4 \equiv \xi_c \,, \quad \psi_6 \equiv \xi_t \,, \quad \psi_8 \equiv \xi_d \,, \quad \psi_{10} \equiv \xi_s \,, \quad \psi_{12} \equiv \xi_b \,, \end{aligned} \tag{5.10}$$

so that, if for instance one selects the top quark interaction terms with A from eq. (5.8), we get

$$\begin{aligned} -\mathcal{L}_{\mathcal{Y}} &= \frac{i}{\sqrt{2}} \left[\left(\bar{\Delta}_2^\dagger\right)_{33} \bar{t} \, P_L \, t + \left(\bar{\Delta}_2^\dagger\right)_{32} \bar{t} \, P_L \, c + \left(\bar{\Delta}_2^\dagger\right)_{31} \bar{t} \, P_L \, u + \text{h.c.} \right] A + \ldots \\ &= \left(y^{56A} \, \psi_5 \, \psi_6 + y^{54A} \, \psi_5 \, \psi_4 + y^{52A} \, \psi_5 \, \psi_2 + \text{h.c.} \right) A + \ldots \,. \end{aligned} \tag{5.11}$$

One can then identify each entry of the y^{IJk} tensors,[16] for example, one has

$$y^{56A} = y^{65A} = \frac{i}{\sqrt{2}} \left(\bar{\Delta}_2^\dagger\right)_{33} ,$$
$$y^{54A} = y^{45A} = \frac{i}{\sqrt{2}} \left(\bar{\Delta}_2^\dagger\right)_{32} . \tag{5.12}$$

Notice that most of the entries of the y^{IJk} tensors are zero — all diagonal terms, $y^{IIk} = 0$, due to the way Weyl mass terms are written; and since we are not considering charge breaking vacua the up and down quark mass matrices will not mix and therefore $y^{IJk} = 0$ for $I \in \{1, \ldots, 6\}$ and simultaneously $J \in \{7, \ldots, 12\}$. Further, the coefficients M^{IJ} used in eq. (3.10) are obtained from the mass terms in the lagrangian through

$$M^{IJ} = -\frac{\partial^2 \mathcal{L}}{\partial \psi_I \partial \psi_J} , \tag{5.13}$$

this derivative being computed with all fields set equal to zero. We therefore have a semi-analytical algorithm to compute one-loop fermionic corrections to scalar mass matrices:

- Obtain, from symmetry arguments, the Yukawa textures for the Γ and Δ matrices.
- Choose vevs v_1 and v_2 and random values for the Yukawa couplings. Numerically obtain the eigenvalues of the quark mass matrices and their diagonalization matrices.
- Compute the quark contributions to the minimization equations using the procedure outlined before eq. (5.6).
- Rotate the Yukawa matrices to the quark mass eigenbasis, eq. (5.7).
- Identify the non-zero entries of the tensors y^{IJk} as per the method described in eq. (5.11).
- With the y^{IJk} and quark masses determined, compute the fermionic contributions to the self-energies, Π^F_{ij}, from eq. (3.10).

We will now apply this procedure to textures which are obtained from U(1) and Z_3 symmetries. To do this, we will use the work of reference [24], where the effect of all 2HDM abelian symmetries on the Yukawa matrices was obtained.

5.1 U(1)-symmetric Yukawa textures

With the U(1) symmetry extended to the Yukawa sector, not just the scalar and gauge sectors, we have a lagrangian invariant under a continuous symmetry. Goldstone's theorem therefore implies that the pseudoscalar must remain massless at loop level. Let us see that this is indeed the case in an explicit simple example, that of a 2HDM with Type-I Yukawa couplings inforced by a U(1) symmetry, i.e. the textures shown in eq. (2.21). Unlike the general case treated above, for Type-I Yukawas a single doublet (in this case Φ_2) couples to fermions. Therefore the symmetry enforces $\Gamma_1 = \Delta_1 = 0$ and the up (down) Yukawa

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[16] The y^{IJk} tensors are symmetric on the indices I, J, due to the fact that the product of two Weyl spinors is also symmetric, i.e. $\xi\chi = \chi\xi$. See [27] for more details.

matrices are directly proportional to the up (down) quark mass matrices — therefore the quark mass eigenbasis is also the basis where the Yukawa matrices are diagonal, and no FCNC occurs. All quark masses are therefore of the form

$$m_q = \frac{\lambda_q}{\sqrt{2}}\, v_2\,, \tag{5.14}$$

where λ_q is a diagonal element of Γ_2/Δ_2 in the quark mass basis. Substantial simplifications then occur:

- Since G^0 is the neutral imaginary component of Φ_1 and this doublet does not couple with fermions, G^0 has no Yukawa interactions.
- The quark masses depend only on v_2 therefore there are no fermionic contributions to the derivatives of the potential with respect to v_1.
- Another consequence is that the tensors y^{IJG^0} are automatically zero, which implies that the contributions to the self-energies, $\Pi^F_{G^0G^0}$ and $\Pi^F_{AG^0}$, are also zero.
- As a consequence the masslessness of the neutral Goldstone boson G^0 at one-loop is automatically confirmed.
- The y^{IJA} tensor becomes extremely simple, leading to no FCNC. The entries pertaining to the top quark, for instance, are given by

  $$y^{56A} = y^{65A} = \frac{i\, m_t}{v_2}\,, \tag{5.15}$$

  and

  $$M^{56} = M^{65} = m_t\,. \tag{5.16}$$

For example, the one-loop contribution of the top quark to the A mass will then be given by (using the definitions of eqs. (3.12) and (5.6))

$$\begin{aligned}\frac{1}{16\pi^2}\, X^{\text{top}}_{AA} &= \Pi^{\text{top}}_{AA} - \frac{(-12)}{2v_2}\,\frac{\partial m_t^2}{\partial v_2}\, A(m_t^2)\\ &= \Pi^{\text{top}}_{AA} + 12\,\frac{m_t^2}{v_2^2}\, A(m_t^2)\,,\end{aligned} \tag{5.17}$$

where the self-energy term is, according to (3.10),

$$\Pi^{\text{top}}_{AA} = 2\times 3\times \mathrm{Re}\left[y^{56A}y_{56A}\right] B_{FF}(m_t^2,m_t^2) + 2\times 3\times \mathrm{Re}\left[y^{56A}y^{56A}M_{56}M_{56}\right] B_{\bar{F}\bar{F}}(m_t^2,m_t^2)\,, \tag{5.18}$$

where the overall factor “2” comes from the sum on the symmetric y^{IJ} tensors indices, and “3” accounts for the colour of the quark. Then, from eqs. (A.6), (A.7) and (A.11), we obtain

$$\begin{aligned}B_{FF}(m_t^2,m_t^2) &= -2\, m_t^2 \log\left(\frac{m_t^2}{\mu^2}\right) - 2\, A(m_t^2)\,,\\ B_{\bar{F}\bar{F}}(m_t^2,m_t^2) &= -2\log\left(\frac{m_t^2}{\mu^2}\right).\end{aligned} \tag{5.19}$$

Therefore, using the values shown above for y^{56A} and M_{56} in eq. (5.18) we find that

$$\Pi^{\text{top}}_{AA} = -12\,\frac{m_t^2}{v_2^2}\,A(m_t^2)\,, \tag{5.20}$$

which implies that $X^{\text{top}}_{AA} = 0$. The same conclusion will apply for the contributions for all other quarks. Thus, in the Type-I U(1) model the pseudoscalar remains massless at one-loop when that symmetry is spontaneously broken, as expected by Goldstone's theorem. Since this demonstration can be made for each quark separately, the conclusions we reached hold for models Type-II, X or Y, where no FCNC occurs in the model.

We tested all possible U(1) Yukawa textures listed in ref. [24] (to wit, those textures for the down and up quarks shown in sections III C.1 to C.3 of that reference). In that paper, textures resulting from the application of abelian global symmetries on the 2HDM lagrangian were obtained, with the demands that the symmetry-constrained Yukawa matrices lead to six massive quarks, and a CKM matrix with no zero entries. These minimal requirements were not tested numerically (meaning, the Yukawa textures were not shown to reproduce the correct values of the quark masses of entries of the CKM matrix). Several textures were found to be possible only under application of a Z_2 symmetry — thus not leading to a massless pseudoscalar — or of a Z_3 symmetry, which we will treat in the next section. Unlike the flavour-preserving textures of Type-I (or II, X and Y), which we showed analytically that lead to a massless pseudoscalar at the one-loop level, for the vast majority of the U(1) textures presented in [24] the calculations of the self-energies can only be made numerically. We use the procedure outlined above in section 5, generating random numbers for the Yukawa matrices' entries and for the vevs;[17] diagonalizing numerically the resulting quark mass matrices, obtaining the right-handed and left-handed rotation matrices defined in eq. (2.17); obtaining the contributions from the quarks to the one-loop minimization conditions through the process explained in eqs. (5.4), (5.5) and (5.6); obtaining the mass-basis Yukawa matrices through eq. (5.7) and identifying the entries of the y^{IJk} tensors following the procedure outlined in eq. (5.11); and finally computing the quark contributions to the pseudoscalar mass matrix following the formulae of eq. (3.10).

The conclusion was the same for all the U(1) Yukawa textures from [24]: if the U(1) symmetry has been spontaneously broken, the one-loop pseudoscalar mass matrix yields two massless eigenvalues, even when including fermionic contributions. This was expected, since it is exactly what Goldstone's theorem predicted. It may be seen as an explicit confirmation of that theorem, but in the context of this paper it serves as a confirmation that our numerical methods of performing the mass corrections at one-loop are correct.

5.2 Z_3-symmetric Yukawa textures

As we discussed in section 2, it is possible to have Yukawa textures which are Z_3-symmetric — those shown in eq. (2.22) are a specific example, they can only be obtained if the angle θ, defined in eq. (2.3), is equal to $2\pi/3$. As shown in eqs. (2.23), the several phases transforming left and right handed quark fields are also constrained to be multiples of $\pm 2\pi/3$. An important

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[17] We restricted ourselves to real entries for the Yukawa couplings, but there should be no differences if complex entries were considered.

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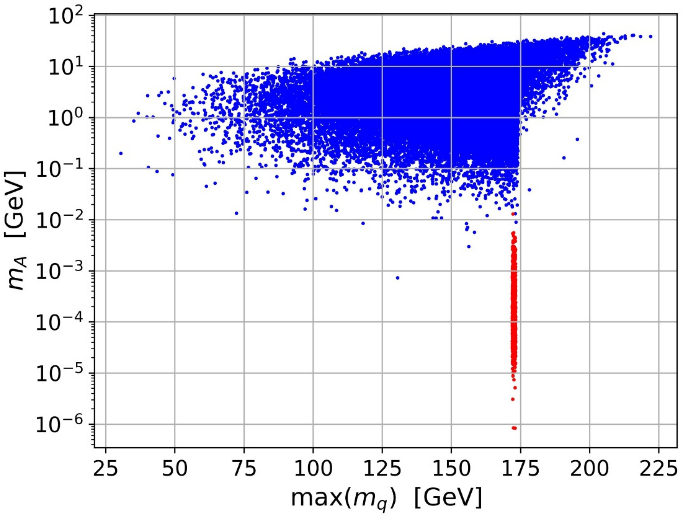

Figure 1. Pseudoscalar mass as a function of the highest quark mass obtained for the Z_3 Yukawa textures of eq. (2.22). In red, points for which all six quark masses were fitted to their experimental values.

point for what we will show below is the observation that the textures of the Γ (or Δ) matrices in eq. (2.22) require $\theta = 2\pi/3$ on their own, without the need to consider symmetry constraints arising from the Δ (Γ) matrices. Other implementations of a Z_3 symmetry, such as the one studied in ref. [26], also require $\theta = 2\pi/3$, but to reach that conclusion one needs to analyze the restrictions arising from non-zero textures in both the up and down Yukawa matrices, not just those of the up sector, or just those of the down one.

We performed a numerical analysis of the one-loop quark contributions to the pseudoscalar mass arising from the Yukawa textures of eq. (2.22). As we did for the U(1) textures, we generated random numbers for the values of the Yukawa couplings (between -1 and 1) and for the vevs v_1 and v_2 such that $v_1^2 + v_2^2 = v^2$ and computed the quark mass matrices, their eigenvalues and right-handed and left-handed rotation matrices. We then calculated the quark contributions to the one-loop minimization conditions and to the pseudoscalar self-energies. In all that follows we chose $\mu = 100\,\text{GeV}$.[18] The results are shown in figure 1, where we show the pseudoscalar mass m_A as a function of the highest quark mass obtained in the numerical procedure described above. In the case of the blue points in the plot we did not require that the quark masses obtained are correct, but in a separate fit we generated red points using a minimization procedure to find the values of the Yukawas which, for a given set of vevs, correctly reproduce all six quark masses within 2-σ intervals of their experimental values, taken from [32]. We computed the self-energies at $s = 0$.[19] We verify that the neutral Goldstone boson continues, as expected, to be massless, but the same cannot now be said for the pseudoscalar.

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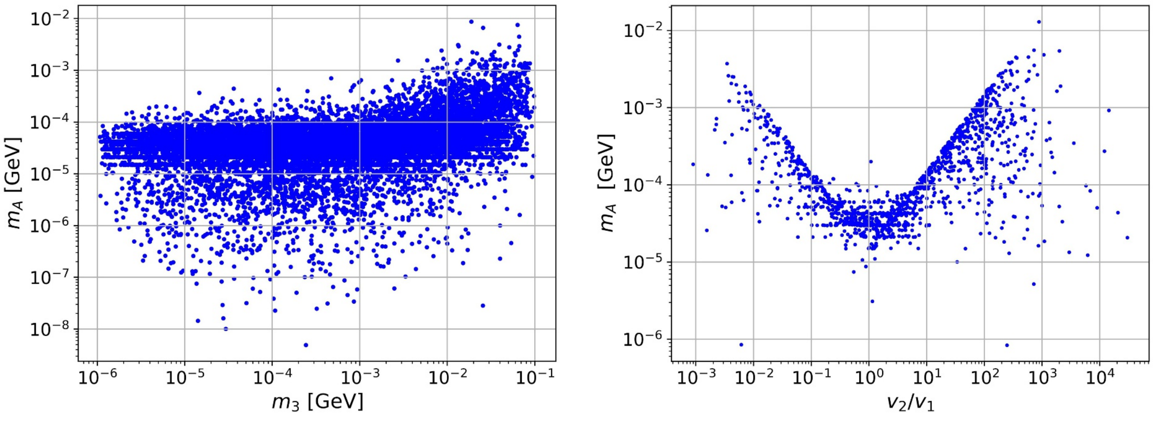
[18] The physical results are independent of this choice, though it is assumed that the value of μ is such that the logarithmic terms in the potential and self-energies are small. For coherence, this also means that the values of all running quantities are being taken at the scale μ.

[19] This is a good approximation [30, 31], and we confirmed its validity performing the iterative procedure mentioned earlier. We found that convergence to $s = m_A^2$ occurs after less than 7 iterations for most cases and that the final value of m_A is usually at most 6% different from the one computed with $s = 0$.



(a) (b)

Figure 2. Scatter plots for the one-loop pseudoscalar mass as a function of (a) Minimum value of the quark mass generated, fixing the highest mass to 10 GeV and allowing the remaining two to differ by several orders of magnitude. Only up quarks considered. (b) Vev ratio v_2/v_1, keeping $v_1^2 + v_2^2 = v^2$. All six quark masses correctly fitted.

The most relevant conclusion from figure 1 is that the one-loop quark contributions force $m_A \neq 0$, thus confirming what is expected from Goldstone's theorem: since the lagrangian has a discrete Z_3 symmetry — not a continuous U(1) one — then a spontaneous breaking of that symmetry ought to not imply a massless pseudoscalar. That in the 2HDM with a Z_3 symmetry that occurs at tree-level is an accident, arising from the fact that the scalar potential is identical for both Z_3 and U(1) symmetries. But here we see how the lagrangian with a discrete symmetry implies a massive pseudoscalar.

The second notable conclusion is that the pseudoscalar mass obtained is much smaller when all six quark masses obtained from the Z_3 Yukawa textures are in agreement with experimental values — this, we believe, is a consequence of the hierarchy of quark masses, which ranges from $m_u \simeq 2.2\,\text{MeV}$ to $m_t \simeq 173\,\text{GeV}$. In order to obtain such diverse orders of magnitude in the quark masses with vevs v_1 and v_2 ranging between 0 and 246 GeV, the entries of the Z_3 Yukawa matrices of eq. (2.22) must also have significant differences in order of magnitude. What we have concluded from our parameter scan is that, in order to correctly reproduce the known spectrum of up and down quark masses, at least one entry of the matrices Γ and another in matrices Δ must be several orders of magnitude (between two and five) below the remaining ones. It is easy to see that, if one (any one) of the non-zero entries in the Yukawa matrices shown in eq. (2.22) is set to zero, the resulting theory is invariant under a full U(1) symmetry, no longer under a discrete Z_3 one. Thus, fitting all quark masses to their real values and therefore reproducing the observed fermion hierarchy forces our Z_3 Yukawa matrices to numerically approach a U(1) texture — which, if exact, would force $m_A = 0$. A corresponding reduction in the pseudoscalar mass is thus obtained.

To verify this, we performed two separate analysis, whose results are shown in figure 2. In plot 2(a), we considered only the contributions from one type of quarks (in this case, the up ones) and fixed one of their masses (m_1) to 10 GeV. The second mass (m_2) we allowed to vary from 0.01 to 1 GeV, and the third mass m_3 was allowed to vary between 10^{-6} and 10^{-3} GeV.

In this way we forced a very hierarchical quark mass spectrum. The effect is clear: as the fermion mass spectrum becomes more hierarchical (meaning, as the smallest quark mass becomes increasingly smaller), the pseudoscalar mass is driven to smaller values. In figure 2(b) we attempted a different fitting procedure, randomly generating vevs v_1 and v_2 correctly reproducing electroweak symmetry breaking, thus satisfying $v_1^2 + v_2^2 = v^2 = (246\text{ GeV})^2$ but including cases with vevs with significant differences in orders of magnitude. In this way we were trying to "shift" the hierarchy in quark masses to a hierarchy in vevs, which would allow for all Yukawa couplings to be of the same order in magnitude, but the conclusion remains: it is not possible to reproduce the experimental values of all six quark masses and not have at least one entry be several orders of magnitude smaller than the rest, which makes the Z_3-invariant matrices be numerically similar to U(1)-invariant ones, thus yielding a much smaller pseudoscalar mass.

This model provides a rather natural mechanism to generate a very low pseudoscalar mass — m_A would be small because it is actually zero at leading order, its non-zero mass the result of radiative corrections. Indeed, we found a rough upper bound of ~ 0.008 GeV on the pseudoscalar mass in this model. Is such a low pseudoscalar mass excluded by experimental axion searches? We can use the exclusion limits shown in figure 90.1 of [32] as a function of the pseudoscalar mass m_A and its coupling to two photons $g_{A\gamma\gamma}$. In the Z_3 model we are studying, the axion would have a diphoton decay through a fermion triangle loop. There are very well known formulae for the width of such a decay (see for instance [33, 34]). Using the definitions from [32] and [34], we obtain

$$g_{A\gamma\gamma} = \frac{\alpha}{6\sqrt{2}\pi}\left|4\frac{\left(\bar{\Delta}_2\right)_{33}}{m_t}\,A^A_{1/2}(\tau_t) + \frac{\left(\bar{\Gamma}_2\right)_{33}}{m_b}\,A^A_{1/2}(\tau_b) + \cdots\right|, \tag{5.21}$$

where $\tau_x = m_A^2/(4m_x^2)$, $A^A_{1/2}(\tau)$ are the well-known pseudoscalar form factors and we explicitly show the top and bottom contributions. With the range of pseudoscalar masses which correctly fit the quark masses (see figure 1) to very good approximation we have $\tau_t \simeq \tau_b \simeq 0$ and in that limit it is easy to obtain $A^A_{1/2}(0) \simeq 2$. For the parameter scan we performed, we obtain $g_{A\gamma\gamma}$ between roughly 10^{-9} and $10^{-3}\,\text{GeV}^{-1}$ for pseudoscalar masses ranging from about 10^2 to 8×10^6 eV and this parameter range is excluded by several astrophysical searches (see [32] for details). A careful consideration of the assumptions undertaken in those searches would be interesting, but falls outside the scope of the current work.

At this stage we should investigate the other Z_3-invariant Yukawa matrices presented in ref. [24]. We did so, and discovered that, out of all the Z_3 invariant textures shown in that paper (to wit, those contained in equations (89) to (95)), the only one that did *not* yield a one-loop massless pseudoscalar is the one shown in our eq. (2.22) (eq. (92) of [24]). Though surprising at first, there is a good reason for this to occur: all the other apparently Z_3 textures shown in that paper can actually be shown to be related by basis transformations to U(1) invariant ones (to be specific, to the textures shown in equations (57) or (60) of [24]). Therefore they are not really Z_3 invariant, but rather invariant under a continuous U(1) symmetry, and as such the pseudoscalar remains massless at one-loop for such Yukawa textures.

An even more interesting situation occurs with the Z_3 Yukawa textures used in ref. [26], which also lead to the pseudoscalar mass remaining equal to zero at the one-loop level. In that paper, the Yukawa matrices are found to be

$$\Gamma_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \times \\ \times & \times & 0 \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} \times & \times & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix},$$

$$\Delta_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \times \\ \times & \times & 0 \end{pmatrix}, \quad \Delta_2 = \begin{pmatrix} 0 & 0 & \times \\ \times & \times & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{5.22}$$

so that the lagrangian is invariant under the choice of phases (defined in eqs. (2.3) and (2.18))

$$\begin{aligned} &\alpha_2 - \beta_3 = 0, \quad \alpha_1 - \beta_1 - \theta = 0 \\ &\alpha_3 - \beta_1 = 0, \quad \alpha_1 - \beta_2 - \theta = 0 \\ &\alpha_3 - \beta_2 = 0, \quad \alpha_3 - \beta_3 - \theta = 0 \end{aligned} \tag{5.23}$$

for the Γ matrices (and all zeros above should be interpreted as integer multiples of 2π) and, for the Δ matrices,

$$\begin{aligned} &\alpha_2 - \gamma_3 = 0, \quad \alpha_1 - \gamma_3 + \theta = 0 \\ &\alpha_3 - \gamma_1 = 0, \quad \alpha_2 - \gamma_1 + \theta = 0 \\ &\alpha_3 - \gamma_2 = 0, \quad \alpha_2 - \gamma_2 + \theta = 0 \,. \end{aligned} \tag{5.24}$$

The equations (5.23) coming from the constraints on the Γ matrices lead to two equations on the α phases,

$$\alpha_1 - \alpha_3 = \theta\,, \quad \alpha_3 - \alpha_2 = \theta + 2n\pi\,, \tag{5.25}$$

for some integer n, which can be satisfied by any value of θ — the Γ matrix textures of eq. (5.22) can therefore be obtained by a generic U(1) symmetry. Likewise, from the Δ matrix constraints in eq. (5.24), we obtain

$$\alpha_1 - \alpha_2 = -\theta\,, \quad \alpha_2 - \alpha_3 = -\theta + 2m\pi\,, \tag{5.26}$$

for any integer m, again equations which might be satisfied for any value of θ — and thus the Δ matrix textures of eq. (5.22) can be obtained by a generic U(1) symmetry. However, when we take eqs. (5.25) and (5.26) together, we immediately obtain that they can only hold together if

$$3\theta = 2n\pi \tag{5.27}$$

for some integer n, which can only be satisfied by $\theta = 2\pi/3 + 2n\pi$. This means that the only way to obtain the combined Γ and Δ textures of eq. (5.22) is with a Z_3 symmetry.

Therefore the Yukawa textures considered in [26] are unquestionably the result of a Z_3 symmetry, but with a difference *vis a vis* those of eq. (2.3) treated in this paper. As we showed in eq. (2.23), the texture of the down-type Yukawa matrices Γ alone forced the angle θ to be $2\pi/3$, and the same can be shown for those Δ matrices. As shown above, however, the Γ matrix textures used in ref. [26], could be reproduced by a U(1) symmetry, and likewise for the corresponding Δ matrix textures. And this is relevant because, given that we are considering neutral minima, the quark contributions to the self-energies in eq. (3.10) are computed separately for up and down quarks. Thus, if the Γ/Δ matrices originating such contributions have textures which are analogous to those obtained from a U(1) symmetry, then they will give the same result as that of a continuous symmetry, preserving the masslessness of the pseudoscalar. Simply put, in the one-loop formulae of eq. (3.10) the up-quark contributions "don't know" about the down-quark ones and therefore, even though the lagrangian of ref. [26] is certainly invariant under a Z_3 symmetry through the interplay between up and down Yukawa matrices, at one-loop even the fermionic contributions to the self-energy will not render the pseudoscalar massive. It is to be expected that with higher order corrections taken into account a non-zero mass contribution from the fermion sector will appear in this model, but that cannot occur at one-loop. It may be an analogous situation to the "propagation" of CP violation from the Yukawa sector to the rest of the theory in the SM [35].

6 Conclusions

Spontaneous breaking of a continuous symmetry yields massless scalars. This is the well-known Goldstone theorem, a crucial part of understanding how the electroweak gauge bosons acquire their masses and longitudinal polarizations in the Higgs mechanism. In the 2HDM, global symmetries are frequently considered to increase the model's predictive power, or to produce interesting phenomenology. One such symmetry is the continuous Peccei-Quinn U(1). When both doublets acquire a vev electroweak symmetry breaking occurs and three Goldstone bosons appear, as usual — but an additional massless pseudoscalar is produced, as expected. Likewise, when imposing a Z_3 symmetry on the 2HDM, a massless pseudoscalar also appears at tree-level, despite that symmetry being a discrete one. This is due to the fact that, the 2HDM potential having at most terms quartic in the scalar fields, imposing a discrete Z_3 symmetry leads to a scalar sector with an accidental continuous U(1). In fact, any Z_N symmetry with $N \geq 3$ leads to the same U(1)-invariant potential. While imposing these different symmetries leads to the same scalar and gauge sectors, they can lead to different fermion sectors. Indeed, there are many different ways of extending both the U(1) and Z_3 symmetries to the fermion sector, with considerably different phenomenologies. Yukawa interactions may distinguish between discrete and continuous symmetries, and one should expect that in a lagrangian invariant under a discrete Z_3 symmetry the pseudoscalar mass will acquire loop corrections stemming from fermions — but remain massless for any U(1)-invariant model.

We tested this assumption by computing the one-loop self-energies to the pseudoscalar mass matrix, showing analytically that, for both U(1) and Z_3 invariant 2HDMs, they yielded two massless particles when only the contributions from the scalar and gauge sectors were taken into account. This was to be expected, since those two sectors of the model cannot distinguish between both symmetries. We then computed the one-loop quark contributions to the pseudoscalar mass matrix for all U(1)-invariant Yukawa textures in the exhaustive list provided in ref. [24], and showed that for all of them the pseudoscalar remained massless at one-loop, as foreseen by Goldstone's theorem. Repeating that calculation for a specific Z_3 Yukawa texture (eq. (95) of ref. [24], eq. (2.22) of this paper) we then found a non-zero pseudoscalar mass (the neutral Goldstone from electroweak symmetry breaking remaining massless, of course). Thus we confirmed that imposing a discrete symmetry on the 2HDM lagrangian did not lead to a massless scalar — the accidental tree-level symmetry of the scalar potential was not verified when one-loop fermion corrections were taken into account. We also found that several symmetries classified as Z_3 in [24] were indeed basis changes from U(1) symmetries, thus producing massless axions even at one-loop.

Two unexpected things were revealed in our calculation, however. The first is that, though Yukawa interactions give rise to a one-loop pseudoscalar mass m_A, the value of m_A is highly, and non-intuitively, dependent on the quark mass spectrum. Indeed, allowing for generic quark masses all of the same order yields one-loop fermion masses as high as $\sim$ 30 GeV, as we see in figure 1 for a maximum quark mass of $\sim$ 200 GeV. We can understand these values by performing a quick estimate, approximating the fermion contributions to the scalar masses by the second derivative of the potential with respect to the vevs; we take $A(x) \simeq x$ in eq. (5.6) and the maximum value v_2 can take, so that

$$m_A \simeq \sqrt{\frac{\partial^2 V_1^F}{\partial v_2^2}} = \frac{1}{8\pi}\sqrt{12\frac{m_q^4}{v^2}} \simeq 22.4 \text{ GeV}\,. \tag{6.1}$$

However, when adjusting all six quark masses to their know values, we see, in the red points of figure 1 that the pseudoscalar mass becomes much smaller, between 10^{-6} and 10^{-2} GeV in our fit. We interpreted this as a consequence of using the Yukawa matrices of eq. (2.22) to reproduce the strong hierarchy of the quark mass spectrum: to obtain such different masses, some of the entries of the Yukawa matrices must be much smaller than others. But when any of the entries of the matrices in eq. (2.22) are zero, the lagrangian ends up invariant under a continuous U(1) symmetry, which leads to $m_A = 0$. The fermion mass hierarchy thus leads to an *approximate* continuous symmetry, thus reducing substantially the order of magnitude of the pseudoscalar mass. The model therefore provides a natural way of generating very small ($\sim$ MeV and lower) axion masses. Since these axions interact with charged fermions they have a loop-induced diphoton decay. The coupling strength of the pseudoscalar-photon interactions determined from those interactions makes the parameter space studied here already excluded by experimental axion searches, though a deeper analysis is warranted (for instance, some axion searches assume the particle behaves like Dark Matter, which may not be a reasonable assumption for the model under discussion).

The second unexpected result was finding a massless pseudoscalar for certain Z_3 Yukawa textures (those of ref. [26], our eq. (5.22)). This is a consequence of the fact that the textures in the down/up Yukawa matrices can be reproduced separately with a U(1) symmetry — it is only when one tries to force *both* down and up Yukawa matrices to have the forms shown in eq. (5.22) that one finds that is only possible for a Z_3 symmetry, not for a generic U(1). But the one-loop pseudoscalar fermionic self-energies have separate contributions from the up and down quarks — hence, if the Yukawa matrices contributing to those self energies have identical textures to others resulting from a U(1) symmetry, the net contribution to pseudoscalar masses will be zero. One expects that Goldstone's theorem is still valid, but only higher order corrections, presumably involving diagrams with both up and down quarks, will eventually result in $m_A \neq 0$. We might then expect that the model studied in [26], if devoid of any soft breaking terms, would produce in a natural way much smaller values for the pseudoscalar mass. Thus, like the consequences of CP violation in the SM on the rest of the lagrangian, the impact of a discrete symmetry in the Yukawa sector may only reflect itself in the pseudoscalar mass at very high orders. On a side note, the work of refs. [36, 37] should be especially appropriate for these questions, as their authors have performed two-loop scalar mass calculations for generic renormalizable theories, using SARAH implementations [38–42]. The tools therein developped should allow for a verification of the results of this work, and a possible way of confirming the two-loop conjecture for the textures of (5.22).

The conclusions reached in this work are valid for any theory with discrete symmetries which lead to accidental continuous ones in the scalar potential at tree-level. Goldstone's theorem assures us that, if the model's lagrangian distinguishes between a continuous and a discrete symmetry in the Yukawa sector, there will be fermionic contributions that will make the pseudoscalar massive. However, it is not guaranteed that will occur already at the one-loop level. The two Z_3 symmetries compared in this paper showed that, if up/down Yukawa matrices are formally identical to those obtained from a U(1) symmetry, m_A will remain zero at least at the one-loop level. An interesting question becomes therefore: other than the textures of eq. (2.22), can there be any other Z_3-symmetric Yukawas that alread yield a non-zero pseudoscalar mass at one-loop? The analysis of ref. [24] was very thorough, but it did not seem to include the textures of ref. [26], for instance. Finally, what applications of interest might there be in these calculations? Though our motivation was essentially a theoretical one, the fact that we found low axion masses arising from symmetries may well be of interest in exploring the allowed parameter space for those particles shown in ref. [32]. In particular, it would be interesting to explore models with discrete symmetries for which the pseudoscalar mass is loop-generated and thus naturally small but where the axion couplings to diphotons is also suppressed (unlike the model we studied here). This would require a suppression of the axion-fermion couplings contributing to $g_{A\gamma\gamma}$ (see eq. (5.21)), which might be possible in a version of the so-called CP3 model.

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A Coupling and integral definitions

Below we define the several integral functions used in the self-energy calculations. Our definitions follow the conventions of [28, 29]:

$$A(x) = x\left[\log\left(\frac{x}{\mu^2}\right) - 1\right], \tag{A.1}$$

$$A_G(x) = 4A(x) - \mathcal{L}_x[xA(x)], \tag{A.2}$$

$$B(x,y) = 2 - r_{sxy}\log\left(\frac{x}{\mu^2}\right) - t_{syx}\log\left(\frac{y}{\mu^2}\right) + \frac{\Delta_{sxy}^{1/2}}{s}\log(t_{xys}), \tag{A.3}$$

$$
\begin{aligned}
B_{SG}(x,y) &= (2x-y+2s)B(x,y)+A(x)-2A(y) \\
&\quad +\mathcal{L}_y[(x+y-s)A(y)-(x-s)^2B(x,y)]\,, && \text{(A.4)}\\
B_{GG}(x,y) &= -\frac{7}{2}B(x,y)+\frac{1}{2}\mathcal{L}_x[xB(x,y)]+\frac{1}{2}\mathcal{L}_y[yB(x,y)]\,, \\
&\quad +\frac{1}{4}\mathcal{L}_x\mathcal{L}_y\{xA(y)+yA(x)+[2s(x+y)-x^2-y^2-s^2]B(x,y)\} && \text{(A.5)}\\
B_{FF}(x,y) &= (x+y-s)B(x,y)-A(x)-A(y)\,, && \text{(A.6)}\\
B_{\bar{F}\bar{F}}(x,y) &= 2B(x,y)\,, && \text{(A.7)}
\end{aligned}
$$

where A_G and B_{GG} are presented for the $\overline{DR}$ renormalization scheme chosen and the following definitions were used:

$$
\begin{aligned}
\Delta_{xyz} &= x^2+y^2+z^2-2xy-2xz-2yz\,,\\
t_{abc} &= \frac{a+b-c+\Delta_{abc}^{1/2}}{2a}\,,\\
r_{abc} &= \frac{a+b-c-\Delta_{abc}^{1/2}}{2a}\,. && \text{(A.8)}
\end{aligned}
$$

Also, we have

$$
\mathcal{L}_x f(x) \equiv \frac{f(x)-f(\xi x)}{x}\,, \tag{A.9}
$$

where $\xi = 0\,,1$ corresponds to the Landau/Feynman gauges. At $s = 0$ we have the following simplified expressions:

$$
\begin{aligned}
B(x,y) &= \frac{A(x)-A(y)}{y-x}\,, && \text{(A.10)}\\
B(x,x) &= -\log\left(\frac{x}{\mu^2}\right)\,, && \text{(A.11)}\\
B_{SG}(x,y) &= 0\,, && \text{(A.12)}
\end{aligned}
$$

where the B_{SG} condition was obtained in the Landau gauge.

Here we present the relevant non-zero couplings used in our calculations (the ones not shown may be obtained from permutations of indices). The scalar trilinear couplings defined in eq. (3.7) are:

$$
\begin{aligned}
\lambda^{G^0H^+G^-} &= -\lambda^{G^0G^+H^-} = -\frac{1}{2}v_2\lambda_4\,,\\
\lambda^{AH^+G^-} &= -\lambda^{AG^+H^-} = \frac{1}{2}v_1\lambda_4\,,\\
\lambda^{G^0G^0h} &= v_1\lambda_1 c_\alpha - v_2\lambda_{34}s_\alpha\,,\\
\lambda^{G^0G^0H} &= v_2\lambda_{34}c_\alpha + v_1\lambda_1 s_\alpha\,,\\
\lambda^{AAh} &= v_1\lambda_{34}c_\alpha - v_2\lambda_2 s_\alpha\,,\\
\lambda^{AAH} &= v_2\lambda_2 c_\alpha + v_1\lambda_{34}s_\alpha\,, && \text{(A.13)}
\end{aligned}
$$

and the quartic scalar couplings are:

$$\begin{aligned}
\lambda^{G^0AH^+H^+} &= \lambda^{G^0AH^-H^-} = -\lambda^{G^0AG^+G^+} = -\lambda^{G^0AG^-G^-} = -\lambda_4 c_\beta s_\beta\,,\\
\lambda^{G^0AH^+G^+} &= \lambda^{G^0AH^-G^-} = \frac{1}{2}\lambda_4(c_\beta^2 - s_\beta^2)\,,\\
\lambda^{G^0G^0H^+H^+} &= \lambda^{G^0G^0H^-H^-} = \lambda_1 c_\beta^2 + \lambda_3 s_\beta^2\,,\\
\lambda^{G^0G^0G^+G^+} &= \lambda^{G^0G^0G^-G^-} = \lambda_3 c_\beta^2 + \lambda_1 s_\beta^2\,,\\
\lambda^{G^0G^0H^+G^+} &= \lambda^{G^0G^0H^-G^-} = (\lambda_1 - \lambda_3) c_\beta s_\beta\,,\\
\lambda^{AAH^+H^+} &= \lambda^{AAH^-H^-} = \lambda_3 c_\beta^2 + \lambda_2 s_\beta^2\,,\\
\lambda^{AAG^+G^+} &= \lambda^{AAG^-G^-} = \lambda_2 c_\beta^2 + \lambda_3 s_\beta^2\,,\\
\lambda^{AAH^+G^+} &= \lambda^{AAH^-G^-} = (\lambda_3 - \lambda_2) c_\beta s_\beta\,,\\
\lambda^{G^0G^0hh} &= \lambda_1 c_\alpha^2 + \lambda_{34} s_\alpha^2\,,\\
\lambda^{G^0G^0HH} &= \lambda_{34} c_\alpha^2 + \lambda_1 s_\alpha^2\,,\\
\lambda^{G^0G^0hH} &= (\lambda_1 - \lambda_{34}) c_\alpha s_\alpha\,,\\
\lambda^{AAhh} &= \lambda_{34} c_\alpha^2 + \lambda_2 s_\alpha^2\,,\\
\lambda^{AAHH} &= \lambda_2 c_\alpha^2 + \lambda_{34} s_\alpha^2\,,\\
\lambda^{AAhH} &= (\lambda_{34} - \lambda_2) c_\alpha s_\alpha\,,\\
\lambda^{G^0G^0G^0G^0} &= 3\lambda_1\,,\\
\lambda^{AAAA} &= 3\lambda_2\,,\\
\lambda^{G^0G^0AA} &= \lambda_{34}\,.
\end{aligned} \tag{A.14}$$

Likewise, the non-zero trilinear gauge couplings in eq. (3.9) are:

$$\begin{aligned}
g^{W^+G^0H^+} &= g^{W^-G^0H^+} = i\,g^{W^+G^0H^-} = -i\,g^{W^-G^0H^-} = \frac{m_W}{v\sqrt{2}}\,c_\beta\,,\\
g^{W^+G^0G^+} &= g^{W^-G^0G^+} = i\,g^{W^+G^0G^-} = -i\,g^{W^-G^0G^-} = \frac{m_W}{v\sqrt{2}}\,s_\beta\,,\\
g^{W^+AH^+} &= g^{W^-AH^+} = i\,g^{W^+AH^-} = -i\,g^{W^-AH^-} = -\frac{m_W}{v\sqrt{2}}\,s_\beta\,,\\
g^{W^+AG^+} &= g^{W^-AG^+} = i\,g^{W^+AG^-} = -i\,g^{W^-AG^-} = \frac{m_W}{v\sqrt{2}}\,c_\beta\,,\\
g^{ZG^0h} &= g^{ZAH} = -\frac{m_Z}{v}\,c_\alpha\,,\\
g^{ZG^0H} &= -g^{ZAh} = -\frac{m_Z}{v}\,s_\alpha\,.
\end{aligned} \tag{A.15}$$

It should be noted that contrarily to the scalar couplings above, g^{aij} are not symmetric under an interchange of scalar indices (see eq. (3.9)), and $g^{abG^0} = g^{abA} = 0$ for all gauge fields a, b. The quartic couplings are:

$$\begin{aligned}
g^{W^+W^-G^0G^0} &= g^{W^+W^-AA} = \frac{2\,m_W^2}{v^2}\,,\\
g^{ZZG^0G^0} &= g^{ZZAA} = \frac{2\,m_Z^2}{v^2}\,.
\end{aligned} \tag{A.16}$$

Finally, for the fermion couplings of eq. (3.8) we have:

$$
\begin{aligned}
y^{1\,2\,G^0} &= \frac{i}{\sqrt{2}}\left(\bar{\Delta}_1^\dagger\right)_{11}, & y^{1\,4\,G^0} &= \frac{i}{\sqrt{2}}\left(\bar{\Delta}_1^\dagger\right)_{12}, & y^{1\,6\,G^0} &= \frac{i}{\sqrt{2}}\left(\bar{\Delta}_1^\dagger\right)_{13}, \\
y^{2\,3\,G^0} &= \frac{i}{\sqrt{2}}\left(\bar{\Delta}_1^\dagger\right)_{21}, & y^{2\,5\,G^0} &= \frac{i}{\sqrt{2}}\left(\bar{\Delta}_1^\dagger\right)_{31}, & y^{3\,4\,G^0} &= \frac{i}{\sqrt{2}}\left(\bar{\Delta}_1^\dagger\right)_{22}, \\
y^{3\,6\,G^0} &= \frac{i}{\sqrt{2}}\left(\bar{\Delta}_1^\dagger\right)_{23}, & y^{4\,5\,G^0} &= \frac{i}{\sqrt{2}}\left(\bar{\Delta}_1^\dagger\right)_{32}, & y^{5\,6\,G^0} &= \frac{i}{\sqrt{2}}\left(\bar{\Delta}_1^\dagger\right)_{33}, \\
y^{7\,8\,G^0} &= \frac{i}{\sqrt{2}}\left(\bar{\Gamma}_1^\dagger\right)_{11}, & y^{7\,10\,G^0} &= \frac{i}{\sqrt{2}}\left(\bar{\Gamma}_1^\dagger\right)_{12}, & y^{7\,12\,G^0} &= \frac{i}{\sqrt{2}}\left(\bar{\Gamma}_1^\dagger\right)_{13}, \\
y^{8\,9\,G^0} &= \frac{i}{\sqrt{2}}\left(\bar{\Gamma}_1^\dagger\right)_{21}, & y^{6\,11\,G^0} &= \frac{i}{\sqrt{2}}\left(\bar{\Gamma}_1^\dagger\right)_{31}, & y^{9\,10\,G^0} &= \frac{i}{\sqrt{2}}\left(\bar{\Gamma}_1^\dagger\right)_{22}, \\
y^{9\,12\,G^0} &= \frac{i}{\sqrt{2}}\left(\bar{\Gamma}_1^\dagger\right)_{23}, & y^{10\,11\,G^0} &= \frac{i}{\sqrt{2}}\left(\bar{\Gamma}_1^\dagger\right)_{32}, & y^{11\,12\,G^0} &= \frac{i}{\sqrt{2}}\left(\bar{\Gamma}_1^\dagger\right)_{33}.
\end{aligned}
\tag{A.17}
$$

The couplings y^{IJA} can be obtained from these by replacing Δ_1 (Γ_1) by Δ_2 (Γ_2). The coefficients M^{IJ}, defined in eq. (5.13), are the masses of the six quarks, so we have:

$$
\begin{aligned}
M^{1\,2} &= m_u, & M^{3\,4} &= m_c, & M^{5\,6} &= m_t, \\
M^{7\,8} &= m_d, & M^{9\,10} &= m_s, & M^{11\,12} &= m_b.
\end{aligned}
\tag{A.18}
$$

Data Availability Statement. This article has no associated data or the data will not be deposited.

Code Availability Statement. This article has no associated code or the code will not be deposited.

References

[1] ATLAS collaboration, *Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC*, *Phys. Lett. B* **716** (2012) 1 [`arXiv:1207.7214`] [INSPIRE].

[2] CMS collaboration, *Observation of a New Boson at a Mass of 125 GeV with the CMS Experiment at the LHC*, *Phys. Lett. B* **716** (2012) 30 [`arXiv:1207.7235`] [INSPIRE].

[3] ATLAS collaboration, *A detailed map of Higgs boson interactions by the ATLAS experiment ten years after the discovery*, *Nature* **607** (2022) 52 [*Erratum ibid.* **612** (2022) E24] [`arXiv:2207.00092`] [INSPIRE].

[4] CMS collaboration, *A portrait of the Higgs boson by the CMS experiment ten years after the discovery*, *Nature* **607** (2022) 60 [`arXiv:2207.00043`] [INSPIRE].

[5] V. Barger et al., *Complex Singlet Extension of the Standard Model*, *Phys. Rev. D* **79** (2009) 015018 [`arXiv:0811.0393`] [INSPIRE].

[6] R.N. Mohapatra and G. Senjanovic, *Neutrino Mass and Spontaneous Parity Nonconservation*, *Phys. Rev. Lett.* **44** (1980) 912 [INSPIRE].

[7] T.D. Lee, *A Theory of Spontaneous T Violation*, *Phys. Rev. D* **8** (1973) 1226 [INSPIRE].

[8] G.C. Branco et al., *Theory and phenomenology of two-Higgs-doublet models*, *Phys. Rept.* **516** (2012) 1 [`arXiv:1106.0034`] [INSPIRE].

[9] N.G. Deshpande and E. Ma, *Pattern of Symmetry Breaking with Two Higgs Doublets*, *Phys. Rev. D* **18** (1978) 2574 [INSPIRE].

[10] R. Barbieri, L.J. Hall and V.S. Rychkov, *Improved naturalness with a heavy Higgs: An alternative road to LHC physics*, *Phys. Rev. D* **74** (2006) 015007 [`hep-ph/0603188`] [INSPIRE].

[11] L. Lopez Honorez, E. Nezri, J.F. Oliver and M.H.G. Tytgat, *The Inert Doublet Model: An Archetype for Dark Matter*, *JCAP* **02** (2007) 028 [`hep-ph/0612275`] [INSPIRE].

[12] Q.-H. Cao, E. Ma and G. Rajasekaran, *Observing the Dark Scalar Doublet and its Impact on the Standard-Model Higgs Boson at Colliders*, *Phys. Rev. D* **76** (2007) 095011 [`arXiv:0708.2939`] [INSPIRE].

[13] S. Davidson and H.E. Haber, *Basis-independent methods for the two-Higgs-doublet model*, *Phys. Rev. D* **72** (2005) 035004 [*Erratum ibid.* **72** (2005) 099902] [`hep-ph/0504050`] [INSPIRE].

[14] S.L. Glashow and S. Weinberg, *Natural Conservation Laws for Neutral Currents*, *Phys. Rev. D* **15** (1977) 1958 [INSPIRE].

[15] E.A. Paschos, *Diagonal Neutral Currents*, *Phys. Rev. D* **15** (1977) 1966 [INSPIRE].

[16] R.D. Peccei and H.R. Quinn, *CP Conservation in the Presence of Instantons*, *Phys. Rev. Lett.* **38** (1977) 1440 [INSPIRE].

[17] I.P. Ivanov, *Minkowski space structure of the Higgs potential in 2HDM*, *Phys. Rev. D* **75** (2007) 035001 [*Erratum ibid.* **76** (2007) 039902] [`hep-ph/0609018`] [INSPIRE].

[18] I.P. Ivanov, *Minkowski space structure of the Higgs potential in 2HDM. II. Minima, symmetries, and topology*, *Phys. Rev. D* **77** (2008) 015017 [`arXiv:0710.3490`] [INSPIRE].

[19] M. Srednicki, *Axion Couplings to Matter. 1. CP Conserving Parts*, *Nucl. Phys. B* **260** (1985) 689 [INSPIRE].

[20] P.S. Bhupal Dev and A. Pilaftsis, *Maximally Symmetric Two Higgs Doublet Model with Natural Standard Model Alignment*, *JHEP* **12** (2014) 024 [*Erratum ibid.* **11** (2015) 147] [`arXiv:1408.3405`] [INSPIRE].

[21] N. Darvishi and A. Pilaftsis, *Quartic Coupling Unification in the Maximally Symmetric 2HDM*, *Phys. Rev. D* **99** (2019) 115014 [`arXiv:1904.06723`] [INSPIRE].

[22] J. Goldstone, *Field Theories with Superconductor Solutions*, *Nuovo Cim.* **19** (1961) 154 [INSPIRE].

[23] J. Goldstone, A. Salam and S. Weinberg, *Broken Symmetries*, *Phys. Rev.* **127** (1962) 965 [INSPIRE].

[24] P.M. Ferreira and J.P. Silva, *Abelian symmetries in the two-Higgs-doublet model with fermions*, *Phys. Rev. D* **83** (2011) 065026 [`arXiv:1012.2874`] [INSPIRE].

[25] P.M. Ferreira, M. Maniatis, O. Nachtmann and J.P. Silva, *CP properties of symmetry-constrained two-Higgs-doublet models*, *JHEP* **08** (2010) 125 [`arXiv:1004.3207`] [INSPIRE].

[26] P.M. Ferreira, L. Lavoura, J.P. Silva and L. Lavoura, *A soft origin for CKM-type CP violation*, *Phys. Lett. B* **704** (2011) 179 [`arXiv:1102.0784`] [INSPIRE].

[27] S.P. Martin, *A Supersymmetry primer*, *Adv. Ser. Direct. High Energy Phys.* **18** (1998) 1 [`hep-ph/9709356`] [INSPIRE].

[28] S.P. Martin, *Two loop scalar self energies in a general renormalizable theory at leading order in gauge couplings*, *Phys. Rev. D* **70** (2004) 016005 [`hep-ph/0312092`] [INSPIRE].

[29] S.P. Martin, *Evaluation of Two Loop Self Energy Basis Integrals Using Differential Equations*, *Phys. Rev. D* **68** (2003) 075002 [`hep-ph/0307101`] [INSPIRE].

[30] J.R. Ellis, G. Ridolfi and F. Zwirner, *Radiative corrections to the masses of supersymmetric Higgs bosons*, *Phys. Lett. B* **257** (1991) 83 [INSPIRE].

[31] J.R. Ellis, G. Ridolfi and F. Zwirner, *On radiative corrections to supersymmetric Higgs boson masses and their implications for LEP searches*, *Phys. Lett. B* **262** (1991) 477 [INSPIRE].

[32] PARTICLE DATA GROUP collaboration, *Review of particle physics*, *Phys. Rev. D* **110** (2024) 030001 [INSPIRE].

[33] A. Djouadi, *The anatomy of electro-weak symmetry breaking. II. The Higgs bosons in the minimal supersymmetric model*, *Phys. Rept.* **459** (2008) 1 [`hep-ph/0503173`] [INSPIRE].

[34] A. Barroso, P.M. Ferreira, R. Santos and J.P. Silva, *Probing the scalar-pseudoscalar mixing in the 125 GeV Higgs particle with current data*, *Phys. Rev. D* **86** (2012) 015022 [`arXiv:1205.4247`] [INSPIRE].

[35] J.R. Ellis and M.K. Gaillard, *Strong and Weak CP Violation*, *Nucl. Phys. B* **150** (1979) 141 [INSPIRE].

[36] J. Braathen and M.D. Goodsell, *Avoiding the Goldstone Boson Catastrophe in general renormalisable field theories at two loops*, *JHEP* **12** (2016) 056 [`arXiv:1609.06977`] [INSPIRE].

[37] J. Braathen, M.D. Goodsell and F. Staub, *Supersymmetric and non-supersymmetric models without catastrophic Goldstone bosons*, *Eur. Phys. J. C* **77** (2017) 757 [`arXiv:1706.05372`] [INSPIRE].

[38] F. Staub, *SARAH*, `arXiv:0806.0538` [INSPIRE].

[39] F. Staub, *From Superpotential to Model Files for FeynArts and CalcHep/CompHep*, *Comput. Phys. Commun.* **181** (2010) 1077 [`arXiv:0909.2863`] [INSPIRE].

[40] F. Staub, *Automatic Calculation of supersymmetric Renormalization Group Equations and Self Energies*, *Comput. Phys. Commun.* **182** (2011) 808 [`arXiv:1002.0840`] [INSPIRE].

[41] F. Staub, *SARAH 3.2: Dirac Gauginos, UFO output, and more*, *Comput. Phys. Commun.* **184** (2013) 1792 [`arXiv:1207.0906`] [INSPIRE].

[42] F. Staub, *SARAH 4: A tool for (not only SUSY) model builders*, *Comput. Phys. Commun.* **185** (2014) 1773 [`arXiv:1309.7223`] [INSPIRE].