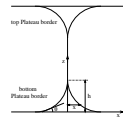


INTRODUCTION

- In confined foams there exist wall Plateau borders (PBs), or menisci, where the films meet the confining substrates.
- What is the shape of a PB of a given size on a surface of a given wettability (i.e., contact angle θ_c)? **Can that surface support a foam?**
- This is important for firefighting foams, containers for foamy foodstuffs, etc.
- We solve the Young-Laplace equation with gravity for a planar film spanning a gap between two horizontal, flat substrates, to predict the shape of the PBs.
- Validate results by comparing with Surface Evolver and experimental data.

THEORY

- Young-Laplace equation for PBs where flat film meets planar substrate:



$$\left[1 + \left(\frac{dx}{dz}\right)^2\right]^{-3/2} \frac{d^2x}{dz^2} = -\frac{\Delta p}{\gamma}$$

where Δp is the pressure difference across the surface and γ is the surface tension.

- Rewrite equation in terms of film inclination $\theta(z)$: boundary conditions are $\theta(0) = \theta_c$, $\theta(h) = \pi/2$.
- Assume hydrostatic PBs, normalise lengths by h and introduce Bond number $Bo = \rho gh^2/\gamma$.
- Analytically exact solutions for bottom (+) and top (-) PBs:

$$x'(z') = \int_{z'}^1 \frac{(1-z'')(\cos\theta_c \pm \frac{Bo}{2}z'') dz''}{\left[1 - (1-z'')^2(\cos\theta_c \pm \frac{Bo}{2}z'')^2\right]^{1/2}}$$

- In zero gravity ($Bo = 0$, top=bottom):

$$x'(z') = \frac{1}{\cos\theta_c} \left\{1 - \left[1 - (1-z')^2 \cos^2\theta_c\right]^{1/2}\right\}$$

SURFACE EVOLVER

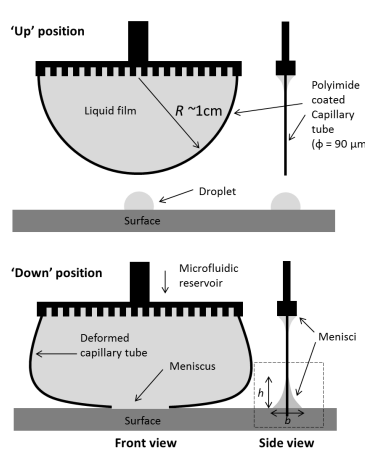
- Discretise each interface and perform direct numerical minimisation of surface energy for a fixed PB area.
- Only half PB simulated, by symmetry.

EXPERIMENT

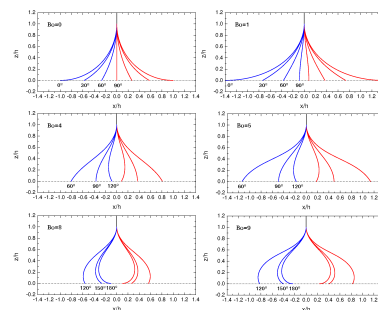
- Contact angle meter (GBX Scientific Instruments, France).
- Commercially available soap solution (Pustefix, Germany), surface tension $\gamma = 28.2 \pm 0.3 \text{ mJ m}^{-2}$.
- Five different solid surfaces:

Material	θ_c (°)
SiO ₂	18.2 ± 2.8
Teflonised polished Si	51.7 ± 0.3
PDMS elastomer	61.0 ± 2.1
Teflonised rough Si	64.0 ± 0.4
Teflonised black Si	109.3 ± 0.3

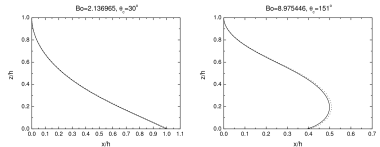
- In-house microfluidic tool consisting of (i) microfluidic reservoir with a number of capillary slots, made of ABS plastic; (ii) thin, flexible, hydrophilic loop which supports liquid film, made of polyimide-coated capillary tubing (Molex, USA). Gives bottom PB shape only, not top.



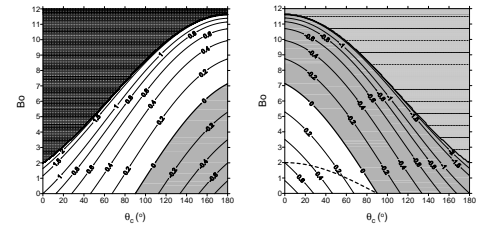
RESULTS



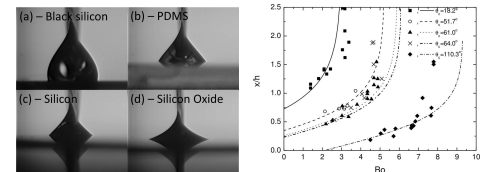
Analytically-calculated PBs at the bottom substrate for various combinations of Bo and θ_c .



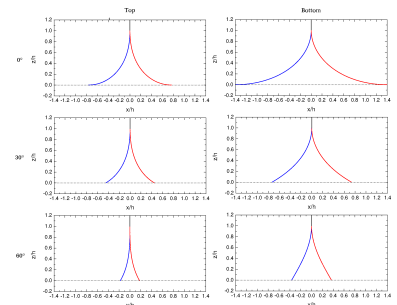
Comparison of bottom PB shapes from analytical theory (solid line) and Surface Evolver (dotted line).



Regions of allowed (white, below dashed line only in right panel) and forbidden (elsewhere) PBs in (θ_c, Bo) space, at the bottom (left) and top (right) substrates. The curves are lines of constant $x'(z' = 0)$.



PBs at four of the five surfaces used in the experiments. Scaled PB half-width $x(z = 0)/h$ vs Bond number, from theory (curves) and experiment (symbols).



Analytically-calculated PBs at the top and bottom substrates, for $Bo = 1$ and three different fixed contact angles. (The PBs at the top substrate are shown inverted for ease of comparison.)

CONCLUSIONS

The combination of a particular surface (θ_c) in contact with a particular foam (ρ and γ) leads to allowed and forbidden wall PBs. Therefore both surface and liquid (foam) properties need to be taken into account in applications where wetting of surfaces by foams is important.