



Research paper

Pantograph-catenary offline hardware-in-the-loop hybrid testing

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ABSTRACT

Hybrid testing boosts the development and analysis of complex mechanical systems by combining physical testing with a computer-modelled environment. In railway dynamics, Hardware-in-the-Loop (HiL) simulation is a hybrid testing method sought after to study complex interactions, particularly between the pantograph and the catenary. Existing pantograph-catenary HiL approaches require real-time catenary models which restrict fidelity and spatial resolution. This paper proposes an iterative offline HiL method that removes this constraint while maintaining hybrid testing capability. The proposed methodology is established through a set of test experiments, considering different pantograph uplift forces and the stagger of the overhead contact wire. The feasibility and convergence of the proposed method is demonstrated for a set of test cases. Advantages and disadvantages of the online and offline approaches are discussed, together with further developments that may be considered to improve them.

1. Introduction

The electrification of rail transport is a key step in addressing the modern mobility requirements, balancing the demand for increases speed with environmental responsibility. One critical aspect of this transport system is the pantograph-catenary interface. The pantograph, mounted on the roof of the railway vehicle, must maintain dependable contact with the overhead line to supply power to the train. Uninterrupted power supply to the traction motors is essential, contact losses not only compromise railway operation, but also increase risk of failures and maintenance costs. Hence the robust operation of the pantograph is essential to guarantee a smooth sliding contact with the overhead catenary, ensuring reliable operation.

The dynamic coupling between the railway pantograph and catenary is an active field of research [1]. Different research projects aim to achieve faster operating speeds and increase compatibility between different pantographs and catenary systems using innovative designs and numerical analysis methods [2,3]. The literature published on the subject is multidisciplinary, concerning topics such as, dynamic performance and contact quality [4], virtual homologation and interoperability [2,5] and condition monitoring [6–8] while spanning disciplines as varied as aerodynamics [9] and wear [10,11].

Although railway research continuously produces innovative solutions, the industry remains conservative in adopting them. Most

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railway acceptance procedures require costly line tests, which slow down the adoption of new solutions [2,5]. Consequently, there is a drive to reduce or remove these testing requirements, replacing them with more efficient, faster, and cheaper alternatives [12].

Different numerical tools have been developed to analyse the dynamic interaction between the pantograph and catenary, playing now a key role in supporting innovation. The employment of these tools is widely accepted by the industry and is established in standards and technical specifications [5], creating opportunities to reduce the extent of experimental testing required by relying more on validated simulation. Software packages, such as PantoCat [13], have the capability to assess new components prior to a physical prototype being produced using numerical models. Other software tools simulate the interaction of a pantograph with the surrounding air to understand the effect of pantograph operating height in the contact with the overhead wires [9].

When a prototype pantograph is available, full numerical tools can support testing only partially. In this case, the testing method must interface with the physical prototype to capture non-linear effects that numerical models cannot fully represent. Thus, Hardware-in-the-Loop (HiL) environments [14–16] are better suited for the later development stages. With the use of HiL, as a hybrid testing method, the modelling flexibility of numerical tools is extended to the testing of physical components through an experimental facility. In general terms, a physical component or mechanism replaces the associated numerical model on the virtual environment, improving simulation accuracy and providing assurance on the reliability of the component.

A usual pantograph-catenary Hardware-in-the-Loop implementation places a pantograph on a specialised test bench where it is subjected to realistic operational conditions. To represent the interaction of the pantograph with the catenary, actuators reproduce the relative movement of the overhead catenary contact wire as the vehicle runs through the railway track. The inputs for the actuators are produced by a numerical model of the catenary simulated in real time, based on the forces applied by the pantograph on the actuators. Other possible uses of HiL tests include testing different components in a pantograph model (dampers, suspensions, etc.), performing reliability tests under real operating conditions, or testing active systems to improve dynamic behaviour. So far, all test benches built for HiL testing are bespoke and have unique configurations and characteristics, which require a customised hybrid testing implementation. Several research institutions have published work on the development and use of their hybrid testing facilities, covering a wide variety of topics. Examples include the research groups at Southwest Jiaotong University [17,18], at Politecnico di Milano [19–21], at Technische Universität Wien [14,22,23], at the Railway Technical Research [24,25] and at the Universitat Politècnica de València [15,26–28]. In addition to this list, there is also the newest high-performance full-scale pantograph testing facility in the University of Huddersfield, known as PANTHER, where the present work was conducted.

Regarding the excitation of the pantograph head to reproduce the movement of the wire, most of the implementations published so far solely consider vertical movement of the wire, not reproducing the contact wire stagger when the train runs along the track and the overhead system. Exceptions to this feature include the facilities at POLIMI [21] and RTRI [29]. Of interest, the work published in [19, 21] shows that accurately representing stagger is essential, as confirmed by comparisons with numerical and line-test results. It is concluded that by performing the hybrid test only in the vertical direction, the contact force statistical parameters are underestimated when compared with pantographs tested in line. In some cases, [17], such as in PANTHER, the movement passed on from the train to the pantograph, through the pantograph base, can also be reproduced during testing. Nevertheless, this feature has not been fully explored, and according to [17,29,30], the influence of these inputs on the contact force is not significant in normal service, when compared to the excitation imposed on the pantograph head.

A crucial element on the implementation of a pantograph-catenary HiL simulation setup is the control architecture employed. Robust control methods improve the accuracy of testing by ensuring the actuator follows the wire motion and by reducing disturbances, modelling errors, and delays [31]. While the first pantograph hybrid simulations [17,18,32] put strong effort on achieving real-time performance due to the constrained computational capabilities, more recent implementations have looked into the effect of actuator dynamics, [25], and time delays [15,28] on the accuracy and stability of the testing. These latter applications use different approaches to deal with the problems affecting the control loop. In Kobayashi et al. [24], the dynamic substructure system (DSS) framework detailed in [33] is employed in pantograph-catenary testing to deal with instabilities originating from delays in the actuator response. The results published demonstrate good correspondence between fully numerical and DSS-supported hybrid testing at span and dropper passing frequencies, up to 20 Hz. In [28], a LQG controller is used to stabilise the system by minimising the difference between the calculated height of the contact point and the value imposed by the vertical actuator on the test bench. In [22,31], the addition of an interface control layer between the virtual environment and the test bench is developed. This layer can have different objectives to ensure the accuracy of the test results. While in [22], the interface controls the force and displacement signals exchanged between the virtual and physical environment to reduce errors in energy and momentum conservation; in [31] the interface goals are split in two tasks: a reference governor adjusts the command to the actuator for accurate reproduction of the contact point motion respecting the limitations of the testing hardware, while a time delay compensator is established to deal with known constant time delays on the control loop. The use of the interface method results in the accurate reproduction of the virtual dynamic characteristics even with time delays.

Another important research subject on pantograph hybrid testing is the numerical representation of the catenary dynamic behaviour. The implementation needs to achieve a balance between its accuracy and the computation effort required to achieve real-time performance, leading to simplifications in comparison to the models used in purely numerical simulations. The first real-time HiL catenary models used are linear and represent short sections of catenary, employing modal superposition to achieve real-time integration with 2-milliseconds timesteps, [17]. Several later implementations expand this methodology through the inclusion of the non-linear dynamics associated with dropper slackening and the use of the shifted back procedure [20] to extend the running length of the pantograph on the short virtual domain. Another methodology applied in [14,23] uses a short time-varying virtual domain that represents the catenary on the vicinity of the pantograph. To remove unwanted reflections from the domain boundaries, this method extends the computational domain with an absorbing boundary layer. This layer is created with the additional objective of reproducing

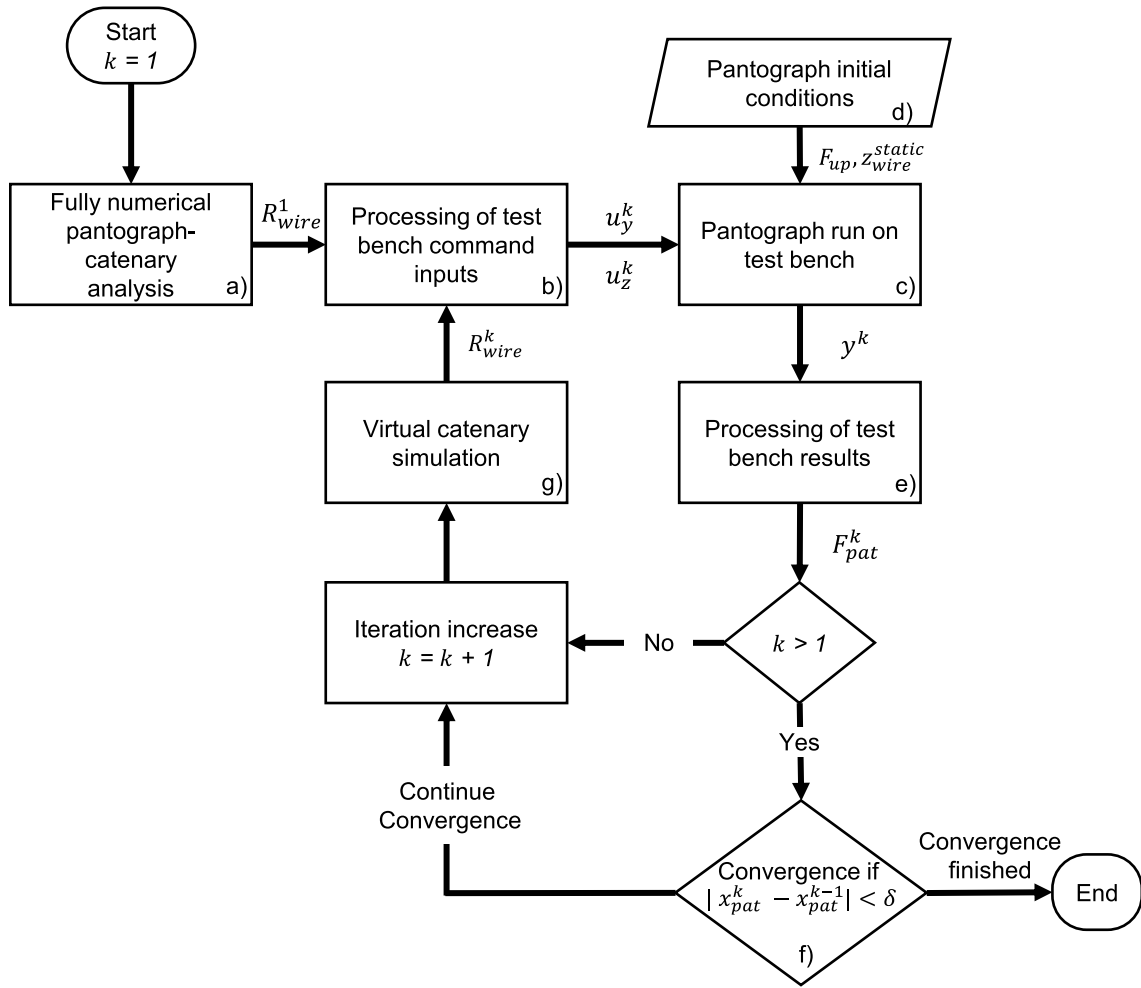


Fig. 1. Diagram of the offline iterative HiL workflow.

the wave propagation dynamics of the contact wire. On the same subject, Universitat Politècnica de València have achieved real time catenary formulations with several methods. The first methodology makes use of a formulation that relates the dynamic characteristics of contact and messenger wires with the analytical results for an infinite string supported by a visco-elastic layer that can be solved analytically [27]. The second method, [28], splits the computational effort of the catenary representation into off-line and on-line stages. The off-line stage is computed prior to the test and solves the bulk of the calculations required, with the results stored in memory. This procedure accelerates the on-line computational stage with fewer calculations to be performed in real time allowing the complete catenary model to be considered, including different spans and compression affecting droppers.

The large proportion of the literature here reviewed employs a full or partial real-time hybrid testing methodology. This means the numerical catenary model is partially or fully calculated in real time. This characteristic imposes limits on the complexity and size of the catenary model due to computing limitations. There is then a gap in capabilities for hybrid testing with large catenary models, especially when more complex case studies are necessary to be considered, such as larger models which can include track curvature and cant, large wire gradients, or additional discrete features [34].

In this paper, a novel iterative hybrid simulation framework is proposed, in which the catenary dynamic behaviour is calculated completely offline prior to a pantograph test bench run. Consequently, no computational restrictions are imposed on the catenary model. This approach is novel in the context of pantograph–catenary testing, despite sharing similarities with established approaches in structural engineering and hybrid simulation research. Recent developments in offline iterative hybrid simulation include methods that refine interface forces or states through iterative correction, improving accuracy in structural engineering testing at the cost of additional computation, [35]. Some concepts here discussed are linked with iterative control strategies for hybrid testing, where real-time feedback loops are complemented by offline iterative procedures to achieve smaller interface errors [36]. The proposed method can find connections with other dynamic sub-structuring methods, such as the real-time HiL, sharing common challenges such as actuator dynamics, delay compensation, and interface stability, [37]. Finally, the offline iterative HiL methodology fits with other offline co-simulation techniques [38]. In the way that physical pantograph testing and catenary simulations are alternated iteratively

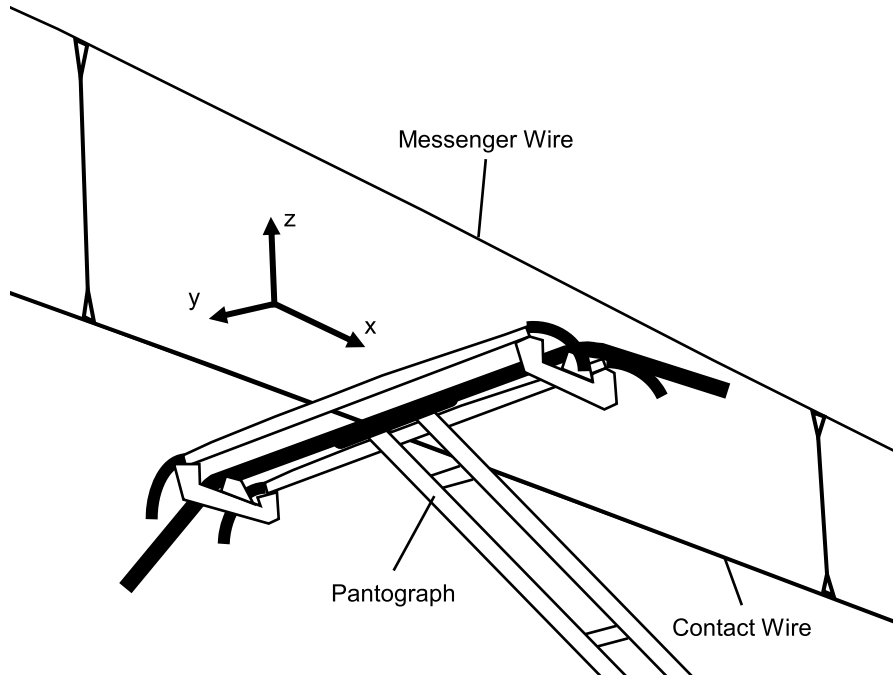


Fig. 2. Coordinate system for contact wire displacement.

until a convergent solution is reached.

In the field of pantograph–catenary testing, the proposed framework shares some similarities with the Hybrid System Response Convergence (HSRC) method presented in [39], although the latter is not fully detailed. A key difference lies in the reliance on a numerical pantograph model during the iterative process: while the HSRC-based method requires such a model, the method proposed in this work enables hybrid testing without any numerical surrogate after the initial estimation of pantograph–catenary dynamics. This distinction also affects convergence speed. Moreover, the proposed approach introduces a stabilisation and acceleration strategy based on the parameter α within the inter-iteration force correction formulation (see Section 2.1.3). As a result, fewer iterations are required to achieve convergence compared to [39]. A testing method similar to the HSRC is employed in [28] to validate online HiL test results, instead of replacing them. This leads to a smaller number of iterations required to achieve convergence in [28], however a real-time application is still required.

The hybrid testing framework proposed in this work contributes to the state of the art with a testing methodology that is independent of real-time catenary model evaluations, being the first step to perform pantograph hybrid testing on extended and more complex numerical catenary models without real-time-driven simplifications. The method also offers the added benefit of removing the time delays of the communication between the physical and virtual systems, while simplifying how the effects of actuator dynamics are compensated to maintain test accuracy. To demonstrate the method feasibility and convergence, it is implemented on a full-scale pantograph test bench at the Institute of Railway Research of the University of Huddersfield, United Kingdom. Different results are presented for cases considering a full-size Finite Element Method (FEM) catenary model, of around 1.5 km, built on a straight track. The iterative convergence procedure inherent to this framework is detailed, technical implementation steps for the experimental procedure defined, and the contact force results analysed.

The structure of this document is as follows, Sections 2 and 3 cover the offline HiL methodology and its application on the test facility. Section 4 discusses the results obtained on selected test cases. The conclusions are summarised in Section 5, along with considerations on the advantages and disadvantages of offline and online (real-time) HiL approaches and further improvements set for future development.

2. Offline HiL hybrid testing methodology

The iterative offline HiL methodology proposed in this work is conceptualised on the premise of eliminating real-time evaluations of the catenary model during testing. Setting this objective avoids restrictions on the complexity and size of the numerical models associated to real-time computation. Hence, the proposed method cycles between independent pantograph tests and catenary simulations. In each iteration, one subsystem is updated using the results of the other until the solution converges. The catenary dynamic behaviour is simulated through a numerical model, while the pantograph is tested on a test bench. In this section the iterative testing cycle is first introduced and detailed on the following sub-sections. A list with the symbols used in this manuscript is compiled in Appendix A.

2.1. Offline hybrid testing scheme

The proposed fixed-point iterative offline HiL testing sequence follows the diagram depicted in Fig. 1. The proposed scheme consists of four main parts detailed below:

2.1.1. Initialisation

In the first iteration ($k = 1$), a fully numerical pantograph–catenary dynamic simulation provides the initial estimate of the interaction, step *a*). It calculates the catenary contact wire coordinates where it interacts with the pantograph (R_{wire}^k), and the resulting contact force, F_{pat}^0 (1), as (1) and (2), respectively:

$$R_{\text{wire}}^k(t) = [x(t), y(t), z(t)]^T, \quad (1)$$

$$\begin{aligned} \begin{bmatrix} R_{\text{wire}}^1(t), F_{\text{pat}}^0(t) \end{bmatrix} &= G(M_{\text{cat}}^{\text{num}}, M_{\text{pat}}^{\text{num}}, s_{\text{pat}}) \\ \text{or} \\ \begin{bmatrix} R_{\text{wire}}^1(t), F_{\text{pat}}^0(t) \end{bmatrix} &= G(M_{\text{cat}}^{\text{num}}, M_{\text{pat}}^F, s_{\text{pat}}) \end{aligned} \quad (2)$$

where $G(\cdot)$ represents the numerical dynamic analysis of the associated inputs. The coordinates of $R_{\text{wire}}^k(t)$ are expressed with respect to the catenary model reference frame (x, y, z), depicted in Fig. 2, with respect to a given time step, t .

As the dynamic analysis is fully numerical in this step, the catenary is represented through its numerical model, $M_{\text{cat}}^{\text{num}}$. The pantograph representation in this initial step can be achieved through its numerical model, $M_{\text{pat}}^{\text{num}}$, or through a numeric fictitious constant traveling force applied on the contact wire, M_{pat}^F . The pantograph numerical model expedites the convergence process, being the option employed in this work. The pantograph position along track, s_{pat} , is the remaining input to the dynamic analysis. The dynamic analysis numerical solver employed in this work, in both steps a) and g), is detailed in Section 3.2.

2.1.2. Test execution

The R_{wire}^k is processed to produce the actuator inputs required for the pantograph test bench run u_z^k and u_y^k in step *b*). To simplify the process, the pantograph–wire contact is to a single point instead of one per strip. This is due to the pantograph lumped mass model employed in this work only having one top mass representing both contact strips. The process starts by decomposing R_{wire}^k into vertical and lateral components r_z^k and r_y^k and subtracting fixed offsets as in (3).

$$\begin{aligned} r_z^k(t) &= R_{\text{wire}}^k(t - \tau_z) \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - \hat{r}_z \\ r_y^k(t) &= R_{\text{wire}}^k(t - \tau_y) \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - \hat{r}_y \end{aligned} \quad (3)$$

In (3), the offsets $\hat{r}_z$ and $\hat{r}_y$ ensure that the average contact wire position matches the centre of the corresponding actuator stroke. These offsets should remain fixed throughout the iterations. τ_y, τ_z represent time offsets introduced to account for the dynamics of the actuator and are calculated each iteration. The calculation of τ_y and τ_z is described in Section 2.2. While on the vertical direction no additional processing is required, Eq. (4), in the lateral direction, the stagger representation requires smoothing.

$$u_z^k(t) = r_z^k(t) \quad (4)$$

The actual stagger profile produces sharp changes in lateral acceleration. An actuator with limited acceleration cannot replicate acceleration changes at higher equivalent train speeds. To represent the contact point horizontal movement while respecting the actuator capabilities, the smoothed input for actuator, $y_{\text{cmd act}}$, is calculated on each span, i.e. distance between two consecutive supports, according to Eqs. (5) and (6)

$$r_{yj}^{\text{sin}}(t) = -\frac{\pi}{2} + \frac{r_{yj}^k(t) - \min(r_{yj}^k(t))}{\max(r_{yj}^k(t)) - \min(r_{yj}^k(t))} \times \pi \quad (5)$$

$$u_y^k(t) = \frac{\max(r_{yj}^k(t)) + \min(r_{yj}^k(t))}{2} + \frac{\max(r_{yj}^k(t)) - \min(r_{yj}^k(t))}{2} \times \sin(r_{yj}^{\text{sin}}(t)) \quad (6)$$

where, the values of r_{yj}^{sin} correspond to a coordinate conversion from the value of the lateral stagger r_{yj}^k on span j to the range between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$. This conversion is done on each span, according to its maximum and minimum values of stagger, $\max(r_{yj}^k), \min(r_{yj}^k)$. An illustration of the smoothing process is depicted Fig. 3 where the displacement and acceleration of the stagger of the contact wire are

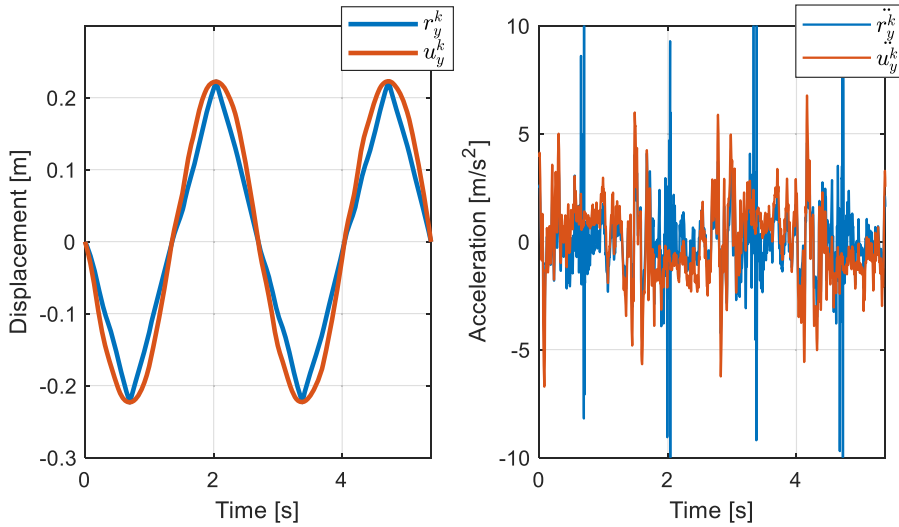


Fig. 3. Lateral movement of the contact wire at the position of contact with the pantograph; displacement (left) and acceleration (right).

presented for a speed of 100 mph (≈ 160 km/h). It is visible that the correction tracks well the lateral acceleration on the straight section of the stagger, reducing the peak acceleration at the direction inversion point from 130 m/s^2 , outside of the plot axis, to a value lower than 5 m/s^2 . The smoothed profile retains the movement amplitude; however, it does not reproduce the geometry exactly.

To evaluate the differences between the original and the smoothed stagger profile under load, the Root Mean Square Error (RMSE) and the maximum error between the original trajectory and the actual actuator trajectory are calculated. Considering the last iteration of the worst-case study case, as defined in Section 4.1, the RMSE for the actuator is of 33.0 mm, while the maximum difference is of 49.8 mm, close to the stagger position. While working under the physical limitations of the actuator performance a compromise is required, it may be possible to further develop this method in the future. Nevertheless, the smoothing preserves the main characteristics of the stagger (amplitude and spatial variation) and should be interpreted as an actuator-feasible approximation of the contact point motion. Considering that the vertical effects are dominant in pantograph-catenary interaction, the differences from the smoothing process are reasonable and this procedure is considered to remain the representative of the lateral actuation.

The actuator inputs are filtered using a low-pass filter with a 20 Hz cut-off frequency before the test to speed up convergence. The initial conditions for the pantograph test are also required for the test execution, step d). The pantograph uplift force, F_{up} , and the initial vertical position of the contact wire, z_{wire}^{static} must be fixed at the start of each test to ensure the convergence of the iterative process. F_{up} is proportional to the pneumatic pressure set on the pantograph raising actuator. However, due to non-linear hysteresis, F_{up} depends on the motion direction of the pantograph. Therefore, the preparation of the pantograph for testing should be the same in each iteration. z_{wire}^{static} is obtained from a static analysis of the catenary model with a point load of magnitude F_{up} applied at the pantograph starting position. The pantograph test run on the instrumented test bench, step c) is run with the inputs from step b) and parameters from step d). During the test, the test bench instrumentation records the contact force at the interface between the pantograph head and the actuators, Y^k .

2.1.3. Simulation update

The force signal recorded on the test bench, Y^k , is post-processed and synchronised, step e), to generate the input for the catenary dynamic simulation, F_{pat}^k . Y^k corresponds to the sum of the forces applied on each contact strip. The conditioning of Y^k reduces noise introduced by the test bench instrumentation. Y_f^k is the result of the filtering of Y^k with a low-pass filter of 20 Hz cut-off frequency, in line with the standards practiced on experimental testing of pantographs, [40,41]. The synchronisation with the simulation time is performed by shifting Y_f^k by a time offset, τ_F . τ_F represents the time shift required to align the actuator displacements with R_{wire}^k . In this work, τ_F is obtained by curve fitting and the synchronised contact force, F_{pat}^k , is calculated through:

$$F_{pat}^k(t) = F_{pat}^{k-1}(t) + \alpha \left(Y_f^k(t - \tau_F) - F_{pat}^{k-1}(t) \right) \quad (7)$$

where the parameter α is introduced to expedite the convergence process as well as prevent sudden large changes and over-corrections that disturb convergence between iterations, providing means to regulate the balance between stability and speed of the convergence process. The α parameter can be adjusted in the range $(0, 1]$ through sensitivity analysis, to fit the complexity of a specific case study. Choosing the largest possible value that still ensures stability allows the convergence process to take less iterations. For this work, preliminary values of the α parameter were considered and a constant α value of 0.25 was set. For $k = 1$, F_{pat}^{k-1} corresponds to F_{pat}^0 . With F_{pat}^k , the iteration counter is increased by one ($k = k + 1$) and the dynamic simulation is repeated (step g). In this case, contrary to step

Algorithm 1

- Offline Iterative HiL for Pantograph-Catenary Interaction.

Inputs: M_{cat}^{num} , s_{pat} , ROI, F_{up} , z_{wire}^{static} , M_{pat}^{num} or M_{pat}^F

Parameters: $a, f_c, \delta_x, \delta_z, \tau_y, \tau_z, \tau_F, \hat{\tau}_y, \hat{\tau}_z$

Outputs: $R_{wire}^*(t)$, $F_{pat}^*(t)$

Initialisation ($k \leftarrow 1$)

Step a)
Estimate pantograph-catenary interaction, extract $R_{wire}^k(t)$ from Eq. (1)

Test execution

Step b)
Calculate τ_z and τ_y to compensate actuator dynamics
Decouple lateral and vertical wire movement by Eq. (3)
Define vertical actuator input, u_z^k , Eq. (4)
Calculate, for each span, the smoothed lateral inputs, u_F^k , by Eqs. (5) and (6)
Filter $u_z^k(t)$ and $u_F^k(t)$ with a low-pass filter with cut-off f_c

Step d)
Apply F_{up} to the pantograph, opened to z_{wire}^{static} height.
%% Prepare pantograph the same way each iteration to avoid hysteresis bias.

Step c)
Bench test with commands $u_z^k(t)$ and $u_F^k(t)$; measure $y^k(t)$

Simulation update

Step e)
Filter $y^k(t)$ with a low-pass filter with cut-off f_c . $\rightarrow y_f^k(t)$
Calculate τ_F to align $y_f^k(t)$ with simulation time
Synchronise and calculate simulation correction by Eq. (7) %% where for $k = 1$, $F_{pat}^{k-1}(t) \leftarrow F_{pat}^0(t)$

$k \leftarrow k + 1$, iteration counter increase

Step g)
Run Updated simulation using Eq. (8), extract $R_{wire}^k(t)$ and go to **Test execution**, Step b)

Convergence check ($k \geq 2$)

Step f)
Calculate X_{pat}^k and X_{pat}^{k-1} by Eq. (10) and Z_{cat}^k and Z_{cat}^{k-1} by Eq. (11)
If $|X_{pat}^k - X_{pat}^{k-1}| < \delta_x \cap \max(|Z_{cat}^k - Z_{cat}^{k-1}|) < \delta_z$ Eq. (9)
STOP
Else
Continue.
end

Termination
Return $R_{wire}^*(t)$, $F_{pat}^*(t)$

a), the dynamic analysis is hybrid, as the catenary is represented by the same numerical model, M_{cat}^{num} , while the pantograph representation is achieved through the experimental measurements, F_{pat}^k :

$$R_{wire}^k(t) = G\left(M_{cat}^{num}, F_{pat}^k(t), s_{pat}\right) \quad (8)$$

The dynamic simulation solver where $G(\cdot)$ is conducted is the same as in step a). The output R_{wire}^k is fed back to step b) closing the iterative loop.

2.1.4. Convergence check and termination

For iterations after the first, $k \geq 2$, the convergence check, step f), is introduced between steps e) and g). A criterion is defined to detect when a converged solution is found. In this work, the criterion is based on the difference in standard deviation of the contact force filtered at 20 Hz of consecutive iterations and the maximum contact wire vertical displacement difference between consecutive iterations, as in:

$$\left| X_{pat}^k - X_{pat}^{k-1} \right| < \delta_x \cap \max(|Z_{cat}^k - Z_{cat}^{k-1}|) < \delta_z \quad (9)$$

where, X_{pat}^k corresponds to the standard deviation calculated as:

$$X_{pat}^k = \sigma\left(F_{pat}^k(t)\right), \text{ with } t \text{ such that } s_{pat}(t) \in \text{ROI} \quad (10)$$

where ROI is a predefined region of interest, following established standards for pantograph–catenary interaction, [40]. Similarly, Z_{cat}^k corresponds to the contact wire displacement calculated as:

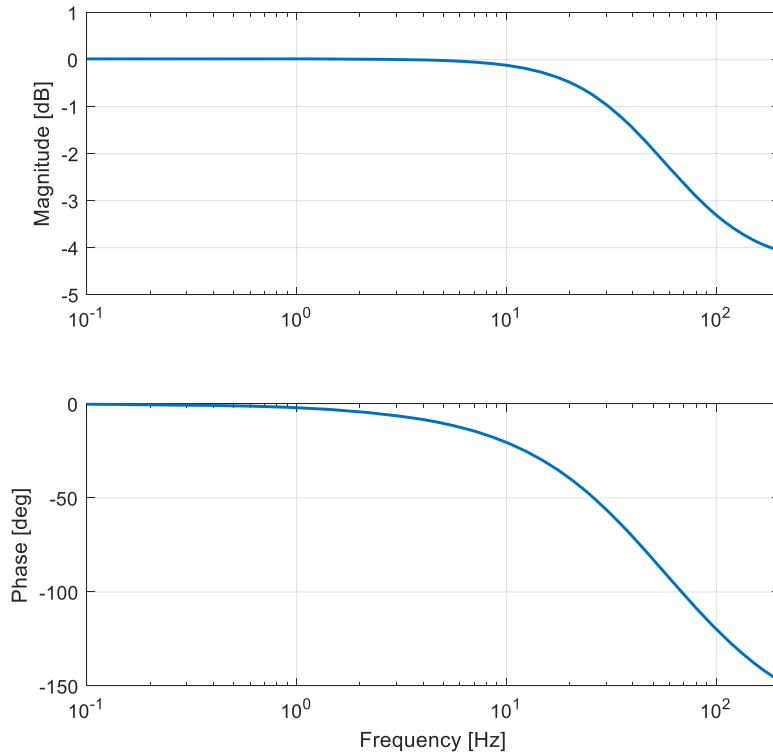


Fig. 4. Magnitude and phase diagrams of the actuator of the test bench.

$$Z_{cat}^k = R_{wire}^k(t) \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \text{ with } t \text{ such that } s_{pat}(t) \in \text{ROI} \quad (11)$$

δ_x and δ_z are the thresholds used to determine when convergence has been achieved. The statistical-based convergence metric is preferred over comparing time histories of force signals, as the convergence of the latter is harder to track. The complement with the tracking of the catenary maximum displacement differences ensures the wire displacement has converged as well. In this work, the values of the thresholds have been defined as $\delta_x = 0.03 \text{ N}$ and $\delta_z = 1e - 3 \text{ m}$.

If convergence is not achieved, the cycle continues. If the convergence criterion is satisfied the signals F_{pat}^* and R_{wire}^* are taken as the converged contact force history and contact wire position.

2.1.5. Offline iterative HiL

The algorithm of the offline iterative process is given in [Algorithm 1](#).

2.2. Compensation of actuator dynamics

In [Eq. \(3\)](#) of step b), τ_y, τ_z represent time offsets introduced in the actuator commands to account for the dynamics of the actuator. These time offsets are calculated at each iteration, synchronising the actuators with the numerical simulation. Their definition requires a dynamic model of the actuator to be available, which can be obtained from either time-domain data, e.g. step response, or frequency-domain tests, e.g. frequency response function. In this work, an empty run with the uncompensated actuator setpoints, i.e. $\tau_y = \tau_z = 0$, is used to determine the actuator dynamic model from the transfer functions relating the commanded inputs to the actuator $In_{act}(s)$, with the actuator response $Out_{act}(s)$, as in [Eq. \(12\)](#).

$$H_{act}(s) = \frac{Out_{act}(s)}{In_{act}(s)} \quad (12)$$

The Bode plot for the hydraulic actuator of the test bench is plotted in [Fig. 4](#).

This actuator bandwidth is 80 Hz, as indicated by the frequency at which the amplitude response reaches -3 dB , while the magnitude at 20 Hz is around -0.4875 dB . The dropper-passing frequency for the study case is around 4.5 Hz, as defined in [Section 3](#). This means that the main frequencies of the catenary dynamic are within the actuator response with magnitude close to 0 dB, or that the actuator does not significantly filter these frequencies. On the present actuator, the relationship $\frac{Out_{act}(s)}{In_{act}(s)}$ can be reduced to a phase /



Fig. 5. Panther, full-scale pantograph test facility.

time delay up to 20 Hz with a maximum magnitude filtering of 5.5%, because the actuator does not significantly attenuate the signal magnitude and the phase lag increases linearly on this frequency range. τ_y, τ_z are the average time-delay imposed by the actuator dynamics on the uncompensated setpoints, which is determined by line fitting in this work. Shifting the inputs in time by these offsets, as indicated in Eq. (3), synchronises the actuators with the numerical simulation time. This corrective action requires the virtual catenary outputs to be slightly longer than the pantograph run, by at least the longest time delay between the two identified, τ_y, τ_z .

To evaluate the proposed compensation method under loading, the RMSE and the maximum error between the desired actuator trajectory, and the actual actuator trajectory are presented. Considering the last iteration of the worst-case study case, as defined in Section 4.1, the RMSE for the vertical actuator is of 0.16 mm, while the maximum difference between r^* and u^* is of 1.35 mm. Following on these results, the compensation procedure is considered appropriate for the case studies and actuator considered, although more complex actuation arrangements may require more complex compensation processes to achieve similar outcomes.

2.3. Convergence analysis

The proposed iterative HiL coupling can be analysed as a fixed-point iteration problem Eq. (13)

$$u_{k+1} = T(u_k) \quad (13)$$

where $T(\cdot)$ represents the combined actions of the physical pantograph experiment, numerical catenary simulation, and force correction. According to the Banach Fixed-Point Theorem, convergence is then guaranteed if $T(\cdot)$ is a contraction mapping i.e. if a Lipschitz constant $0 \leq L < 1$ exists such that:

$$|T(a) - T(b)| < L |a - b|, \text{ for any } a, b \quad (14)$$

This theorem ensures both existence and uniqueness of the converged solution. It also provides an upper bound for the convergence rate, with smaller values of L implying faster correction between iterations. Linearising the pantograph and catenary dynamics yields

$$T \approx K_{pat} / K_{cat} \quad (15)$$

where K_{pat} and K_{cat} denote an equivalent pantograph and catenary vertical stiffness. Thus, convergence depends on the pantograph-to-catenary stiffness ratio. The convergence of the proposed method is analysed empirically in Section 4.1 for all the cases tested.

3. Application of the offline hil hybrid method

The iterative offline HiL methodology proposed in Section 2 has been applied to the PANTHER pantograph test facility at the University of Huddersfield, coupled with the pantograph-catenary numerical analysis software, *PantoCat*. A specific pantograph, the Brecknell Willis HSX, was employed on the four case studies considered. This section describes these elements in the context of their application within the proposed methodology.

3.1. Pantograph test bench

The Pantograph Huddersfield Experimental Rig, known as PANTHER, is the full-scale pantograph testing facility presented in Fig. 5.

PANTHER has a comprehensive set of actuators and features, some of which are not employed in this work and are therefore not

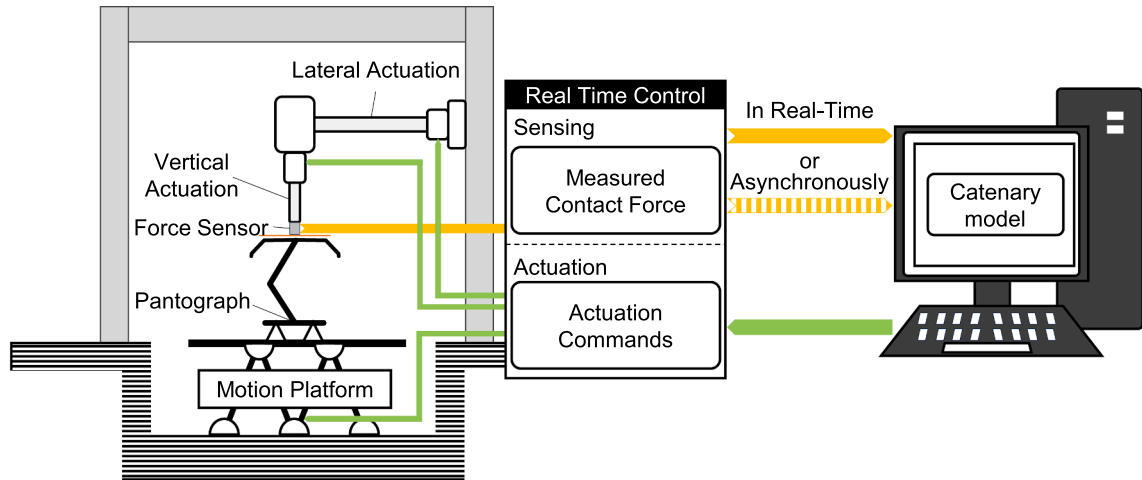


Fig. 6. Schematic of pantograph test bench operation with virtual catenary.

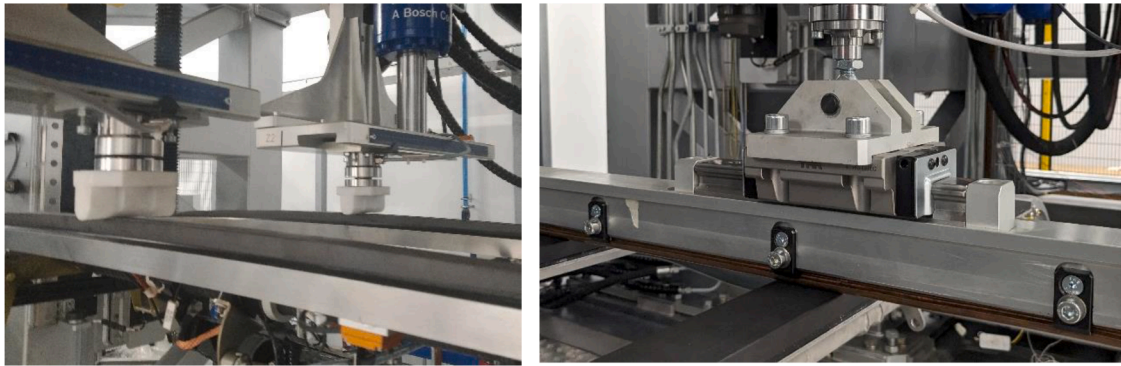


Fig. 7. Assembly with different attachments for actuator / pantograph head interface. Left: PTFE patches attachment, right: contact wire attachment.

highlighted here for simplicity. A simplified diagram of PANTHER running an offline/online test is depicted in Fig. 6.

A pantograph installed in PANTHER interacts with the test bench through three actuation groups: the motion platform at the base, the vertical head actuation, and the lateral head actuation. Each of these actuation groups is summarised below.

The lateral and vertical actuation at the pantograph head are each decoupled into two linear actuator stages: one hydraulic stage aimed at reproducing short and fast movements, and one electric tasked with the larger displacements. In this work, only the hydraulic stage in each direction, vertical and lateral, is required to reproduce the wire movement on straight track. In this configuration the test bench can handle accelerations up to 4 g in the vertical direction and 2.7 g in the lateral direction. Future work may split vertical motion into two stages: hydraulic for fast movements and electric for large-amplitude gradients, such as in level crossings and overbridges.

The vertical hydraulic stage is composed of two independent actuators, each instrumented with a load cell to measure the contact force between the pantograph and the actuator, and a vertical accelerometer to provide inertial compensation of the load cell readings. The interface of the actuators with the pantograph head is achieved through one of the two attachments depicted in Fig. 7. In this work, white PTFE contact patches are used as these not only have lower inertia but also serve as sacrificial wear elements, preserving the surface of the pantograph contact strips.

A six-degrees-of-freedom hydraulic motion platform can simulate train-roof motion and vibration. In this work the motion platform is only used as a flexible floor mounting plate where the pantograph is installed to achieve the correct opening range set to be tested.

The bench real-time controller, depicted in Fig. 6, controls all the actuators and ensures the data logging of all sensors. The test bench controller can either connect to a real-time catenary model through an external real-time machine, enabling a fully online HiL setup, or be asynchronously fed with pre-computed actuator setpoints, as used in this work.

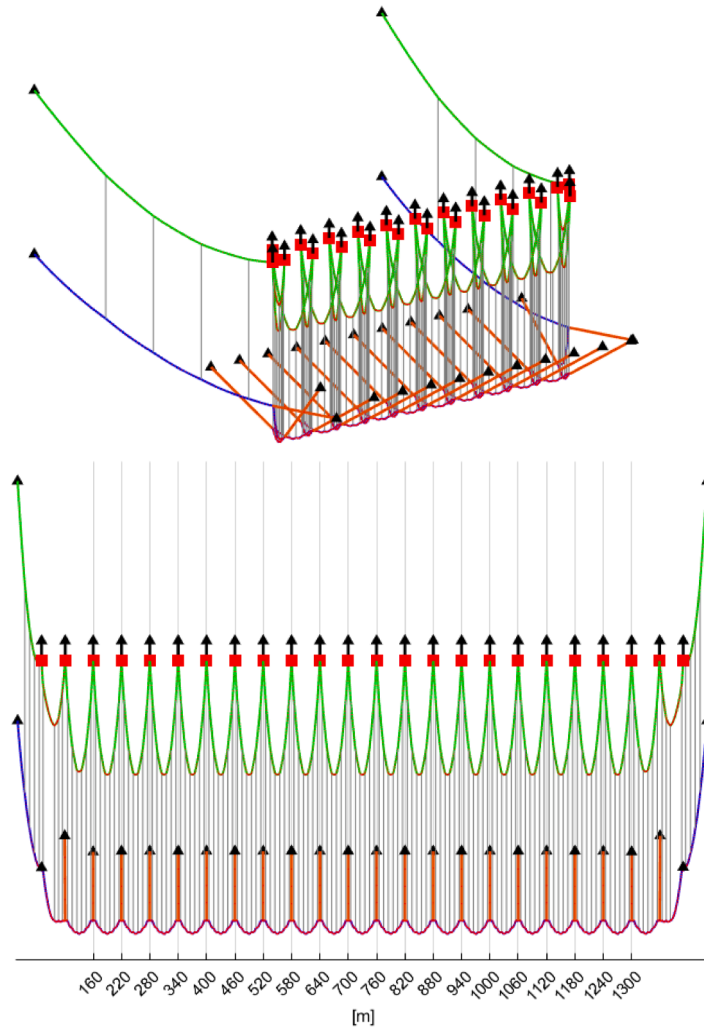


Fig. 8. FE mesh of the catenary model. 3D perspective view (top) and side view (bottom).

3.2. Catenary numerical model environment

An independent numerical tool capable of performing pantograph-catenary or contact-force-catenary simulations is a critical part of the offline iterative methodology here proposed (steps *a.* and *g.* in Fig. 1). In this work, PantoCat, a EN50318 [5] certified pantograph-catenary dynamic analysis tool, [42], was employed to perform these simulations [13,43].

The catenary model is built using a three-dimensional Finite Element (FE) formulation, [44,45], where the contact and messenger wires and the steady arms are represented by beam elements. Droppers are modelled as bar elements under tension and their compressed state eliminated when the dropper is slacked. Lumped masses are included in the model representing clamping elements between droppers, wires and steady arms. The messenger wire supports are reproduced by spring-damper suspension elements. Both ends of the contact and messenger wire, and the end of the steady arms not attached to the contact wire, are set with a fixed point constraint [13,34,46]. This ensures an accurate catenary geometry and correct wire motion relative to the track. This numerical representation is important for subsequently converting the simulated wire position into actuator displacement commands for the test bench. Since the offline HiL methodology relies on the numerical catenary model to represent the infrastructure subsystem, any changes in catenary tension, stiffness distribution or system configuration are directly reflected in the calculated contact-wire response. Therefore, different overhead line designs may be considered without modification of the iterative methodology itself.

In the first iteration of the cycle, the catenary model is coupled with a fully numerical pantograph model, using a penalty contact formulation to represent the pantograph-catenary interaction force [13]. On subsequent iterations, the contact force derived from the values recorded during the pantograph test bench run, F_{pat}^i , is applied on the catenary model as a moving contact force applied along the catenary contact wire. The longitudinal position of the pantograph along the catenary remains the same throughout the iterations, so that only the magnitude of the force changes from one iteration to the next.

To illustrate the proposed methodology, a case study was defined considering a simple type of catenary design, modelled in

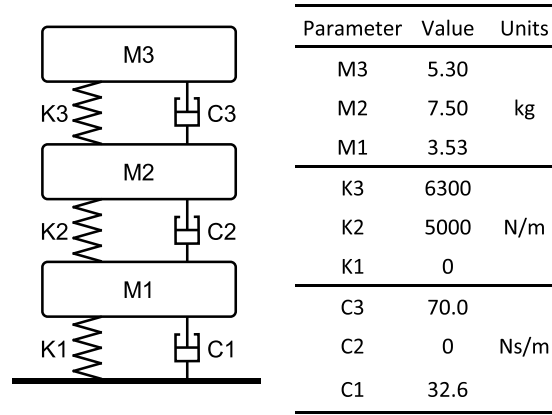


Fig. 9. Lumped-mass model of the HSX pantograph.

PantoCat [13]. The model is built according to Network Rail Master Series design (UKMS100), used in the United Kingdom for speeds of up to 100 mph [47]. The catenary FE model is represented in Fig. 8. The main parameters of the catenary components, geometric and modelling parameters are provided in Appendix B.

3.3. Pantograph under test

The tests in this work employed a Brecknell Willis HSX pantograph, installed on the test bench. This pantograph is commonly used in the United Kingdom railway network and is able to reach operational speeds of up to 125 mph (≈ 200 km/h). The pantograph is raised via a pneumatic actuator placed at its base. The pneumatic system can supply the actuator with two distinct working pressure settings, selected according to specific operating conditions. Each pressure setting is controlled by an adjustable pneumatic regulator, typically configured to provide mean uplift forces of 70 N and 90 N. In this work, both pressure configurations are tested using the iterative offline HiL and its results are compared in Section 4.2.

The offline iterative approach here employed requires not only the physical pantograph but also a numerical model of itself to start the first iteration (step *a*) in Fig. 1). The fidelity of this numerical representation affects the number of iterations required for convergence. Pantograph–catenary dynamic analysis is commonly performed through one of two types of pantograph numerical models, the lumped mass (LM) model or the Multibody (MB) model [9,43]. Despite both modelling approaches having different advantages and disadvantages, both options represent the pantograph operation with simplifications and assumptions, while the full dynamic behaviour is only available in the physical pantograph.

In this work, a three-stage LM pantograph model is used as a starting point for the offline HiL testing, as detailed in Fig. 9 [48]. The model parameters have been identified by matching the model dynamic transfer function with one obtained by testing on a physical pantograph, following standardized procedures [5]. Because this model uses a single top mass representing the motion of both contact strips, in this work, both vertical actuators follow the wire movement computed by the dynamic-analysis software as if acting on a single contact strip.

4. Test results

To analyse the convergence of the proposed methodology and its application on a test bench, a set of test cases was selected. These cases are intended to demonstrate the feasibility and convergence of the methodology under the conditions considered, rather than its performance for all pantograph–catenary systems. Four hybrid tests were performed, considering pantograph static uplift forces of 70 N and 90 N. For each uplift configuration, tests were carried out both with and without the use of the lateral actuator to reproduce the contact wire stagger.

As described in Section 3.3, the pantograph utilized in this test is the HSX. The catenary model considered is detailed in Section 3.2, concerning a UKMS100 type catenary system set in straight track. In all test cases, the train speed considered is 100 mph, which is the maximum rated speed for the catenary. The results obtained are analysed over a region of interest encompassing 5 spans located at the middle of the catenary, from 580 m to 880 m. A 20 Hz low-pass filter is applied to obtain the statistical contact force results presented in this work, in accordance with the applicable standard EN50367, [40]. The following subsections analyse the results from perspectives relevant to the proposed methodology.

4.1. Convergence process

To analyse convergence of the proposed methodology, the test case considering a pantograph with a 70 N static uplift force with lateral actuator movement is here analysed. The remaining cases have presented similar convergence behaviour. However, this case presents the least favourable condition for achieving good contact quality. The test case including the stagger movement is additionally

Table 1

Comparison of the statistical parameters of the contact force and maximum correction factors for each offline HiL iterations for the case with 70 N and stagger.

Stage	Contact Force [N]					σ/F_m	Max F diff [N]	Max cat diff [mm]
	$F_{\max}$	$F_{\min}$	Amp	F_m	σ			
LM	102.60	32.23	70.37	63.21	13.20	0.209	-	-
It 1	98.39	33.75	64.64	63.08	13.82	0.219	-	-
It 2	95.94	32.77	63.17	63.14	13.35	0.211	6.68	2.43
It 3	98.75	33.11	65.64	63.10	13.41	0.212	5.67	2.13
It 4	98.70	32.68	66.02	63.32	14.00	0.221	4.38	1.75
It 5	99.51	31.02	68.49	63.29	14.50	0.229	3.32	1.35
It 6	99.55	29.15	70.40	63.36	14.74	0.233	2.34	1.22
It 7	102.37	28.76	73.61	63.18	14.83	0.235	2.53	0.97
It 8	102.55	28.48	74.07	63.27	14.64	0.231	2.21	0.86
It 9	102.96	28.99	73.97	63.37	14.62	0.231	1.79	1.04
It 10	101.05	27.78	73.27	63.62	14.57	0.229	1.95	1.22
It 11	99.86	27.23	72.63	63.35	14.74	0.233	2.13	1.25
It 12	103.75	28.47	75.28	63.40	14.82	0.234	2.11	0.81
It 13	108.19	28.17	80.02	63.32	14.87	0.235	1.70	0.88
It 14	104.41	27.48	76.93	63.43	15.04	0.237	2.01	1.01
It 15	104.00	28.00	76.00	63.48	15.13	0.238	2.17	0.91
It 16	106.49	28.49	78.00	63.54	15.00	0.236	1.89	0.93
It 17	106.21	27.26	78.95	63.52	14.99	0.236	2.07	0.96

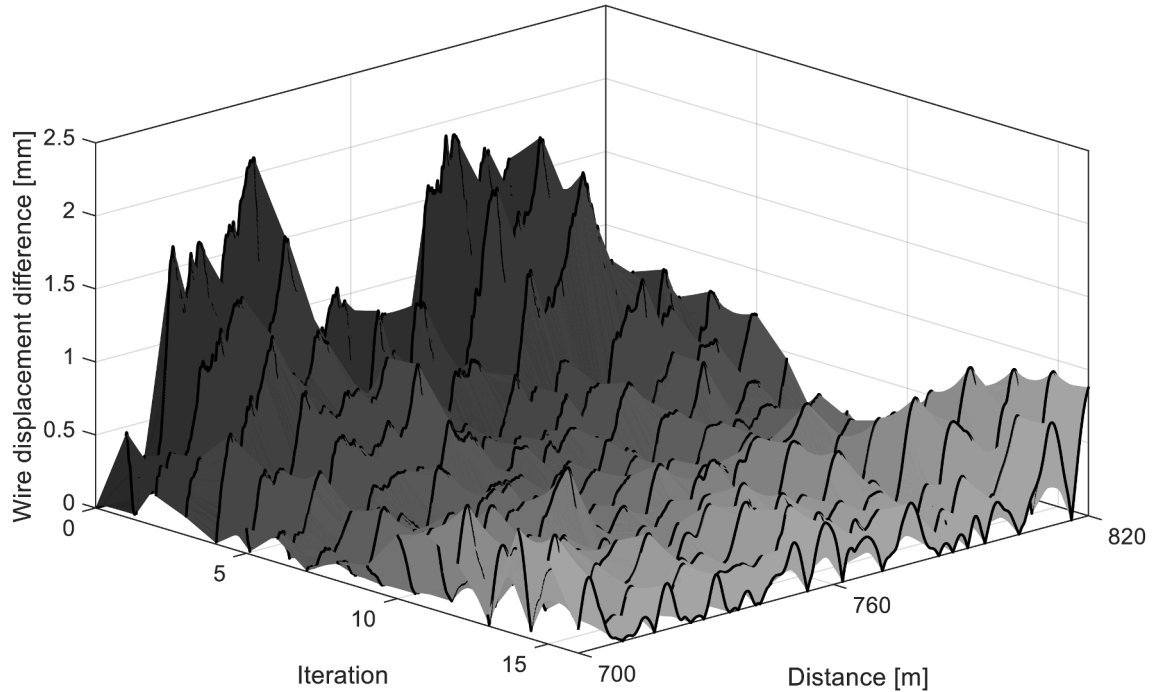


Fig. 10. Magnitude of the catenary displacement corrections between consecutive iterations for the iterative offline HiL on the pantograph with 70 N uplift and stagger.

regarded as the most demanding scenario, as it requires synchronisation of the actuation movement in more than one direction.

Table 1 compiles the statistical parameters of the contact force and the indicative convergence values for the initial numerical simulation and subsequent offline HiL iteration. On the table, the maximum and minimum forces, amplitude of the force, mean force, standard deviation, and the standard-deviation-to-mean ratio are displayed in the columns. The last two columns indicate the maximum difference in force and catenary displacement between sequential iterations, at any given location inside the selected zone of interest. The first column (*Stage*) identifies the testing stage: “LM” regards the contact force evaluated on the full numerical simulation using a pantograph lumped mass model, concerning the starting step in the first iteration; “It k ” concerns the signal F_{pat}^k of iteration k , i. e. contact force obtained from the test bench run after filtering and processing.

Table 1 shows that the initial differences decrease in magnitude as the number of iterations increases. Iteration-to-iteration

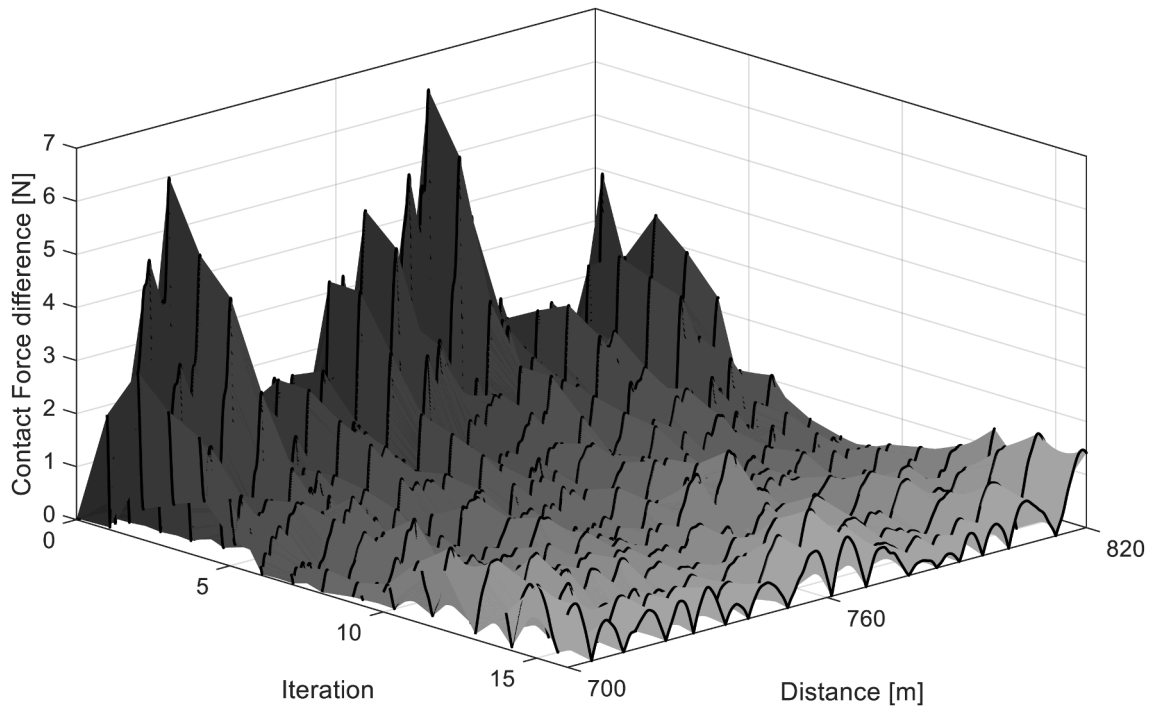


Fig. 11. Magnitude of the contact force change between consecutive iterations.

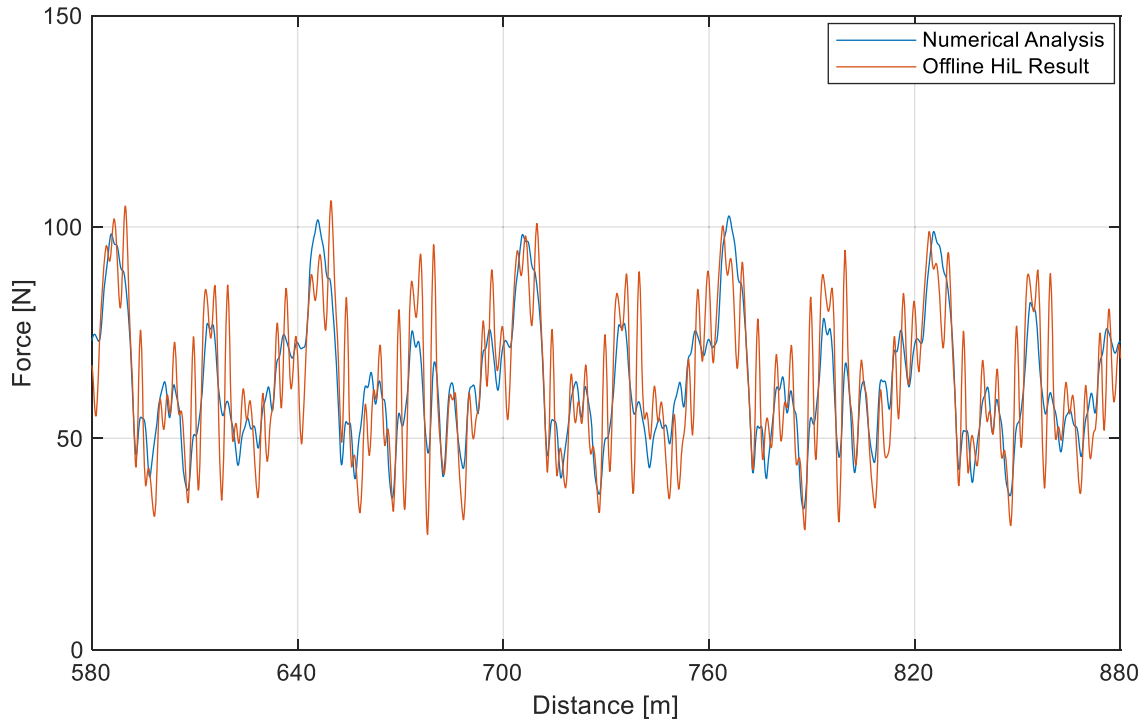


Fig. 12. Comparison between contact force of numerical analysis with LM model and offline HiL results for the 70 N and stagger case.

differences are larger at the start and decrease towards the final cycles. In the last two columns, the maximum value differences stabilise at approximately 2 N of force and <1 mm of displacement.

In the final iteration, the contact force statistical parameters present higher maximum, amplitude, and standard deviation values, as

Table 2

Summary of the convergence of the statistical parameters of all test cases.

Test Case	Contact Force [N]					σ/F_m	Max cat diff [mm]	Cycles for convergence
	F_{max}	F_{min}	Amp	F_m	σ			
With Stagger - 70 N								
It 16	106.49	28.49	78.00	63.54	15.00	0.236	0.93	17
It 17	106.21	27.26	78.95	63.52	14.99	0.236	0.96	
With Stagger - 90 N								
It 14	122.02	33.44	88.58	82.41	16.00	0.194	0.89	15
It 15	121.16	33.75	87.41	82.45	15.99	0.194	0.72	
No Stagger - 70 N								
It 12	104.45	32.08	72.37	64.13	14.38	0.224	0.64	13
It 13	104.37	31.48	72.89	63.96	14.35	0.224	0.55	
No Stagger - 90 N								
It 14	120.12	38.52	81.60	81.29	15.90	0.196	0.30	15
It 15	120.40	37.91	82.50	81.38	15.93	0.196	0.30	

Table 3

Statistical parameters of the contact force of 70 N and 90 N uplift force configuration with stagger.

Test Case	Contact Force [N]					σ/F_m
	F_{max}	F_{min}	Amp	F_m	σ	
70 N	106.21	27.26	78.95	63.52	14.99	0.24
90 N	121.16	33.75	87.41	82.45	15.99	0.19

well as a larger standard-deviation-to-mean ratio, compared with the lumped-mass model results. This behaviour is consistent with the corresponding reduction in minimum contact force. No contact losses are observed at any point during the testing.

Fig. 10 presents the absolute value of the vertical corrections of the wire displacement between consecutive iterations over time, for the case considered. The plot encompasses two spans, in the middle of the ROI [700,820] m, instead of the full ROI to improve readability of the three-dimensional surface.

The decay in the magnitude of the initial corrections as the number of iterations increases can be clearly observed. While the initial magnitude corrections can be higher than two millimetres, on the last iterations of the test the wire displacement corrections are around half a millimetre, except on localised peaks. The corrections are not of uniform magnitude across the complete model, with some areas requiring more correction than others. These results indicate convergence for the considered case.

The same behaviour seen in Fig. 10 is visible in Fig. 11, which shows the absolute value of the contact force differences between consecutive iterations. Fig. 11 uses the same Distance range as Fig. 10 to improve readability of the three-dimensional surface.

In this figure, the initial corrections reach values higher than 6 N, decreasing the last iterations to magnitudes of 2 N or lower, as shown in Table 1.

Fig. 12 presents the contact force obtained from the full numerical simulation with the lumped mass pantograph model, alongside with the final iteration results of the iterative offline HiL test as a function of the pantograph position. The tick marks on the horizontal axis identify the position of the catenary supports.

The comparison shows that the pantograph exhibits slightly more oscillatory behaviour than predicted by the LM pantograph model. However, the overall force peaks and valleys of the force profile are already present on the numerical simulation, with a force peak right after the catenary support and another after the middle of the span with lower amplitude. This is a result of using a good pantograph model to do the initial step of the cycle.

A summary of the convergence obtained for all the cases is available on Table 2. On each case, the statistical parameters, the maximum catenary displacement difference for the last two iterations, and the number of iterations required for convergence are presented. No contact losses were recorded in any of the cases.

It can be observed that the case with 70 N, considering wire stagger, is the one which required more iterations to achieve a convergent state. Based on the analysis from Section 2.3, results obtained in Table 2 indicate that $T(\cdot)$ is a contraction mapping for the considered scenarios. For the most demanding scenario, 70 N uplift with stagger, the corrections reduce from values exceeding 6 N (force) and 2 mm (displacement) to below 2 N and 1 mm respectively in the final iterations, stabilising after approximately 15–17 iterations. The remaining cases (e.g. higher uplift or no stagger) converge even faster, typically within 13–15 iterations. The values on the last two iterations of the test are similar across Table 2, confirming stable convergence of the offline HiL methodology.

Further comparison between cases is provided in the next sections, where only the final results are considered.

4.2. Effect of pantograph uplift

The pantograph employed in the tests is capable of operating under two static uplift force settings, one set to 70 N and the other to 90 N. Hence, both conditions were tested and analysed. Table 3 presents the contact force statistical parameters obtained for each case considering lateral wire movement, or stagger.

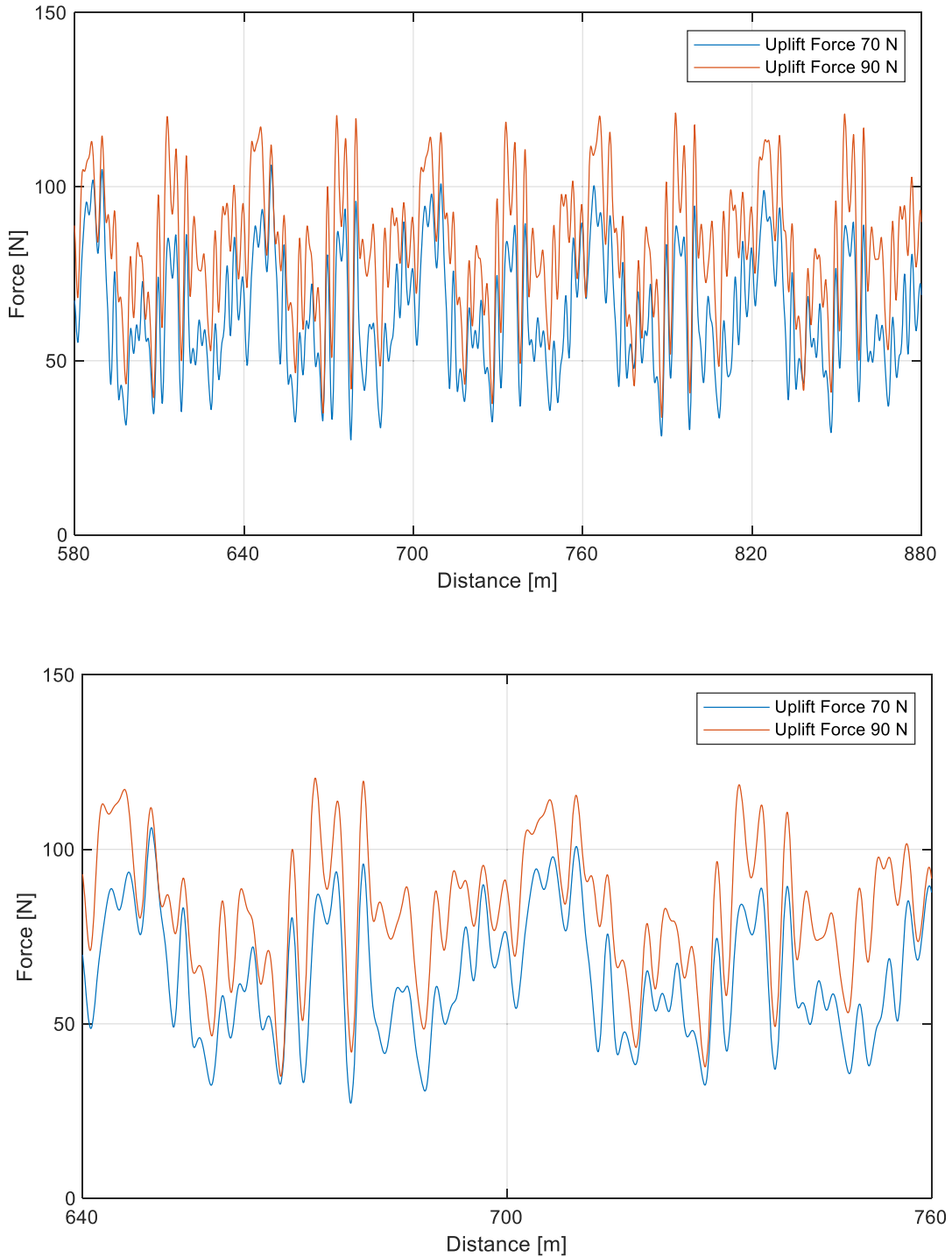


Fig. 13. Contact force of 70 N and 90 N test run with stagger (top) with close up on two spans (bottom).

The 90 N case displays higher contact force maximum, minimum, amplitude, mean and standard deviation than the 70 N case. However, the ratio between standard deviation and mean force, σ/F_m , decreases, indicating that although the standard deviation is higher in the 90 N, the contact force oscillation around the mean contact force is lower. It can be observed that the F_m values are lower than the static uplift force for which the pantograph was set in each case. To understand this behaviour it is important to note that the pantograph static uplift force was not calibrated to achieve a specific target mean force. Instead, the pantograph was tuned independently following the procedure set out in EN 50206-1 [49] and in the manufacturer manual to determine its static uplift force. In

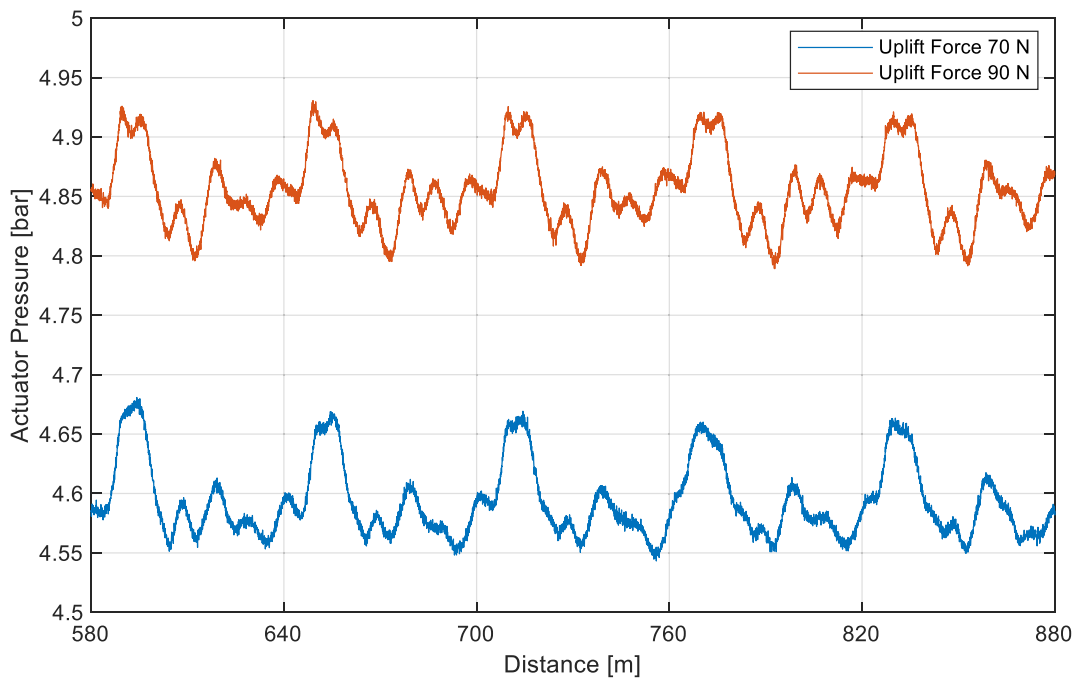


Fig. 14. Actuator pressure of the pantograph in 70 N and 90 N uplift force configuration.

this procedure, the uplift force corresponds to the mean value measured across the full operating range of the pantograph, during both extension and retraction.

The contact force along the pantograph position is presented in Fig. 13, for both the 70 and 90 N cases. The top plot presents the total area of interest covering 5 spans, and the second plot shows a close-up view of 2 spans. In both cases, the tick marks in the horizontal axis match the position of the supports.

At each instant, the contact force is higher in the 90 N case. The peaks and valleys appear in similar regions in both cases, with one contact-force increase just after the support and another near mid-span.

The pantograph tested is equipped with a pressure sensor that measures the pneumatic pressure inside the actuator. The actuator pressure in both uplift configurations is displayed on Fig. 14 as a function of the pantograph position.

It can be observed that the pantograph pneumatic pressure has a clear offset, of approximately 0.25 bar, between the 70 N and 90 N test. This offset is a direct result of the pressure setting of the pneumatic regulator associated with the actuator. Disregarding this offset, both pressure curves present a similar pattern, where the regulator aims to maintain a target pressure inside the actuator cylinder while reacting to the dynamic loads imposed on the pantograph by the interaction with the catenary. Comparing the pressure time histories of both test cases, the peaks remain in the same location, with the valleys exhibiting slightly different oscillation patterns.

4.3. Effect of stagger

The tests discussed in the previous sections consider the catenary contact wire stagger, reproduced on the test bench through the lateral actuator. The stagger is a deliberately built-in feature on modern catenary system designs, with the main objective of distributing the pantograph contact strip wear along its length. A pantograph with a head suspension similar to the HSX a uniform vertical stiffness along the width of the contact strip. Hence, the pantograph response might significantly change depending on the lateral position of the wire and the corresponding contact point along the pantograph contact strip.

The present section discusses the influence of reproducing, or not, the catenary stagger movement on the test bench. When stagger is not employed, the lateral actuator is positioned at the centre of the pantograph contact strip. Fig. 15 presents the lateral and vertical actuator displacements for both 70 N test cases.

While the actuator vertical displacement profiles remain similar, differences of a few millimetres are observable. The wire stagger motion is not triangular due to the conservative approach taken to limit acceleration near the supports described in Section 2.1.2.

Fig. 16 presents the contact force history of both 70 N cases inside the selected area of interest. Both contact forces follow the same trend with slight differences in peak magnitudes. To aid the interpretation of the contact forces, Table 4, extracted from Table 2, compiles the statistical parameters for the contact forces on the last iteration of all tests.

The tests with stagger display higher force amplitude (higher maximum and lower minimum), whereas the differences in the mean force are small, although slightly more evident in the 90 N case. Looking closely to the ratio between standard deviation and mean force, while the ratio increases with stagger on the 70 N case, on the 90 N case the ratio is only marginally lower.

Despite the differences above not being overly significant, they are observable they are observable and indicate a measurable effect

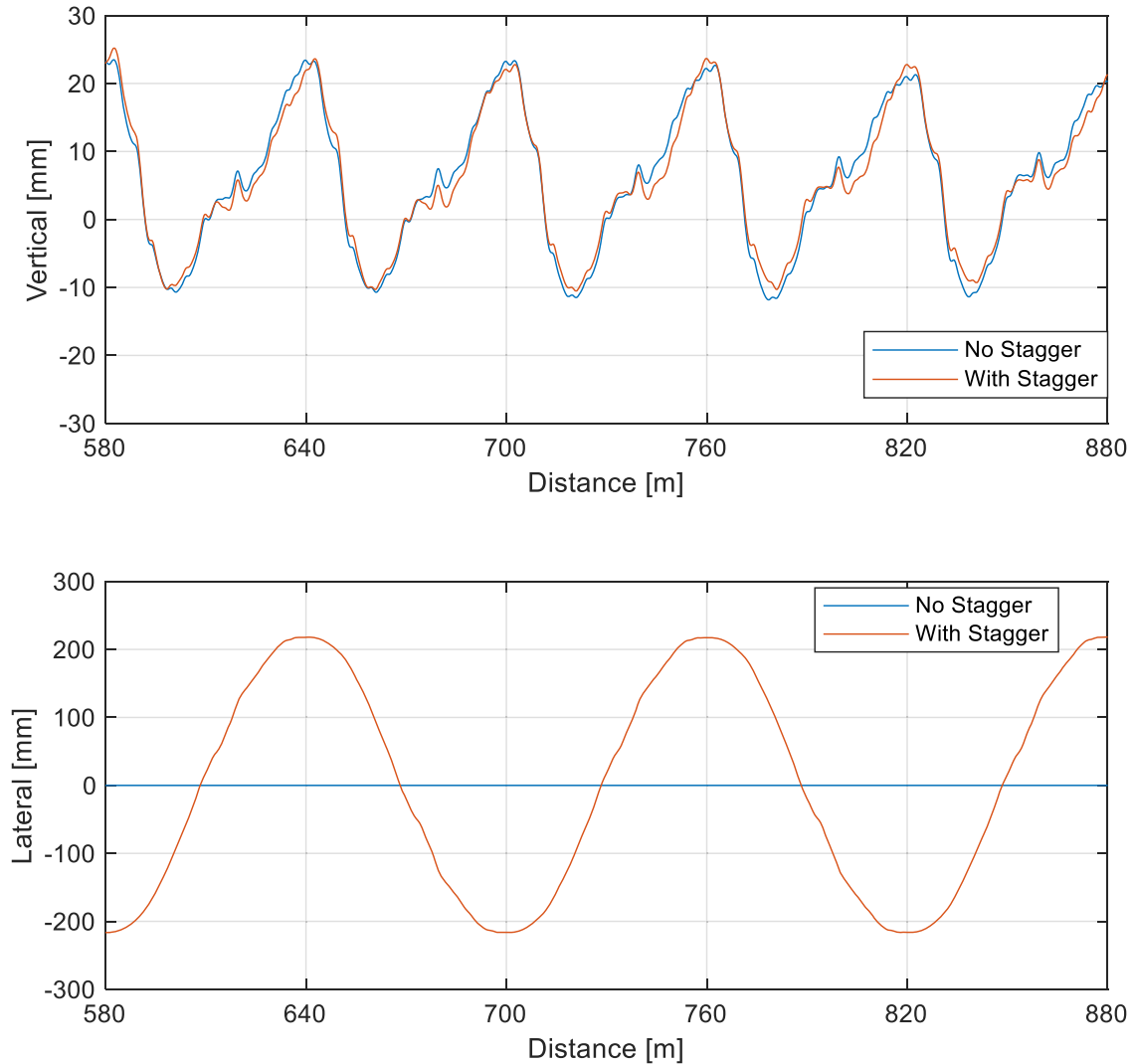


Fig. 15. Vertical (top) and lateral (bottom) movement of the actuation in test with and without wire stagger (lateral movement) for the case with 70 N uplift.

on the resulting contact force on this pantograph. Consequently, the reproduction of the wire lateral movement should be included whenever applicable. Note that these results may not be extendable to other pantograph systems, as different pantographs have different head suspension configurations and different spring-damper characteristics. Depending on these characteristics, there might be pantographs more or less sensitive to this phenomenon. It is also worth noting that in these tests, while on some tests the stagger movement was not reproduced by the test bench, the catenary model and simulation always included the wire stagger.

5. Conclusions

In this work, a new iterative offline HiL test methodology for the hybrid testing of pantograph-catenary interaction is proposed. The main aim of this method is to eliminate real-time calculation of the catenary behaviour, thereby removing restrictions on numerical model size or complexity. The proposed procedure can employ full FEM catenary models capable of incorporating relevant nonlinear dynamics (dropper slackening, wire irregularities, discrete wire geometry variations). The method also simplifies compensation for actuation time delay imposed by the actuator dynamics and control, as this is not required to be conducted in real time and there is prior knowledge of the actuator movements.

The methodology is implemented on a full-scale pantograph test facility, PANTHER. The FEM model of the entire catenary system is analysed in PantoCat. Four test cases are considered to demonstrate the feasibility and convergence of the proposed testing method and understand different testing considerations. Convergence analysis of the results obtained at each iteration is presented for the most demanding case tested. The convergence of the method is shown by a magnitude decay of the differences of the wire displacement and

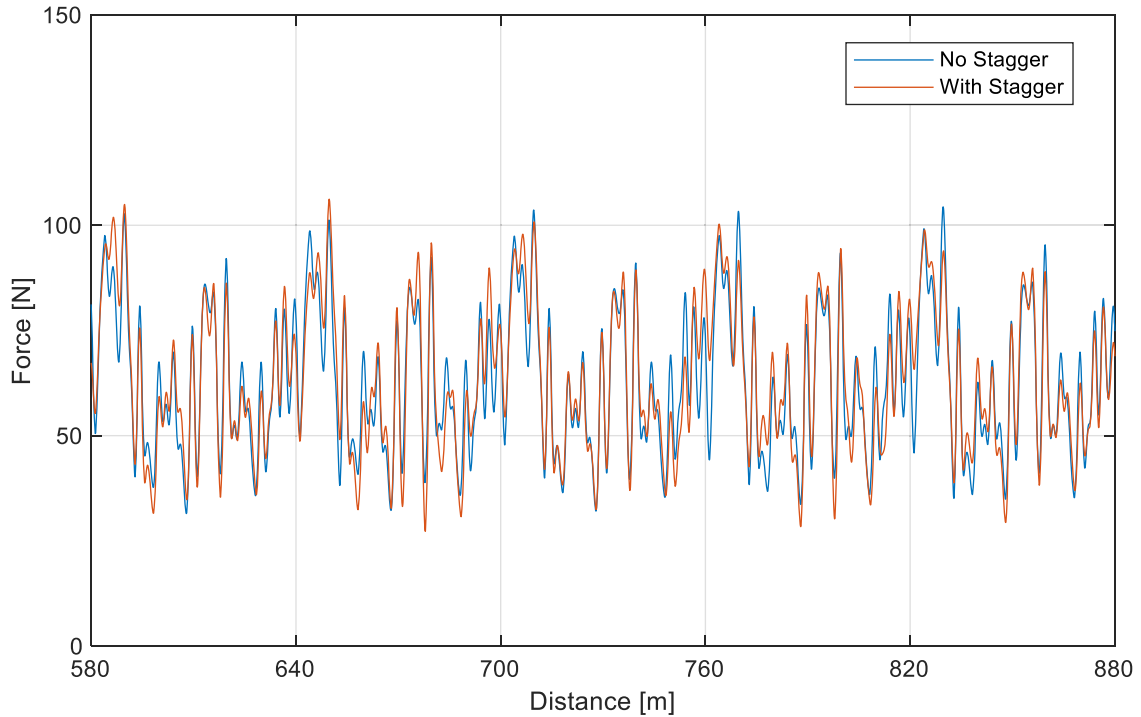


Fig. 16. Contact force of test run with and without stagger for the 70 N uplift case.

Table 4

Comparison of the statistical parameters of the contact force between tests with and without stagger.

Test Case	Contact Force [N]					σ/F_m
	F_{max}	F_{min}	Amp	F_m	σ	
Uplift 70 N						
No Stagger	104.37	31.48	72.89	63.96	14.35	0.22
Stagger	106.21	27.26	78.95	63.52	14.99	0.24
Uplift 90 N						
No Stagger	120.40	37.91	82.50	81.38	15.93	0.20
Stagger	121.16	33.75	87.41	82.45	15.99	0.19

contact force evaluated between consecutive iterations. The LM pantograph model is shown to be an appropriate starting point to the methodology, as the main contact force characteristics are present in the first estimation, although with lower oscillations.

The analysis of the four cases considered looks into the effect of the pantograph uplift force, and stagger on the statistical parameters of the contact force and its time history. Regarding uplift force, an increase in uplift is associated with reduced contact force oscillation for a single pantograph, as indicated by the lower ratio of the standard deviation to mean contact force in the 90 N case compared with the 70 N case. The results indicate that reproducing the wire stagger affects the contact-force statistical parameters. For the pantograph analysed, disregarding the stagger movement can lead to differences in the evaluated contact force oscillations. These results may not be generalisable to other pantographs or other case scenarios.

The main limitation of the offline iterative methodology is the additional time required to perform multiple iterations, unlike real-time HiL which is completed in a single run. Furthermore, convergence is critically reliant on guaranteeing the same initial test conditions at every iteration. Nevertheless, the ability to analyse more complex catenary models without restrictions imposed by real-time computation is a significant advantage that warrants further investigation. Future developments of the offline iterative HiL methodology will focus on optimising the process, refining stagger reproduction by moving from actuator-feasible contact-point motion towards a more geometrically accurate representation and exploring additional applications. Further case studies will assess the robustness and repeatability of the methodology, its capability to handle more complex scenarios, and its validation against line-test measurements. Future studies should also investigate the application of the proposed methodology to very-high-speed railway systems (>350 km/h), their corresponding catenary infrastructures, and rigid conductor rail systems, where different tension levels, stiffness characteristics, and pantograph dynamic requirements may influence convergence behaviour and testing performance. Additional topics include the introduction of wire vertical gradients, e.g. in level crossings and tunnels; adding discrete feature on the catenary system such as neutral sections and section insulator; adding arbitrary track geometry and curvature; and developing novel

approaches to account for multiple-pantograph operation.

Moreover, now that this implementation has been successfully demonstrated, some deviations to the original method can be also explored. For instance, evaluation of the effect of starting the first iteration with a constant force instead of a lumped-mass model; assessing how noise and actuator bandwidth affect convergence; exploring alternative stopping criteria; and studying the influence of the alpha parameter in more detail.

CRedit authorship contribution statement

J.P. Santos: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Resources, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **P. Antunes:** Writing – review & editing, Supervision, Software, Project administration, Funding acquisition, Conceptualization. **J. Pombo:** Supervision, Project administration, Funding acquisition. **J.M. Rebelo:** Software, Resources, Investigation. **A. Schirrer:** Writing – review & editing, Supervision, Methodology, Formal analysis. **S. Jakubek:** Writing – review & editing, Supervision, Methodology, Formal analysis. **M. Tur:** Writing – review & editing, Supervision, Methodology, Formal analysis. **S. Gregori:** Writing – review & editing, Supervision, Methodology, Formal analysis.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Table of Symbols

Symbol	Units	Description
Operators		
$G(\dots)$	-	Numerical dynamic analysis
$\max(\dots)$	-	Operator maximum
$\min(\dots)$	-	Operator minimum
$\sin(\dots)$	-	Operator sine
$T(\dots)$	-	Iterative operator representing pantograph experiment, numerical simulation and force correction
$\sigma(\dots)$	-	Standard Deviation
Models		
M_{cat}^{num}	-	Full finite element method (FEM) catenary model
M_{pat}^{num}	-	Numerical pantograph model (Lumped mass or Multibody)
M_{pat}^F	N	Moving constant force representing the pantograph response
$H_{act}(s)$	-	Transfer function of the actuator (frequency response to commanded input)
Signals		
$F_{pat}^0(t)$	N	Force output from the pantograph-catenary dynamic analysis in iteration $k = 1$
$F_{pat}^k(t)$	N	Force input applied to the catenary model in iteration k after force processing
$F_{pat}^e(t)$	N	Contact force history result obtained through convergence
$R_{wire}^k(t)$	m	3D coordinates of the contact wire position above the pantograph on iteration k
$R_{wire}^e(t)$	m	Contact wire position result obtained through convergence
$r_{yj}^k(t)$	m	Unprocessed lateral actuator displacements in span j
$r_{yj}^{sin}(t)$	rad	Coordinate-converted lateral stagger signal used in stagger smoothing
$r_y^k(t), r_z^k(t)$	m	Unprocessed lateral and vertical actuator displacements required to follow the contact wire movement
$u_y^k(t), u_z^k(t)$	m	Lateral and vertical test bench actuator command in iteration k
$S_{pat}(t)$	m	Pantograph longitudinal position along the track
$Y^k(t)$	N	Contact force measured at test bench in iteration k
$Y_f^k(t)$	N	Measured force filtered with low-pass filter
$Z_{cat}^k(t)$	m	Vertical contact wire position above the pantograph on iteration k
$In_{act}(s)$	-	Input signal to estimate actuator transfer function
$Out_{act}(s)$	-	Output signal to estimate actuator transfer function
Parameters		
F_{up}	N	Static pantograph uplift force
f_c	Hz	Low-pass filter cutoff frequency
j	-	Span number (catenary region between consecutive supports)
k	-	Iteration counter

(continued on next page)

(continued)

Symbol	Units	Description
$\hat{r}_y, \hat{r}_z$	m	Lateral and vertical actuator offsets to make average wire height at the centre of the actuator stroke
ROI	m	Region of interest used for statistical evaluation of contact force
t	s	Time step in simulation or test environment.
X_{pat}^k	N	Standard deviation of filtered force on ROI in iteration k
x, y, z	-	Global coordinate axes of the catenary reference frame
$z_{\text{wire}}^{\text{static}}$	m	Static vertical contact wire position under applied uplift
M1, M2, M3	kg	Inertial parameter of Lumped mass model
K1, K2, K3	N/m	Link stiffness parameter of Lumped mass model
C1, C2, C3	Ns/m	Link damping parameter of Lumped mass model
α	-	Contact force correction factor, between (0,1]
α_R	s^{-1}	Rayleigh model: mass-proportional damping coefficient
β_R	s	Rayleigh model: stiffness-proportional damping coefficient
δ_X	N	Convergence threshold for standard deviation difference
δ_Z	m	Convergence threshold for catenary displacement difference
τ_F	s	Time shift aligning test bench measured force with virtual simulation
τ_y, τ_z	s	Time delay compensation factor for lateral and vertical actuator dynamics

Appendix B. Geometric and modelling catenary parameters

The catenary model employed in this work represents the standard British design for conventional speed, named Network Rail Master Series (UKMS100), as shown in Fig. 8. Table B.1 compiles the geometric parameters of the catenary model, including the stagger at the support, and heights of contact and messenger wires.

Table B1

Geometric parameters of the catenary model.

Support	Span	Span Length (m)	Stagger (m)	CW Height (m)	MW Height (m)
1	1	50	3.7	5.7	6.9
2	2	50	-0.23	4.95	6
3	3	60	-0.23	4.7	6
4	4	60	0.23	4.7	6
5	5	60	-0.23	4.7	6
6	6	60	0.23	4.7	6
7	7	60	-0.23	4.7	6
8	8	60	0.23	4.7	6
9	9	60	-0.23	4.7	6
10	10	60	0.23	4.7	6
11	11	60	-0.23	4.7	6
12	12	60	0.23	4.7	6
13	13	60	-0.23	4.7	6
14	14	60	0.23	4.7	6
15	15	60	-0.23	4.7	6
16	16	60	0.23	4.7	6
17	17	60	-0.23	4.7	6
18	18	60	0.23	4.7	6
19	19	60	-0.23	4.7	6
20	20	60	0.23	4.7	6
21	21	60	-0.23	4.7	6
22	22	60	0.23	4.7	6
23	23	60	-0.23	4.7	6
24	24	50	0.23	4.7	6
25	25	50	0.23	4.95	6
26	-	-	3.7	5.7	6.9

The majority of the spans include 6 droppers, except spans 1, 2, 24 and 25 that have 4 droppers. They are spaced in multiples of 5 m. The main parameters of the catenary components are supplied in Table B.2.

Table B2

Design parameters of the UKMS100 catenary system.

Parameter	Unit	Contact Wire	Messenger Wire	Droppers	Steady Arms
Cross Section	m^2	1.07E-04	6.58E-05	1.00E-05	3.11E-04

(continued on next page)

Table B2 (continued)

Parameter	Unit	Contact Wire	Messenger Wire	Droppers	Steady Arms
Linear Mass	kg/m	9.52E-01	5.96E-01	9.30E-02	1.36E+00
Young Modulus	Pa	1.15E+11	1.13E+11	1.13E+11	7.10E+10
Tension	N	1.10E+04	1.10E+04	N.A.	N.A.

Additionally, the maximum FE element size on the wires is 0.7 m, and the damping of all the elements are reproduced by a proportional Rayleigh model with coefficients $\alpha_R = 0.0125\text{s}^{-1}$ and $\beta_R = 1e^{-4}\text{s}$.

Data availability

Data will be made available on request.

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