

# Learning & Teaching

*Mathematics Teacher: Learning and Teaching PK-12*, is NCTM's newest journal that reflects the current practices of mathematics education, as well as maintains a knowledge base of practice and policy in looking at the future of the field. Content is aimed at preschool to 12th grade with peer-reviewed and invited articles. *MTLT* is published monthly.

ARTICLE TITLE:

AUTHOR NAMES:

DIGITAL OBJECT IDENTIFIER:

VOLUME:

ISSUE NUMBER:

## Mission Statement

The National Council of Teachers of Mathematics advocates for high-quality mathematics teaching and learning for each and every student.

Approved by the NCTM Board of Directors on July 15, 2017.

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NATIONAL COUNCIL OF  
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# Improve It!

## Understanding Shape's Structures

A look at first graders' abilities to build and improve their geometry constructions while moving between and connecting 2D and 3D representations.

Joana Conceição and Margarida Rodrigues

**“Can you build these shapes?”** “How can you do it?” “Can you explain it?” How often do we hear these questions in a classroom? Probably not so much. Building two-dimensional (2D) and three-dimensional (3D) shapes fosters children's abilities to understand spatial relationships, when they are challenged to construct, analyze, and improve their constructions and the mental models they possess of them. Composing shapes prompts children to look into structures and the relationships within, promoting higher levels of reasoning. As they move between 2D and 3D representations, they relate parts of a shape and connect different representations, leading to a deep understanding of mathematical concepts (National Council of Teachers of Mathematics [NCTM], 2017).

Can You Build It? is an engaging *Improve It!* task that asks children to construct 3D shapes based on 2D depictions. Students' initial 3D creations often need revision, and as they build and re-build, they develop spatial relationships. This task was implemented in a first-grade class, connected to composing 3D geometric shapes to address the Common Core State Standard 1.GA.2 (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Moving constantly between 2D and 3D leads to a stronger spatial sense that helps students to unveil the shapes' structures, which is an important topic in mathematics and recommended as a mathematical practice.

Looking at the Can You Build It? task (see Task Sheet), what sort of constructions can teachers

anticipate from children? What relationships can children see and use to improve their constructions? The structure of shapes presents growing levels of complexity: Shape A has only one layer of cubes, Shape B has different layers, and Shape C has different levels and cannot be fully seen. Teachers can use a larger set of shapes. We emphasize the importance of using manipulatives so children can build, analyze, and improve. This approach fosters spatial relationships that lead to more complex, abstract, and powerful mental models.

Because the task is primarily about students constructing and exploring, the lesson was organized into three stages: (1) presenting the task to students (10 minutes); (2) students constructing and improving constructions, alone or in peer groups (30–45 minutes); and (3) a whole-class conversation with students presenting and discussing strategies (15 minutes).

### BUILDING AND IMPROVING

Questioning students can encourage them to explore their reasoning, to reflect, and to improve spatial relationships:

- Where are you starting from (and why)?
- What do you see in the drawing that helps you build the 3D shape?
- What do you notice between the 2D and 3D shapes?
- What part of your construction matches this part of the diagram? (and vice versa)



- Is there more than one possible 3D shape that fits the 2D drawing? Why?
- What is the same and different with each new shape?
- Did you check your construction to see if it matches the drawing? How?
- Is it possible to build it using fewer steps?

As Matney et al. (2020) highlight, while talking about it, students improve spatial vocabulary.

### BUILDING ONE LEVEL CONSTRUCTION

Student 1 built shape A (see Task Sheet) with an extra cube in two parallel sides, probably as a result of double-counting the cubes in the corners, making the construction with a two-cube size free space (Figure 1).

Student 1 looked at the construction several times but could not grasp what was different. Student 2

constructed the shape without leaving any empty space in the middle (Figure 2).

Both students seemed to have doubts. Student 2 looked at Student 1's construction:

*Student 2:* It has only one hole in the middle. [points to the picture]

[Student 1 tries once more, placing a cube in the empty space (Figure 3). Again, Student 2 challenges Student 1.]

*Student 2:* Four?! [points to one side of the shape]

[Student 1 takes out some cubes from the shape and observes its structure.]

*Student 1:* [claiming instantly] I know! Look, I know! [She finishes the construction.] 3 - 2 - 1 [points at each side of the construction]

**Figure 1** First Attempt by Student 1



**Figure 2** First Attempt by Student 2



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doi:10.5951/MTLT.2023.0035

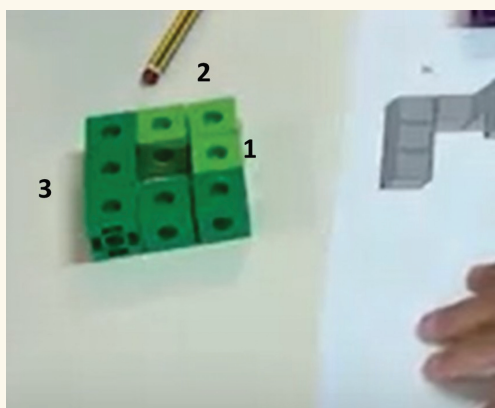
As they completed the shape, they pointed and named the number of cubes in each part: 3 - 2 - 1 - 2 (Figure 4). During whole-class discussion, some students recognized these same challenges. All of them used the size of the empty space as a spatial relationship.

Here, the teacher did not interfere, and the students were able to challenge each other to improve their shapes. Students saw different relationships, reflected on them, and discussed it. Reflecting and discussing helped students to improve their constructions.

**Figure 3** Second Attempt by Student 1



**Figure 4** Final Construction by Student 1



## BUILDING COORDINATED VERTICAL AND HORIZONTAL LAYERS

The second shape had two challenges: a corner where cubes intersect each other, and two opposite towers. Student 3's construction showed two incongruities: (1) both towers had the same height; and (2) a cube was missing between the corner and left side tower. While comparing 2D and 3D representations, Student 3 noticed the difference in the towers' heights and instantly removed one cube. However, it was particularly challenging for her to understand that a cube was missing (Figure 5).

To overcome this challenge, Student 3 organized the shape in composite, and the teacher asked her to connect each part in the depiction with the shape she had assembled.

*Teacher:* This one, tell me how did you do it? Which parts does this one have [2D picture] that this one also has [3D shape]? How are they equal?

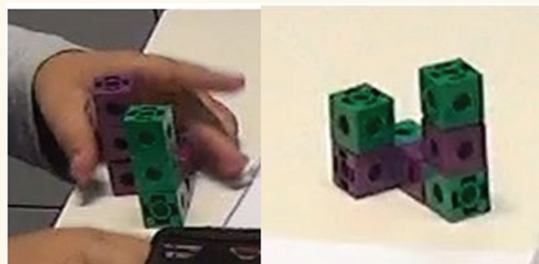
*Student 3:* These are here. These are here, which is this one and this one. [She moves between 2D and 3D to connect both representations.]

*Teacher:* Wait, which ones are these? [Points to the image.] These ones are these [Moves from 2D to 3D] What about these? [points to the cube Student 3 has missed, in the picture]

*Student 3:* Are these... [points to 3D]

*Teacher:* But you told me those were these... [moves from 3D to 2D]

**Figure 5** First and Second Attempts by Student 3





Along with questioning, Student 3 matched each composite in both 3D and 2D representations (Figure 6) and saw where the cube was missing. Then, she disassembled one part of the construction to include the cube.

### BUILDING UP MISSING INFORMATION

Mostly because of the hidden cubes lying below the second and third horizontal layers—but also because of the way the two sides intersect each other—the third shape was the most challenging. Furthermore, the spatial relationships that are visible in each form of representation might be interpreted differently.

In Figure 7, Student 1 based her construction on what she saw of the three layers, in a different arrangement. When she tried to relate both representations, she could not fill the information gap because she was unable to extract enough information to make both constructions coherent.

In Figure 8, Student 4 explained his process showing the first layer, from bottom to top, having the construction upside-down. After, he rotated the shape to its initial position and removed the cube on top to show the four cubes in the middle.

Student 4 said, “You put these on bottom, four here and this one on top.” He structured the shape using three layers, which is a form of structuring that underlies many mathematical structures. Student 5 also explained how he grasped the shape’s structure and explained some inferences he made: “First we

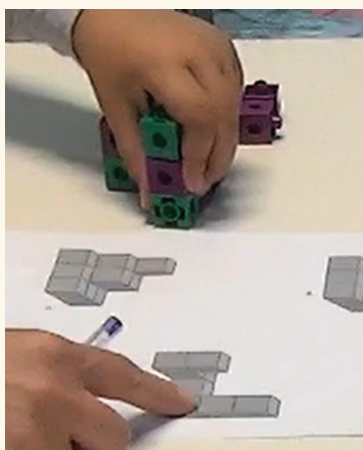
have placed these three and here two [bottom layer], but I have noticed that there must be more four to support these [middle layer]. After this, I made these four [middle layer].” Student 5 then removed the cube on top to show the cube under that one, in the middle layer. Students who found interpreting the picture challenging were able to build it during Student’s 5 explanation, as they saw the layers separated and what was hidden under.

### CONCLUSION

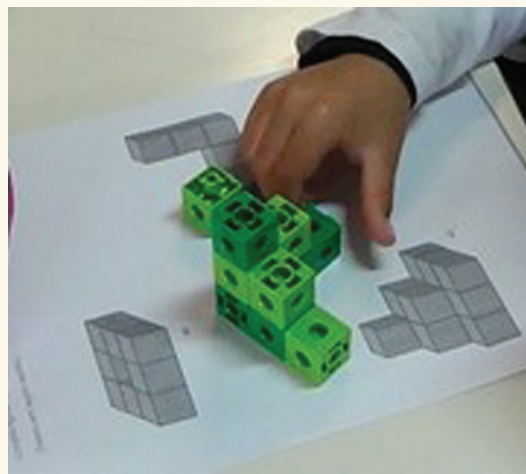
A task like this creates opportunities for children to engage in constructing a shape, improving the relationships they use to build it and refining the mental models they possess of it. To “read” a 2D picture of a 3D shape, children decompose the shape, mentally, and recompose it, physically, keeping in mind the congruency between shapes. This requires understanding part-whole relationships underlying each form of representation to connect both representations spatially. Hence, working with shapes must be intentional.

In the *Can You Build It?* task, students were able to explain and share their reasoning processes with their peers, helping those who were not able to interpret the 2D pictures. Student interactions must be stimulated so they feel comfortable to challenge, to question and be questioned, and to learn from each other. Teacher’s questions also help students to match 2D and 3D representations, promoting their reflection and their ability

**Figure 6** Student 3 Compares 3D and 2D Representations



**Figure 7** Construction of Shape C by Student 1



**Figure 8** Student 4 Explains His Process to Build Shape C



to improve the constructions by deepening their awareness of shape structures.

We agree with Matney et al. (2020) when they argue that students must represent information mentally to construct it physically. Students' mental

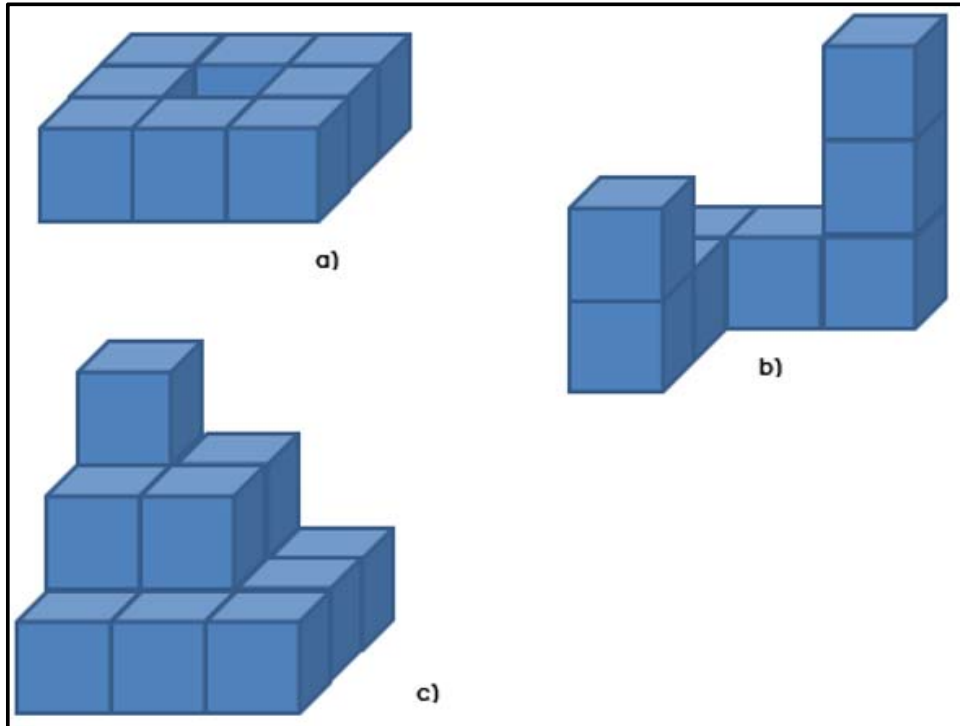
images become physical images, and through responding to questions and talking with peers, they are able to improve their 3D constructions, while also improving their spatial sense: something essential in geometry. —

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## Improve It! Task: Can You Build It?

Here are three images of 3D shapes.



Can you build these shapes using wood cubes?