

# Characterization of sheet formability limits using a novel diagonal-cruciform test specimen

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## ABSTRACT

This paper introduces an innovative diagonal-cruciform test specimen that significantly enhances the characterization of formability limits in sheet metal forming. The specimen's unique design features a reticular two-dimensional geometric structure, with four triangular arms connecting at the center, which effectively induce biaxial tension stress states when subjected to uniaxial loading. Furthermore, the incorporation of machined spherical cups at its center to locally reduce thickness ensures that damage accumulates in this region. Experimental strain loading paths are captured using digital image correlation (DIC) and analyzed with in-house software developed specifically for research and education on material formability. The software identifies and plots the onsets of necking and fracture in principal strain space, and results prove that the diagonal-cruciform specimen is highly effective in generating stable biaxial tension strain loading paths in C11000 copper sheets, operating under friction-independent conditions without the necessity for specialized testing equipment. The fracture limits are subsequently validated by comparing them against the strain loading paths obtained from a single-point incremental sheet-formed part up to failure. The investigation confirms the versatility and robustness of the novel diagonal-cruciform test specimen for evaluating the formability of C11000 copper sheets and provides valuable insights into its potential application across the broader field of sheet formability characterization.

## 1. Introduction

Copper alloys, with their excellent electrical and thermal conductivity, corrosion resistance, and durability, are increasingly finding applications in industries such as electronics, automotive, aerospace, and construction. The growth of renewable energy and emerging technologies like electric and hybrid-electric vehicles or smart grids has led to a surge in demand for copper, particularly in sustainable solutions and products (Huang et al., 2020). As industries continue to innovate, the versatility and reliability of copper alloys make them a vital material for future technological developments.

Formability, the maximum deformation a metal can undergo during a manufacturing process without yielding defects, is a critical concept in metal forming, as it allows engineers to assess whether the desired shape

can be achieved without compromising the material's integrity. In the realm of sheet metal forming, understanding the formability limits is essential for optimizing material consumption, reducing defects, and ensuring the production of high-quality components.

In sheet metal forming, formability limits are typically defined by the onsets of plastic instability and fracture. Plastic instability is related to the appearance of diffuse or localized necking, while fracture is determined by the initiation and propagation of cracks that develop due to the accumulation of ductile damage in highly deformed regions (Nielsen and Martins, 2021). This understanding of formability is crucial, as it represents the final stage of damage evolution over time, where damage is treated as a product property that accumulates across processes (Tekkaya et al., 2020).

Formability limits of sheet materials due to necking are commonly

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represented in principal strain space by the forming limit curve (FLC), illustrated by the purple line in Fig. 1a. This curve can be obtained through standard experimental procedures based on Nakajima (Nakajima et al., 1968) or Marciniak (Marciniak, 1965) tests, which are normalized according to the ISO 12004-2 (2021) standard. In contrast, there is no standard for the determination of the fracture limits, which are commonly obtained by a combination of Nakajima tests and uniaxial loading tests with various geometries, including tensile (T), double-notched tensile (DNT), and double-notched shear tests (DNST) (Isik et al., 2014). The fracture forming limit (FFL), shown as the red line in Fig. 1a, relates explicitly to crack opening due to tension, known as mode I of fracture mechanics, and is associated with damage accumulation following the ductile fracture criterion developed by McClintock (1968), as introduced by Martins et al. (2014).

The Nakajima and Marciniak test specimens stand out for their ability to induce fractures through biaxial tension. In conjunction with these standard test specimens, an alternative design known as the 'cruciform test specimen' emerged with Shiratori and Ikegami's (1967) pioneering work in the late 1960s. To facilitate fractures under biaxial tension, numerous researchers have successfully introduced machined spherical cups on one or both sides of alternative cruciform test specimens to create a thickness transition region and concentrate stress at the thinner areas, effectively triggering crack initiation at the center of the specimen (Song et al., 2017; Zhang et al., 2021). However, although the use of cruciform test specimens mitigates the challenges associated with the Nakajima and Marciniak tests, particularly their sensitivity to friction, it is essential to recognize that all these tests require access to specialized, dedicated machines, which are far more expensive than universal testing machines. Cruciform test specimens represent a substantial advancement in formability characterization, but cost constraints challenge their widespread implementation.

To effectively address this problem, the authors have recently introduced a diagonal-cruciform (DC) test specimen (Sampaio et al., 2024a), as shown in Fig. 1b. The specimen can generate strain loading paths under biaxial tension using a universal testing machine without being affected by friction sensitivity and without requiring specialized setups and equipment, as in the case of conventional cruciform, Marciniak's, and Nakajima's testing procedures. This is accomplished by using a two-dimensional reticular structure design with triangular arms that converge at a central point and decompose the external force exerted at the ends of the specimen (see Fig. 1b for additional details). The spherical cups machined on both sides of the specimen create a

symmetric thickness reduction in the material, forcing ductile damage to concentrate in the central region, as proposed in previous studies (Zhang et al., 2021).

This paper aims to evaluate the suitability of the DC test specimen to establish the FFL by incorporating data from tensile and DNT tests. Additionally, it seeks to validate the use of the strain acceleration method, which was recently introduced by the authors (Sampaio et al., 2024b), for detecting the onset of diffuse necking, in both DC and DNT test specimens, broadening the method's applicability beyond just tensile test specimens.

Strain loading paths are assessed using digital image correlation (DIC) (Peters and Ranson, 1982), and the output data is analyzed with software recently developed by the authors for research and educational purposes in material formability, which automatically converts strain-time evolutions into strain loading paths in both principal strain and effective strain vs. stress triaxiality spaces (Sampaio et al., 2024b).

The work is carried out in UNS C11000 copper sheets, and the FFL obtained from DC, tensile, and DNT test specimens is validated through comparisons with experimental data obtained from single-point incremental sheet-forming of a benchmark truncated circular-to-square pyramidal part made from the same material. The process is ideal for validating the FFL because, unlike other sheet metal forming processes, it fails by fracture without prior necking, enabling a direct comparison solely focused on fracture.

## 2. Material and methods

### 2.1. Mechanical characterization

The experimental work was carried out on UNS C11000 copper sheets with a thickness of 2 mm, having a minimum chemical composition of 99.9 % copper, and a maximum of 0.04 %–0.06 % oxygen. Tensile test specimens were extracted from these sheets in the rolling direction according to the (ASTM E8, 2022) standard, using water jet cutting. This process resulted in average surface roughness values of 5  $\mu\text{m}$  across all cut areas of the sheets from which the specimens were taken.

The specimens were then subjected to annealing heat treatment at 500 °C for 1.5 h, cooling in furnace down to  $\sim 200$  °C and final air cooling to room temperature. The decision to anneal the tensile test specimens was due to the necessity of annealing the C11000 copper sheets to decrease the forces in single-point incremental forming, which

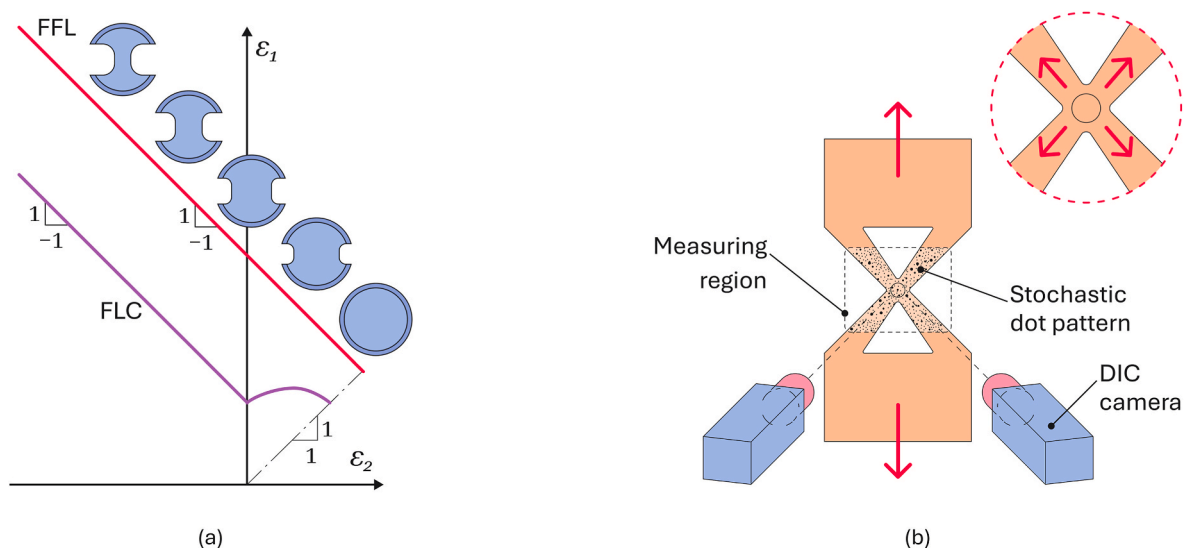


**Fig. 1.** (a) Forming limit diagram for sheet forming obtained from Nakajima test specimens and (b) a schematic representation of the diagonal-cruciform sheet test specimen with a detail that illustrates the working principle derived from the decomposition of the external uniaxial force along its arms.

was later utilized to validate the FFL obtained from DC, tensile, and DNT test specimens.

Annealing decreases work hardening and improves ductility through recovery and recrystallization. This decreases anisotropy compared to the cold-rolled condition (as-supplied) condition and justifies the extraction of tensile specimens from the sheets exclusively in the rolling direction.

The flow stress of the annealed C11000 copper sheets depicted in Fig. 2 was determined from tensile tests conducted at room temperature according to the (ASTM E8, 2022) standard and using an Instron 4507 universal testing machine. The data was approximated with a Ludwik–Hollomon strain hardening model, which is expressed as follows,

$$\sigma = 520\epsilon^{0.5} \quad (1)$$

The strain values used to create the flow curve shown in Fig. 2 were obtained from tensile tests conducted with a DIC system from Dantec Dynamics (model Q-400 3D). Details about this system can be found in the following sub-section of the paper.

Table 1 provides the average values of the elasticity modulus  $E$  yield stress  $\sigma_Y$ , ultimate tensile strength  $\sigma_{UTS}$ , and area reduction at fracture  $A$  of the annealed C11000 copper sheets obtained from the tensile tests.

## 2.2. Formability characterization

The formability characterization of the C11000 copper sheets utilized tensile, DNT, and the newly proposed DC test specimens (Fig. 3), and was performed on the same universal testing machine previously used for material characterization. Similar to the tensile tests, the other formability test specimens were also extracted from the supplied sheets along the rolling direction using water jet cutting. The DC specimens were subsequently machined on each side at the center with a ball-nose end mill with a diameter of 10 mm to create two spherical cups (refer to the detail in Fig. 3c).

All the specimens underwent the annealing treatment that was previously described.

The formability tests were performed at room temperature with a crosshead velocity of 5 mm/min, and the experimental evolution of the in-plane strains over time was obtained using a commercial DIC system from Dantec Dynamics (model Q-400 3D). This system features two 6-megapixel resolution cameras equipped with 50.2 mm focal length lenses and an f/8 aperture, allowing for image capture at a shutter frequency of 10 Hz.

The main parameters used in data acquisition and analysis conform to the best practices guide presented by Jones and Iadicola (2018) and are summarized in Table 2. The measurement uncertainty for the DIC strain measurements is 0.002.

The areas being measured on the specimens were covered with a stochastic black dot pattern, which accounted for approximately 40 % of

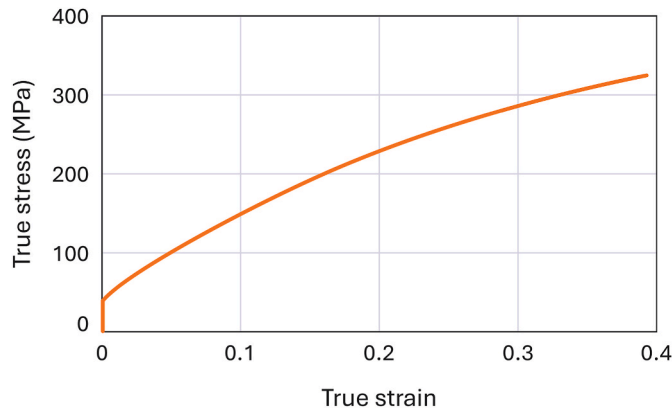


Fig. 2. Flow stress of the annealed C11000 copper sheet.

Table 1

Summary of the mechanical properties of the annealed C11000 copper sheets.

| $E$ (GPa) | $\sigma_Y$ (MPa) | $\sigma_{UTS}$ (MPa) | $A$ (%) |
|-----------|------------------|----------------------|---------|
| 110       | 47.5             | 216                  | 48.5    |

a pre-painted white background and were illuminated by a spotlight during testing. Since the regions of interest in the DC and DNT specimens are significantly smaller than those in the tensile specimens, 10 mm spacers were used with the lenses to reduce the minimum focusing distance. This adjustment enabled the capturing of smaller areas with relatively higher image quality.

Post-processing of the output data from DIC was done using in-house software recently developed by the authors (Sampaio et al., 2024b). The onset time of diffuse necking  $t_{diff}$  was obtained using the strain acceleration method, recently proposed by the authors (Sampaio et al., 2024b), which uses the experimental evolution of the minor in-plane strain to detect a local extreme in the second derivative of the minor principal in-plane strain with respect to time  $\ddot{\epsilon}_2(t)$  at the edges of the sheet surface region where diffuse necking is likely to occur,

$$t_{diff} = \arg \min_t \left( \frac{d^2 |\epsilon_2(t)|}{dt^2} \right) \quad (2)$$

The strain acceleration method is an alternative to determining the peak force in a force-time evolution, as described by Thamm et al. (2023), because the latter relies on load cells for quantitative force measurements, which can be challenging or even impossible to use in certain situations. For instance, when serrations occur in force-time evolutions due to localized deformation bands, such as those arising from the Portevin-Le Chatelier effect (Zavattieri et al., 2009), or when force plateaus make it difficult to identify the onset time of diffuse necking  $t_{diff}$ .

The onset of localized necking  $t_{loc}$  was identified using the time-dependent method proposed by Martínez-Donaire et al. (2009), which detects a maximum in the major principal in-plane strain rate  $\dot{\epsilon}_1(t)$  at a point in an adjacent region to where localized necking occurs,

$$t_{loc} = \arg \max_t \left( \frac{d\epsilon_1(t)}{dt} \right) \quad (3)$$

The conjugate strain pair at fracture ( $\epsilon_{1f}, \epsilon_{2f}$ ) was determined by measuring the thickness of the fractured specimen (Embury and Duncan, 1981), as strain loading paths that exhibit localized necking are characterized by a kink (sharp change of strain ratio) in principal strain space (Atkins, 1996). This kink indicates a shift in the loading path toward a plane strain characteristic loading ( $d\epsilon_2 \approx 0$ ), and can be expressed by the following relations,

$$\epsilon_{2f} = \epsilon_{2^*} \quad (4)$$

$$\epsilon_{1f} = -\epsilon_{2f} - \epsilon_{3f} = -\epsilon_{2f} - \ln \frac{t_f}{t_0} \quad (5)$$

In equation (4),  $\epsilon_{2^*}$  denotes the minor in-plane strain at the onset of diffuse necking  $\epsilon_{2n}$ , or, if applicable, the minor in-plane strain at the onset of localized necking  $\epsilon_{2loc}$ , as obtained from the DIC system. In equation (5), the subscript  $f$  refers to the onset of fracture, while the initial thickness and final sheet thickness at fracture are denoted as  $t_0$ , and  $t_f$ , respectively.

To measure  $t_f$  the specimens need to be cut and mounted in resin to ensure that the fractured surfaces are parallel to the microscope. The sheet thickness at fracture  $t_f = \sum_{i=1}^{n_m} t_{fi} / n_m$  was determined by taking the average value of several  $n_m$  individual thickness measurements  $t_{fi}$  along the crack using a Mitutoyo TM-505 B toolmaker's microscope with a precision of  $\pm 0.001$  mm.

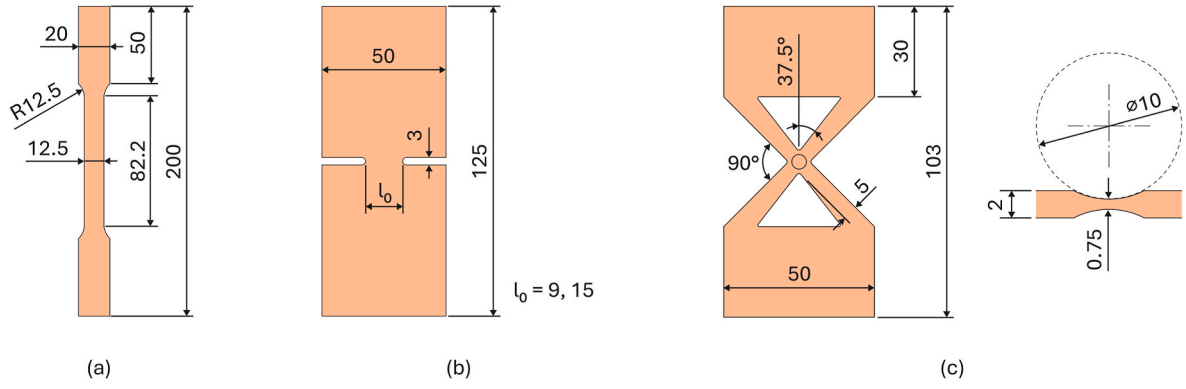


Fig. 3. Geometry of (a) tensile, (b) double-notched tensile (DNT), and (c) diagonal-cruciform (DC) test specimens (dimensions in mm).

Table 2

DIC software and main parameters used in data acquisition and analysis.

| DIC software                     | INSTR 4D from DANTEC DYNAMICS               |
|----------------------------------|---------------------------------------------|
| Image filtering                  | Not applicable                              |
| Subset size                      | $17 \times 17$ pixels <sup>2</sup>          |
| Step size                        | 15 pixels                                   |
| Subset shape function            | Affine and distortion shape transformations |
| Data processing and filtering    |                                             |
| - Pre-filtering of displacements | Not applicable                              |
| - Strain formulation             | Logarithmic                                 |
| - Virtual strain gauge size      | Automatic based on the subset size          |
| - Post-filtering of strains      | Not applicable                              |
| Noise-floor and bias             | Not applicable (3D full field system)       |

### 3. Results and discussion

#### 3.1. Diffuse necking

Fig. 4 presents the application of the strain acceleration method to the tensile, DNT, and DC specimens, enabling the identification of the onset time of diffuse necking  $t_{diff}$ . The right vertical axis is displayed on a normalized scale to support the computational implementation of the strain acceleration method and to render values independent of the original data scale.

Furthermore, Fig. 4 enables a comparison between the results obtained with this method (indicated by the dashed green vertical line) and those derived from the maximum force criterion (indicated by the dashed red vertical line), where the onset time of diffuse necking  $t_{diff}$  is the moment when material continues to deform under a decreasing

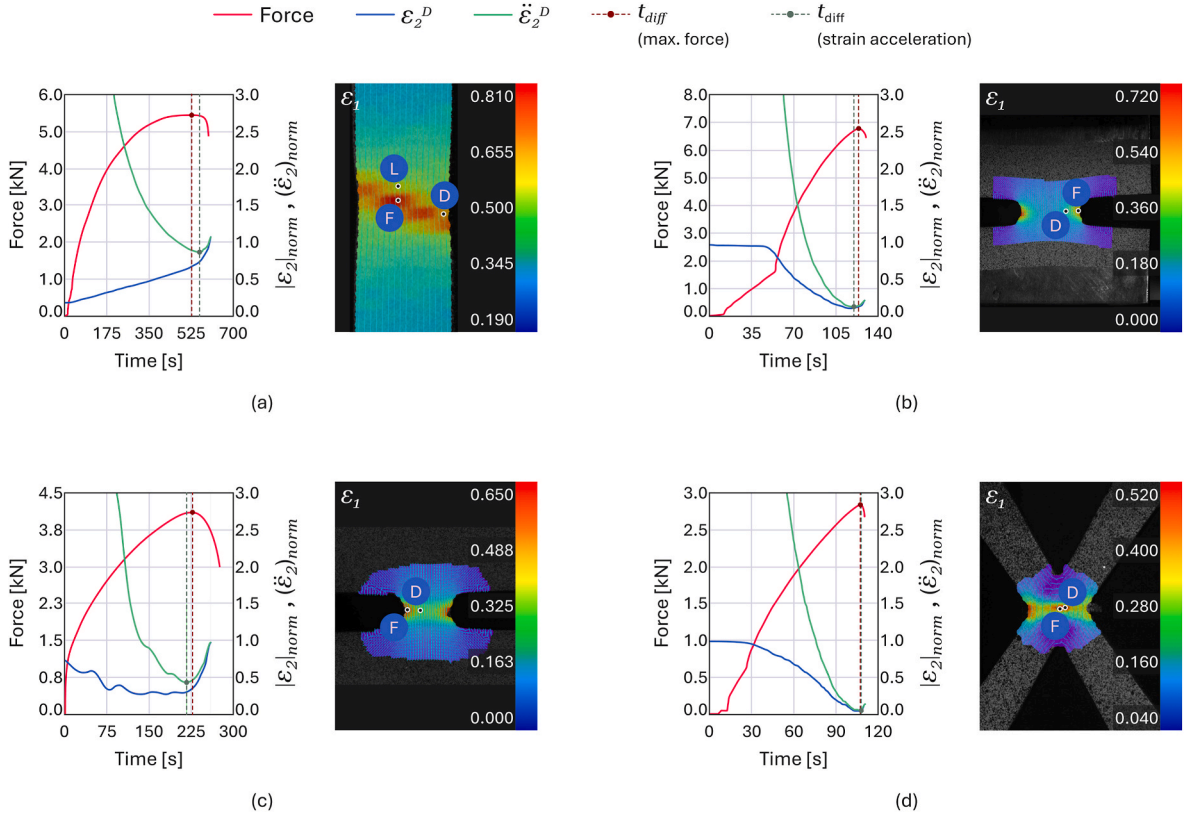


Fig. 4. Application of the strain acceleration method (represented by the dashed green vertical line) and the maximum force criterion (represented by the dashed red vertical line) to determine the onset of diffuse necking in (a) tensile, (b) double-notched tensile with  $l_0 = 15$  mm, (c) double-notched tensile with  $l_0 = 9$  mm, and (d) diagonal-cruciform test specimens.

force, leading to plastic instability (i.e., unstable loading conditions).

The right-hand side pictures of Fig. 4 display the distributions of the major in-plane principal strain  $\varepsilon_1$  at the instant of time immediately before fracture initiation, as captured by the DIC system. The strain loading paths and the strain measurements at the onset of fracture were taken at point F, while the strains at the onset of diffuse necking were assessed at point D. Additionally, when applicable, the strains at the onset of localized necking were evaluated at point L, as shown in the pictures.

Table 3 summarizes the values of  $t_{diff}$  obtained using both the strain acceleration method and the maximum force criterion. The low relative error values indicate that the strain acceleration method can be effectively applicable across the range of stress states encountered in sheet metal forming, from uniaxial to biaxial tension.

It is important to note that identifying the onset time of diffuse necking  $t_{diff}$  in the DC specimens is somewhat dependent on the positioning of the evaluation point 'D' (Fig. 4d). As this point is moved further from the center, the strain acceleration method may result in a slightly less accurate determination of the onset time of diffuse necking,  $t_{diff}$ , or may even fail to identify it altogether when the point approaches the region where plastic deformation is consistently uniform. In such cases, significant changes in strain acceleration are not observed. This sensitivity arises from thickness reduction in the spherical cups at the center of the specimen, which leads to more concentrated deformation and makes diffuse necking more pronounced in that region.

The relatively larger errors in the tensile tests, while still within an acceptable range, are likely due to the extended plateau observed in the force–time curve around the maximum force. This plateau, which lasts for nearly 30 % of the test duration (during which the force remains above 98 % of its maximum value), can result in an imprecise identification of the onset time of diffuse necking  $t_{diff}$  when relying on the maximum force criterion.

Once  $t_{diff}$  is identified, the corresponding pairs of in-plane principal strain values at the onset of diffuse necking  $(\varepsilon_1, \varepsilon_2)_{diff}$  are automatically computed using the software recently developed by the authors (Sampaio et al., 2024b). This is achieved by evaluating the strain–time functions  $\varepsilon_1(t)$  and  $\varepsilon_2(t)$  at  $t = t_{diff}$ . The resulting values are presented in the following section, where the strain loading paths are plotted, and the forming limit diagram is constructed.

### 3.2. Forming limit diagram

The developed software automatically combines the strain–time evolutions to generate strain loading paths in principal strain space,  $\varepsilon_1 = f(\varepsilon_2)$ . It also determines the onset of localized necking following the time-dependent method proposed by Martínez-Donaire et al. (2009) whenever the major principal strain–time evolution  $\varepsilon_1 = f(t)$  at the neighborhood of the most strained region on the sheet surface is provided (only tensile test specimens in the case of this paper). The results are portrayed in Fig. 5, along with photographs of representative fractured test specimens of the four different geometries tested.

The software determined the conjugate pairs of principal strains at fracture according to equations (4) and (5) after providing the specimens' initial  $t_0$  and fracture  $t_f$  thicknesses, with the latter being an

average between three different measurements ( $n_m = 3$ , refer to Section 2.2).

The strain loading paths derived from the tensile specimens exhibit a trend of  $\beta = d\varepsilon_2/d\varepsilon_1 = -1/2$ , typical of uniaxial tension states until the onset of localized necking. Beyond this point, the strain loading path is assumed to follow a trend characteristic of plane strain ( $\beta = 0$ ) (Embury and Duncan, 1981; Atkins, 1996).

On the other hand, the strain loading paths derived from the DNT test specimens (both the ones with  $l_0 = 9$  mm and the ones with  $l_0 = 15$  mm) show a trend that is somewhat closer to plane strain conditions compared to the tensile test specimens, with  $\beta \approx -1/4$  up to the onset of diffuse necking. In contrast to the tensile specimens, neither of the DNT specimens displays a band of higher values for the major in-plane principal strain  $\varepsilon_1$  in the distributions obtained from DIC (refer to Fig. 4b and c). This observation is the reason why the authors did not evaluate localized necking for these specimens. As a result, the characteristic trend of plane strain was applied immediately after the onset of diffuse necking and continued until fracture.

The DC specimens, which fall within the first quadrant of the principal strain space, exhibit strain loading paths that demonstrate biaxial stress conditions throughout the entire test. Initially, these paths closely align with the characteristic trend of equibiaxial tension ( $\beta = 1$ ), but they later shift to a more vertical orientation. As previously noted by the authors when discussing the diagonal-cruciform specimen, this shift occurs because the strain field changes from  $\varepsilon_x > \varepsilon_y$  to  $\varepsilon_y > \varepsilon_x$  after the initial deformation stages. This change is accompanied by a gradual alteration in the reticular structure geometry of the specimen, causing the internal axial forces along the arms to progressively rotate toward the longitudinal direction. Consequently, higher strain levels are exerted in the longitudinal direction compared to the transverse direction.

Similar to the DNT test specimens, the major principal strain distributions in the DC specimens do not permit the evaluation of localized necking. Therefore, the strain loading paths are assumed to follow a plane strain deformation trend from the onset of diffuse necking up until fracture.

The resulting FFL was traced, considering the theoretical slope of '–1', and can be described by the following expression,

$$\varepsilon_1 = -\varepsilon_2 + 2.07 \quad (7)$$

The Hill (1952) and Swift (1952) models were employed, using the previously determined hardening exponent  $n = 0.5$ , to represent the limits for localized and diffuse necking in Fig. 5, respectively, thereby approximating the FLC (refer to the purple dashed and solid lines). As observed, the onsets of localized necking identified in the tensile tests (indicated by the '×' markers in Fig. 5) align well with the Hill-defined limit line. In contrast, the onsets of diffuse necking (represented by the '+' symbols in Fig. 5) fall slightly below the Swift-defined limit for the tensile and DC specimens and slightly above it for the DNT specimens.

Considering the small relative errors noted between the strain acceleration method and the maximum force criterion discussed in the previous section, and the observation that the strain acceleration method detected these onsets slightly later than the maximum force for specimens below the limit and the opposite for those above it, it is evident that the forming limit associated with diffuse necking of the annealed C11000 copper sheet is not accurately represented by the theoretical Swift model.

Nevertheless, aside from one DC test, the onsets of diffuse necking fall within an acceptable margin of error from the FLC. The greater variability in the values for the DC specimen is attributed to the machined thickness reduction region mentioned earlier.

Fig. 6 shows the loading paths in the effective strain vs. stress triaxiality space, together with the limits associated with diffuse and localized necking and fracture, as determined by the software (Sampaio et al., 2024b). The loading paths are shown as  $\bar{\varepsilon} = f(\bar{\eta})$  functions, with  $\bar{\eta}$  being the weighted average measure of stress triaxiality, calculated as

**Table 3**

Onset time of diffuse necking  $t_{diff}$  for various specimen geometries.

| Specimen             | Strain acceleration method $t_{diff}$ (s) | Maximum force criterion $t_{diff}$ (s) | Relative error |
|----------------------|-------------------------------------------|----------------------------------------|----------------|
| Tensile              | 561.2                                     | 529.2                                  | 6.05 %         |
| DNT ( $l_0 = 15$ mm) | 120.4                                     | 123.9                                  | 2.80 %         |
| DNT ( $l_0 = 9$ mm)  | 216.8                                     | 227.3                                  | 4.64 %         |
| DC                   | 106.9                                     | 107.3                                  | 0.42 %         |

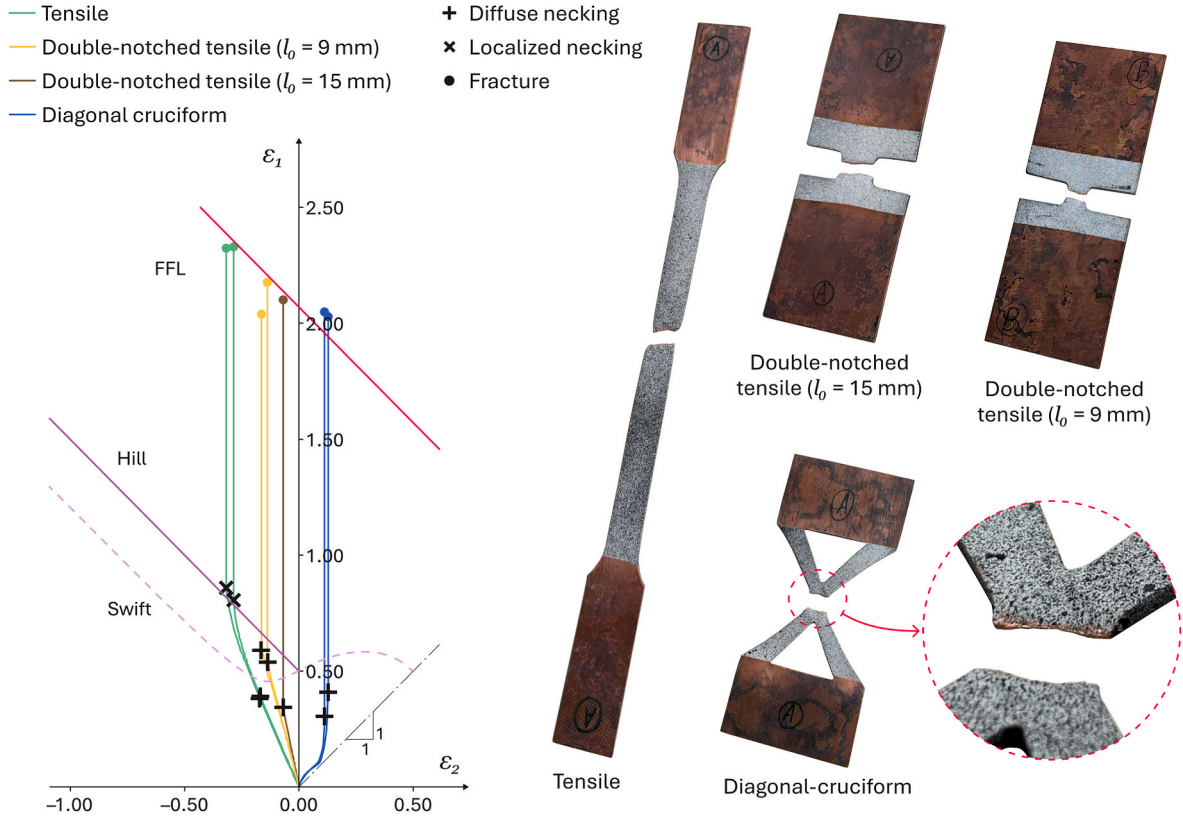


**Fig. 5.** Strain loading paths and forming limit diagram of the C11000 copper alloy obtained using tensile, double-notched tensile (DNT) and diagonal-cruciform (DC) test specimens.

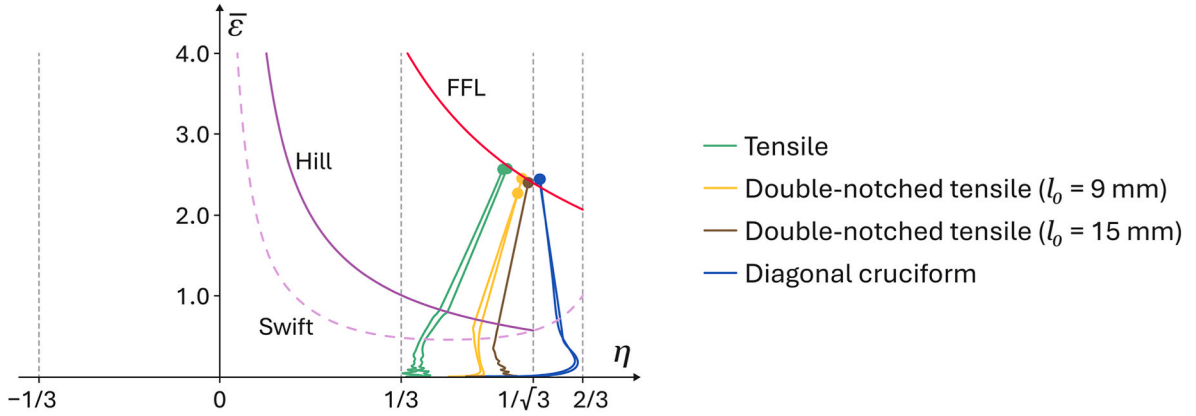


**Fig. 6.** Loading paths and forming limits related to necking and fracture in the effective strain vs. stress triaxiality space.

follows,

$$\bar{\eta} = \frac{\sum_{i=1}^N (\eta_i \Delta \bar{\epsilon}_i)}{\bar{\epsilon}} \quad (8)$$

where  $N$  represents the total number of points, or measurements,  $\eta_i$  denotes the instantaneous stress triaxiality at a specific time instant  $i$ , and  $\Delta \bar{\epsilon}_i$  represents the increment of effective strain between the instants of time  $i$  and  $i-1$ . The instantaneous triaxiality values  $\eta_i$  can be calculated from the instantaneous slopes  $\beta_i = (d\epsilon_2/d\epsilon_1)_i$  and the anisotropy coefficient  $r$  as follows (Martins et al., 2014),

$$\eta_i = \frac{\sqrt{1+2r}(1+\beta_i)}{3\sqrt{1+\frac{2r}{1+r}\beta_i+\beta_i^2}} \quad (9)$$

The weighted average measure of stress triaxiality  $\bar{\eta}$  introduced by the authors (Sampaio et al., 2023) does not carry physical meaning, but it enhances the calibration of critical damage values when applying uncoupled fracture criteria in the context of the effective strain vs. stress triaxiality space, since it accounts for the history of stress triaxiality. Consequently, the weighted average stress triaxiality  $\bar{\eta}$  makes experimental calibration of critical damage sensitive to variations in stress states during plastic deformation. Mathematically, equation (8) can be understood as a weak form of equation (9), where the instantaneous stress triaxiality  $\eta_i$  values are considered only in an average sense over

the complete loading path.

The fracture limit is modelled using the simplified version of the McClintock (1968) fracture criterion, where  $D_{crit} = 1.38$  is used in order to produce an equivalent limit to the FFL presented in equation (7),

$$D_{crit} = \int_0^{\bar{\epsilon}_f} \eta d\bar{\epsilon} \quad (10)$$

### 3.3. Validation through incremental sheet forming

The application of single-point incremental sheet forming on annealed C11000 copper sheets with a thickness of 2 mm was carried out to validate the previously calibrated fracture forming limit (FFL). Single-point incremental forming is a versatile sheet metal forming process that enables dieless fabrication of complex parts, offering significant benefits, such as reduced tooling costs and shorter lead times, making it ideal to produce small batch sizes of customized components (Jeswiet et al., 2005).

The selected geometry for this experiment was a benchmark truncated circular-to-square pyramidal loft. As illustrated in the photographs of the formed part (Fig. 7a), the shape begins with a circular base and gradually transitions to a square cross-section as the depth of forming increases. The starting blank was a square C11000 copper sheet measuring 120 mm on each side and 2 mm thickness. It underwent the same annealing treatment described in Section 2.1 and fracture occurred at a formed depth of approximately 55 mm (Fig. 7b).

The process was implemented using a Haas Mini Mill 2 vertical CNC machining center with a ball-ended forming tool that had a diameter of 12 mm. The backing plate featured a hole with a diameter of 100 mm, into which the part was formed. Additional process parameters are summarized in Table 4.

The initial blanks were electrochemically etched with a grid of circles, each having an initial diameter of  $d_0 = 2.5$  mm, and the in-plane

strains ( $\epsilon_1, \epsilon_2$ ) of the incremental sheet formed parts were determined using circle grid analysis (CGA). This method calculates strains based on experimental measurements of the major  $l$  and minor  $w$  axes of the ellipses resulting from the deformed circles, as proposed by Keeler (1965).

$$\epsilon_1 = \ln \frac{l}{d_0} \quad (11)$$

$$\epsilon_2 = \ln \frac{w}{d_0} \quad (12)$$

The measurements were performed along meridional directions, from one edge to the center of the sheet blanks, at the moment when cracks were triggered. A computerized digital camera measurement system, model GPA-100, from ASAME was used.

The strains at fracture ( $\epsilon_{1f}, \epsilon_{2f}$ ) could not be obtained from CGA because even very small circles placed near the cracks will provide values that cannot be considered the true fracture strains. Instead, the strains at fracture - also known as the 'gauge length' fracture strains - were calculated from thickness measurements of the cracked region (Embury and Duncan, 1981) using equations (4) and (5). The minor in-plane fracture strain  $\epsilon_{2f}$  (4) was calculated using equation (12) after measuring the minor axis of the ellipse that resulted from the plastic deformation and fracture of the circle during the test. The calculation assumed plane strain characteristics after this last measurement, even though necking was not observed during incremental sheet forming (Jeswiet, 2005). The procedure for measuring thickness and calculating the major  $\epsilon_{1f}$  and thickness  $\epsilon_{3f}$  strains along the cracks followed that described in Section 2.2.

The resulting strain loading path (Fig. 7c) was plotted in principal strain space, showing an excellent agreement with the FFL previously characterized by using tensile, DNT, and DC test specimens (Section 3.2).

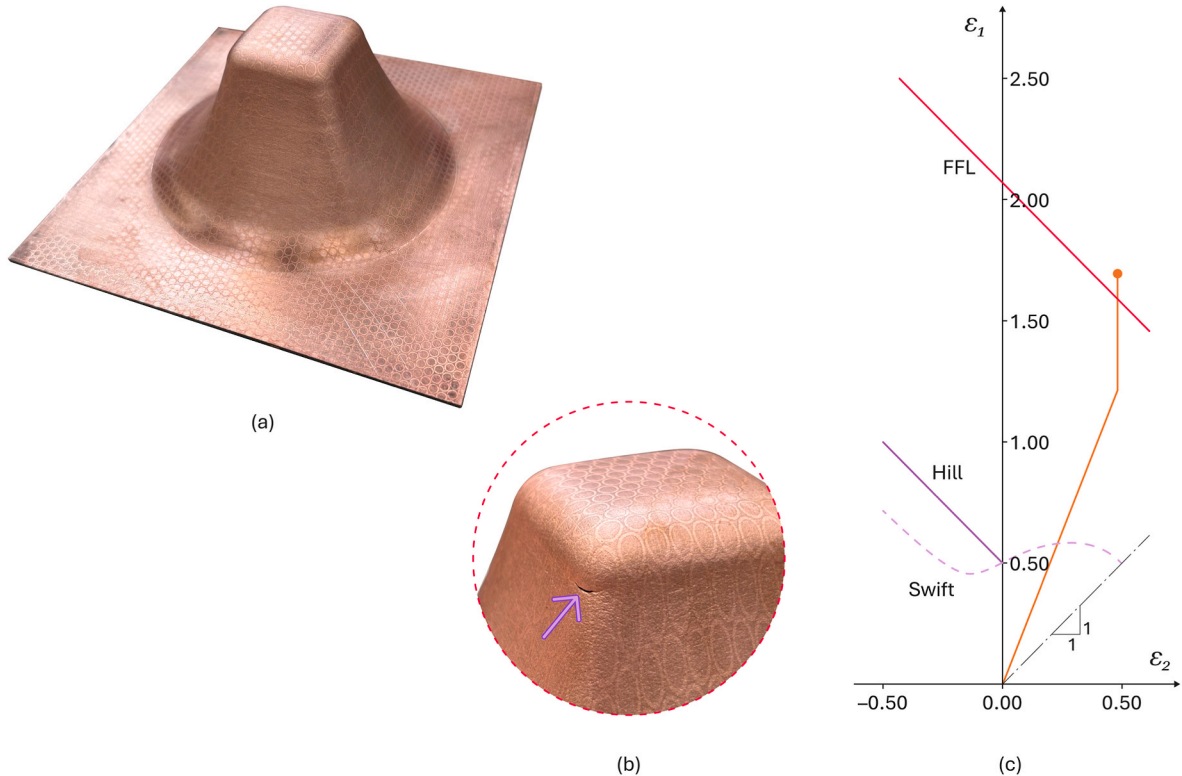


**Fig. 7.** (a) Incremental sheet formed part without fracture and (b) detail of the fracture in a more deformed part. (c) Strain loading path at fracture plotted in principal strain space.

**Table 4**

Process parameters for the single-point incremental sheet forming.

| Spindle speed | Feed rate (mm/min) | Tool path strategy | Maximum stepdown (mm) | Tool path | Lubricant               |
|---------------|--------------------|--------------------|-----------------------|-----------|-------------------------|
| Free rotation | 1000               | Helix ramp         | 0.2                   | Spiral    | Castrol Iloform™ TDN 81 |

#### 4. Conclusions

The newly proposed diagonal-cruciform test specimen effectively induced strain loading paths under biaxial stress conditions during the uniaxial loading of C11000 copper test specimens. The effectiveness of the specimen design was confirmed by integrating these results with those from tensile and double-notched tensile tests, which allowed for the definition of the fracture forming limit (FFL) without requiring specialized experimental setups or multiaxial testing equipment.

The onset of diffuse necking was identified using the strain acceleration method recently proposed by the authors. In this method, the local minimum in the second derivative of the minor principal in-plane strain with respect to time served as a reliable indicator for the onset of diffuse necking, showing close agreement with results obtained from the maximum force criterion. Comparisons between the two methods indicated relative errors of 6.05 % for tensile specimens, 2.80 % for double-notched tensile specimens with  $l_0 = 15$  mm, 4.64 % for double-notched tensile specimens with  $l_0 = 9$  mm, and 0.42 % for diagonal-cruciform specimens. These findings indicate that the strain acceleration method can be effectively applied across various stress states in sheet metal forming. The slightly higher relative errors observed in the tensile test specimens are attributed to the prolonged plateau in the force-time evolution around the maximum force.

Single-point incremental forming of a benchmark truncated circular-to-square pyramidal loft made from C11000 copper sheets with 2 mm thickness was carried out to validate the overall characterization procedure. Results showed that the resulting strain loading paths leading to fracture closely match the fracture forming limit (FFL) previously characterized using tensile, double-notched tensile, and diagonal-cruciform test specimens.

#### CRediT authorship contribution statement

**Rui F.V. Sampaio:** Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Eduardo B.G. Dias:** Investigation, Formal analysis, Data curation. **João M.A. Viegas:** Resources, Investigation, Formal analysis, Data curation. **João P.M. Pragana:** Writing – review & editing, Visualization, Validation, Methodology, Investigation, Formal analysis, Conceptualization. **Ivo M.F. Bragança:** Writing – review & editing, Validation, Methodology, Investigation, Conceptualization. **Carlos M.A. Silva:** Writing – review & editing, Validation, Supervision, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Paulo A.F. Martins:** Writing – review & editing, Writing – original draft, Supervision, Resources, Investigation, Conceptualization.

#### Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

The author is an Editorial Board Member/Editor-in-Chief/Associate Editor/Guest Editor for this journal and was not involved in the editorial review or the decision to publish this article.

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#### Data availability

Data will be made available on request.

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