Chapter 1
A Budget Setting Problem

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Abstract Consider a typical agency relation involving a capital owner and a manager. The principal (i.e., the capital owner) has a potential budget to assign to investment projects. The effective amount of investment will be a share of the potential level, given the specific form of interaction that will be established between the principal and the agent (i.e., the manager). The budget setting problem originating from this relation is evaluated from the point of view of the manager, who wants to maximize the received budget, in an intertemporal basis. The optimal control problem is subject to a constraint, which indicates how the assigned budget evolves over time. In this constraint, a matching function takes a central role; the arguments of the function are the agent's effort to absorb new funds and the financial resources the principal has available but has not yet channeled to the manager.

Keywords: Budget setting, Agency relation, Optimal control, Intertemporal optimization, Dynamic analysis.

1.1 Introduction

Agency relations and the information asymmetries they enclose are a main topic of economic analysis. Pioneer work on this subject by Akerlof (1970), Spence (1973), Stiglitz (1974), Jensen and Meckling (1976) and Fama and Jensen (1983), among others, has launched a prolific literature that intends to explain how a *principal* selects an *agent* to act on her / his behalf and to pursue her / his goals. Because principal and agent have different interests and the access to relevant information probably differs among them, there are potential costs involved in this relation, for both players.

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This paper proposes a simple intertemporal optimization model that deals with agency relations in a specific context, namely concerning the interaction of a capital owner with a team of managers that will undertake a series of investment projects over time. The team is selected *a priori*, and therefore we will not be concerned with adverse selection issues. The problem arises when the capital owner has to decide how much financial resources to allocate to the agent. The principal has a given potential budget to assign, but she / he will release the funds only against new business proposals. If the managers do not make any new proposal, the principal will progressively cut the access to funds. From the point of view of the managers, they will be interested in accessing the largest possible portion of the potential budget, however collecting additional funds has a cost, related to the design of business proposals the managers will have to present to the capital owner.

Budget setting is a relevant theme of research in economics, management and management accounting. There are several approaches to this issue in the literature, for instance the ones proposed by Brekelmans (2000) and Gox and Wagenhofer (2007). Our analysis differs from the mentioned studies because it takes a dynamic scenario and because it focus on the matching between the will of the capital owner in transferring funds to new projects and the effort made by managers to present new investment proposals. The matching process is essential for our analysis; we will assume a matching function that is adapted from typical labor market search and matching theory (see Pissarides (1979)).

The techniques employed to solve the intertemporal optimization problem respect the conventional tools used to assess dynamic behavior in low dimensional systems (associated to our maximization problem there will be only one dynamic constraint). See, for instance, Walde (2011) for a thorough analysis of the tools employed to explore this kind of modeling structure.

The remainder of the paper is organized as follows. Section 2 presents and describes the model; section 3 derives optimality conditions and characterizes the steady-state; section 4 approaches the stability of the derived dynamic system; section 5 presents a small numerical illustration; section 6 concludes.

1.2 The Model

Consider a continuous time modeling structure, in which a capital owner has a given potential budget to assign to investment projects. The capital owner has already chosen the team of managers that will undertake the projects and, in the specific stage we are considering, the decision under evaluation is how much financial resources should be transferred to the hands of the managers in order to achieve the best possible outcome from the point of view of the involved parties. This is an agency relation, involving a principal - agent relationship, where the capital owner will assume the role of the principal and the managers will be the agent. Principal and agent may have different or even contradictory interests and, thus, it is vital to understand how they will relate.

The maximum budget available for investment in the activities to be pursued by the hired managers is an invariant in time amount B. Not all of this budget will, presumably, be allocated to the agent and, therefore, we define share $b(t) \in (0,1)$ as the percentage of budget B that at time t is allocated to project development. Under this simple scenario, the two involved entities will have decisions to make. The investor will be concerned with the efficiency with which the managers allocate the financial resources they receive, and wants to choose, in each period, the share of B that best serves this intent. The managers want to receive the highest possible amount of resources they can get, however they will have to incur in a cost to obtain them. Let the cost of acquisition of funds be given by variable $x(t)$. This variable may be interpreted as the costs the agent in the relation must have to support in order to convince the capital owner to release funds. Thus, variable $x(t)$ may be associated with the preparation of proposals by the agent to present projects that the principal will perceive as profitable and, thus, worth releasing additional funds. We should remark that $x(t)$ is a control variable from the point of view of the managers; they choose how much they want to expend in order to obtain the best possible outcome, from their viewpoint, which consists in maximizing, on an intertemporal basis, the difference between the received budget and the project proposal costs.

The mechanism through which funds are assigned to the managers is the result of a matching process, between the agent's project proposals effort, measured by the value of variable $x(t)$, and the budget share that at a given time period the capital owner can potentially assign to the investment projects; this corresponds to the funds, from budget B, not yet allocated to those projects, i.e., $[1-b(t)]B$. The matching process is expressed under the form of a matching function with the following features.

Definition 1. Take a real-valued function $f : \mathbb{R}_+^2 \longrightarrow \mathbb{R}_+$. Function f is a matching function, such that $y(t) = f\{x(t), [1-b(t)]B\}$. The output of the function, $y(t)$, represents the budget transferred from the principal to the agent, at period t, given the values of the respective inputs. Function f contemplates the following properties:

i) f is continuous and differentiable;

ii) f is an increasing function in both arguments: $f_x > 0$, $f_{(1-b)B} > 0$;

iii) f is subject to decreasing marginal returns, relating each of its inputs: $f_{xx} < 0$, $f_{(1-b)B,(1-b)B} < 0$;

iv) f is homogeneous of degree 1: $f\{\varepsilon x(t), \varepsilon[1-b(t)]B\} = \varepsilon f\{x(t), [1-b(t)]B\}$, $\forall \varepsilon > 0$;

v) Both inputs are essential to reach a positive output, i.e., $f\{0, [1-b(t)]B\} = f[x(t), 0] = 0$.

An explicit functional form that obeys the listed properties is, for instance, a Cobb-Douglas specification of the matching process, i.e., $f\{x(t), [1-b(t)]B\} = Ax(t)^{\alpha}\{[1-b(t)]B\}^{1-\alpha}$, $A > 0$, $\alpha \in (0,1)$. The mentioned properties have, all of them, an intuitive interpretation; most importantly, they indicate that a stronger effort by the team of managers in presenting new project proposals and a larger available budget produce, evidently, a better matching result, that is translated in a more generous fund transfer. Logically, there are diminishing returns in this relation: with higher values of each of the inputs, matching will continue to occur but at a progressively lower rate. Observe, as well, that the inputs in the matching function are both indispensable to deliver a meaningful outcome: if the managers do not present any investment proposal, they will receive no funds even though these are potentially available; similarly, if the capital owner has no funds left to assign to this class of projects, the matching result will be null no matter how much effort the managers employ in presenting new proposals.

Another central assumption that we take is that the capital owner forces the agent to present innovative projects in order to continue to receive funds. To impose this behavior on the part of the agent, the principal will automatically cut a share $\lambda \in (0,1)$ of the budget assigned in the previous period from the budget of the current period. The agent can only recover this value by presenting new projects; if it fails to do so, the budget will shrink over time and converge, in the long-run, to zero. The capital owner is not interested in maintaining an agency relation with managers who do not show a will to innovate and this mechanism functions as a way to impose the presentation of new projects in case the managers want to continue to receive funds to develop business activities.

Under the above assumptions, the following differential equation will characterize the dynamics of budget assignment,

$$\dot{b}(t) = f\{x(t), [1-b(t)]B\} - \lambda b(t),\ b(0) \text{ given} \tag{1.1}$$

In the proposed context, the agent wants to maximize, in an intertemporal basis, the value of its available financial resources. We designate these resources by $\theta(t)$ and define them as the difference between the received budget and the costs incurred to propose new projects, i.e., $\theta(t) := b(t)B - x(t)$. Since the problem is of a dynamic nature, the objective function of the managers will be $\Theta(t) := \int_0^\infty \theta(t)\exp(-\rho t)dt$. Parameter $\rho > 0$ is the rate at which the future is discounted to the present. An infinite horizon is considered because we have not established an ending date for the agency relation; furthermore, the consideration of a positive discount rate makes far in time outcomes negligible from the current period point of view.

The above reasoning has conducted us to an optimal control problem, that the agent will want to solve, in which the value of $\Theta(t)$ is maximized, given resource constraint (1.1). The problem is relevant, from an economic point of view, because it contemplates a trade-off: a low effort in searching for new funds will not allow the managers to access a high budget; an excessively strong effort to collect new funds may lead to a higher budget but at an exaggerated cost; somewhere in the middle, the optimal solution will be found: by maximizing $\Theta(t)$, the agent will arrive to optimal trajectories for the two endogenous variables of this setup: the control variable $x(t)$, and the state variable $b(t)$.

Worthwhile noticing, in this environment, is the specific role of variable $b(t)$, the share of the potential budget that is delivered to the managers. This is not a control variable either for the principal or for the agent. Its value is the result of a pre-specified rule through which the capital owner attributes funds. It is the choice of the agent, which acts optimally, and the choice of the principal, concerning the values of the overall budget (B) and the rate at which it cuts managers' funds (λ), that will determine the specific path one will encounter for the assigned budget.

The next section will present the steps required to solve the optimal control problem.

1.3 Solution of the Optimal Control Problem

As described in the previous section, a team of managers is interested in addressing the following optimal control problem,

$$\begin{aligned} &\underset{x(t)}{Max} \int_0^\infty \theta(t)\exp(-\rho t)dt \\ \text{subject to } &: \\ \dot{b}(t) &= f\{x(t),[1-b(t)]B\} - \lambda b(t) \\ &b(0) \text{ given} \\ \theta(t) &:= b(t)B - x(t) \end{aligned}$$

Computation of first-order conditions allows to find an equation of motion for variable $x(t)$ that is valid under the agent's optimal behavior.

Proposition 1. *Assume a Cobb-Douglas matching function. If the agent maximizes, intertemporally, the value of its financial resources, the time trajectory of its control variable, $x(t)$, will be governed by the following law of motion,*

$$\dot{x}(t) = \frac{1}{1-\alpha}\left(\rho + \lambda\frac{1-\alpha b(t)}{1-b(t)} - \alpha A\left\{\frac{[1-b(t)]B}{x(t)}\right\}^{1-\alpha}\right)x(t) \tag{1.2}$$

Proof. To arrive to the solution of the optimization problem, we start by presenting the respective current-value Hamiltonian function,

$$H[x(t),b(t)] = b(t)B - x(t) + p(t)\left(f\{x(t),[1-b(t)]B\} - \lambda b(t)\right)$$

With the Hamiltonian function a new variable is introduced, namely the co-state or shadow-price variable $p(t)$, which can be interpreted as a kind of Lagrange multiplier for this dynamic setting. From the Hamiltonian, we draw the first-order optimality conditions of the problem,

$$H_x = 0 \Rightarrow p(t)f_x = 1$$

$$\dot{p}(t) = \rho p(t) - H_b \Rightarrow \dot{p}(t) = (\rho+\lambda)p(t) - B - f_b$$

The transversality condition $\underset{t\to\infty}{lim} p(t)\exp(-\rho t)b(t) = 0$ must be satisfied as well. The first-order conditions correspond, under Cobb-Douglas matching, to

$$\alpha A\left\{\frac{[1-b(t)]B}{x(t)}\right\}^{1-\alpha}p(t)=1 \tag{1.3}$$

$$\dot{p}(t)=(\rho+\lambda)p(t)-B+(1-\alpha)A\left[\frac{x(t)}{1-b(t)}\right]^{\alpha}B^{1-\alpha} \tag{1.4}$$

The differentiation of (1.3) with respect to time yields

$$\frac{\dot{p}(t)}{p(t)}=(1-\alpha)\left[\frac{\dot{x}(t)}{x(t)}+\frac{\dot{b}(t)}{1-b(t)}\right] \tag{1.5}$$

By replacing the price motion by the corresponding expression in (1.4) and the motion of the budget share as given by (1.1), we can transform expression (1.5) in a dynamic equation for the control variable $x(t)$. After some computation, we arrive to differential equation (1.2) as presented in the proposition.

At this stage, we are in the possession of the information required to analyze the optimal dynamics of the problem under evaluation. A two-dimensional system, involving two endogenous variables is composed by equations (1.1) and (1.2). The study of the dynamics requires looking at the steady-state outcome and respective stability properties. For now, in this section, we concentrate on the steady-state properties.

As it is usual in this kind of model, we define the steady-state as the long-run position for which the system eventually tends and where the respective endogenous variables have stopped growing, i.e., it will correspond to the pair of values $(x^*,b^*)=\{(x^*,b^*):\dot{x}(t)=0;\dot{b}(t)=0\}$. Explicit solutions for (x^*,b^*) with respect to the parameters of the model, are not feasible to present. Nevertheless, the following relations are straightforward to obtain and will be useful when approaching the stability result,

- From (1.1):

$$A\left(x^*\right)^{\alpha}\left[(1-b^*)B\right]^{1-\alpha}=\lambda b^* \tag{1.6}$$

- From (1.2):

$$\alpha A\left[\frac{(1-b^*)B}{x^*}\right]^{1-\alpha}=\rho+\lambda\frac{1-\alpha b^*}{1-b^*} \tag{1.7}$$

The above conditions allow to state the following result

Proposition 2. *In the budget setting problem with a Cobb-Douglas matching function, a steady-state exists and it is unique.*

Proof. Equation (1.6) may be solved in order to x^*, what delivers the outcome

$$x^*=\left\{\frac{\lambda b^*}{A\left[(1-b^*)B\right]^{1-\alpha}}\right\}^{1/\alpha} \tag{1.8}$$

Replacing the value of x^* as presented above into (1.7), one obtains an equilibrium equation solely for steady-state value b^*,

$$\alpha A^{1/\alpha}\left[\frac{(1-b^*)B}{\lambda b^*}\right]^{\frac{1-\alpha}{\alpha}} = \rho + \lambda\frac{1-\alpha b^*}{1-b^*} \tag{1.9}$$

Although condition (1.9) does not allow to obtain an explicit value for b^*, it is straightforward to observe that it has a solution and that this solution is unique. The left hand side (lhs) of the condition is a decreasing function, while the right hand side (rhs) is increasing, as the first derivatives show,

$$\frac{d(\text{lhs})}{db^*} = -(1-\alpha)A^{1/\alpha}\left[\frac{(1-b^*)B}{\lambda b^*}\right]^{\frac{1-2\alpha}{\alpha}} \frac{B}{\lambda\,(b^*)^2} < 0$$

$$\frac{d(\text{rhs})}{db^*} = \lambda\frac{1-\alpha}{(1-b^*)^2} > 0$$

Thus, the lhs of (1.9) is a decreasing function, starting at infinity, for $b^* = 0$, and falling towards zero as b^* grows to its maximum value, $b^* = 1$. The rhs of (1.9) is an increasing function such that rhs$= \rho + \lambda$ for $b^* = 0$ and rhs$\to \infty$ as b^* tends to 1. This reasoning implies that the lhs and the rhs will necessarily cross and that they will cross only once in the domain defined for b^*. This proves the claim in the proposition: only one value of $b^* \in (0,1)$ satisfies the conditions underlying the proposed analytical setup. Once in possession of the equilibrium value of the budget share, the steady-state level of x^* is straightforward to find, given (1.8).

In the following section, the system's stability will be addressed.

1.4 Stability

Having arrived to a unique steady-state point (x^*,b^*), we can now address the stability properties of such steady-state. To undertake this study, we first need to linearize the system of dynamic equations in the vicinity of (x^*,b^*). For such, one has to compute the respective Jacobian matrix, which is composed by the derivatives of each of the equations with respect to each of the endogenous variables, duly evaluated in the steady-state.

The respective computation leads to the following outcome,

$$J = \begin{bmatrix} -\lambda\frac{1-\alpha b^*}{1-b^*} & \alpha\lambda\frac{b^*}{x^*} \\ \lambda\frac{x^*}{(1-b^*)^2} + \frac{x^*}{1-b^*}\left(\rho + \lambda\frac{1-\alpha b^*}{1-b^*}\right) & \rho + \lambda\frac{1-\alpha b^*}{1-b^*} \end{bmatrix} \tag{1.10}$$

Matrix J in (1.10) is the Jacobian matrix of the linearized system,

$$\begin{bmatrix} \dot{b}(t) \\ \dot{x}(t) \end{bmatrix} = J \times \begin{bmatrix} b(t) - b^* \\ x(t) - x^* \end{bmatrix} \tag{1.11}$$

The stability properties of the steady-state will be dependent on the signs of the eigenvalues of matrix J. Negative eigenvalues correspond to stable dimensions and positive eigenvalues are associated with unstable dimensions. The evaluation of the matrix conducts to the following result,

Proposition 3. *The dynamic system underlying the budget setting problem, as formulated, delivers a saddle-path stable equilibrium.*

Proof. One can arrive to the signs of the eigenvalues by computing the trace and the determinant of matrix (1.10). The value of the trace is immediately found by looking at the matrix: $Tr(J) = \rho$; the determinant will take the expression $Det(J) = -\frac{\lambda}{1-b^*}\left(\rho + \frac{\lambda}{1-b^*}\right)$.

Clearly, the trace is a positive value, while the determinant is negative, meaning that one of the eigenvalues is positive while the other is necessarily negative.[2] In this circumstance, the two-dimensional space we are dealing with involves a stable dimension and an unstable dimension, i.e., the equilibrium is saddle-path stable. The eigenvalues could also be computed directly from the matrix. In the case of this system they are relatively straightforward to derive: $\lambda_1 = -\frac{\lambda}{1-b^*} < 0$; $\lambda_2 = \rho + \frac{\lambda}{1-b^*} > 0$.

The saddle-path stable equilibrium is a convenient result in the type of optimal control problem we have just addressed. Because we have two kinds of variables, a state variable and a control variable, saddle-path stability is sufficient to guarantee convergence from any initial state (x_0, b_0) in the vicinity of the steady-state towards this second position, i.e., to point (x^*, b^*). The agent can adjust the value of x^* in order to place the system over the stable arm and, as a result, guarantee the stability of the equilibrium.

An additional step can be taken in the analysis of the stability properties. Namely, one might compute the expression of the stable trajectory. The generic expression of the stable path is given by

$$x(t) - x^* = \frac{p_2}{p_1}\left[b(t) - b^*\right]$$

Elements p_1 and p_2 in the slope of the expression correspond to the elements of the eigenvector of J that relate to the negative eigenvalue (the one that represents stability). The calculus of the elements of the eigenvalue imply the following outcome:

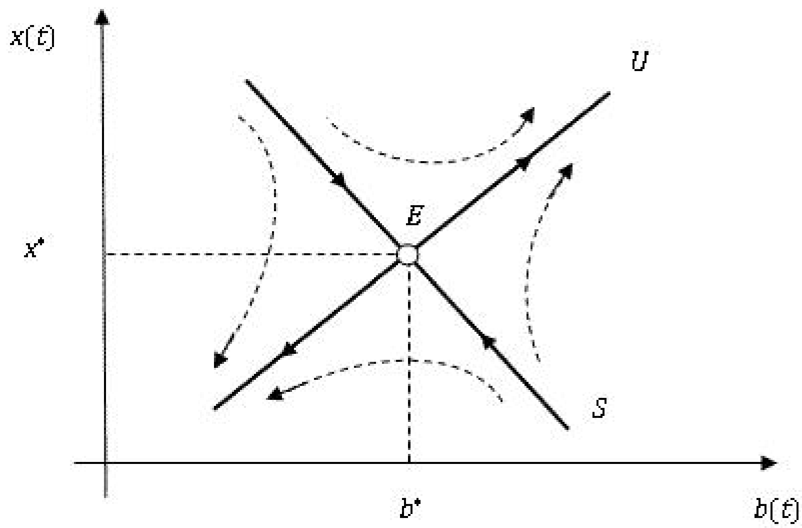
$$x(t) - x^* = -\frac{x^*}{1-b^*}\left[b(t) - b^*\right]$$

[2] Recall that, for any square matrix of order 2, $Tr(J) = \lambda_1 + \lambda_2$ and $Det(J) = \lambda_1\lambda_2$, for λ_1 and λ_2 the eigenvalues of the matrix.

or, equivalently,

$$x(t) = \frac{x^*}{1-b^*}\,[1-b(t)] \tag{1.12}$$

The expression of the stable trajectory, (1.12), indicates that once the agent has adjusted her / his behavior in terms of the resources allocated to design new investment proposals, the path that will be followed and that will lead to the long-term equilibrium point is negatively sloped, i.e., as $x(t)$ increases, the budget share assigned to the managers, $b(t)$, will decline. This can be interpreted as follows: starting, for instance, at a point (x_0, b_0) for which $x_0 > x^*$ and $b_0 < b^*$, the convergence to the steady-state will be characterized by a process in which the costs incurred to gather additional budget will diminish as the budget share grows until reaching their steady-state values. Figure 1 sketches the dynamic relation between the two variables of interest.



Next section provides a small numerical example to illustrate the dynamics of the model.

1.5 Numerical Illustration

Assume the following array of parameter values: $\alpha = 0.75$; $A = 0.1$; $B = 100$; $\lambda = 0.1$; $\rho = 0.05$. The value of α indicates that three quarters of the matching process depends on the project proposals and only one quarter is attributable to the available budget; the value of λ states that at each period 10% of the budget previously assigned to the agent is withdrawn by the capital owner; the value chosen for ρ imposes a 5% intertemporal discount rate. The other values are not specially relevant and just determine the scale of the analysis.

With specific numerical values, it is possible to compute the steady-state pair (x^*, b^*). The evaluation of condition (1.9) leads to result $b^* = 0.769$, i.e., 76.9% of the budget the capital owner has available for the agent's activities is effectively channeled for the projects, given the matching process. The steady-state value of the cost proposal variable is obtainable from (1.8), $x^* = 0.247$.

If we change parameter values, the steady-state will be subject to perturbations. The direction of such perturbations should be intuitive. Table 1 indicates the impact over equilibrium of changing the value of each parameter, one at a time,

	$\alpha = 0.8$	$A = 0.12$	$B = 105$	$\lambda = 0.11$	$\rho = 0.075$
b^*	0.717	0.823	0.773	0.740	0.739
x^*	0.286	0.232	0.247	0.257	0.225

Table 1 - Steady-state results under different parametrizations.

The table considers, for each case represented in a column, a different combination of parameters; the original parameterization is maintained with exception of the indicated change. This allows us to understand what is the impact over both endogenous variables, in terms of their steady-state values, when those changes occur. Results are intuitive: if the matching depends relatively more on the agent's effort in gathering additional funds, this will make b^* to decrease and x^* to increase; when the efficiency of the overall matching process increases (larger A), this implies an increase in b^* and a fall in x^* (a larger equilibrium budget is obtained with less resources allocated to attain such goal); the increase in the overall budget, relatively to the benchmark situation, does not change x^*, but makes b^* to increase; a larger automatic cut in the assigned budget will imply a new steady-state locus such that b^* falls and x^* increases; finally, a higher discount rate will signify that less attention will be given to the far future and this translates in a fall of both equilibrium values.

Let us return to the benchmark parametrization. With these values, we confirm the existence of a saddle-path stable equilibrium in this specific case, because the eigenvalues of the Jacobian matrix are $\lambda_1 = -\frac{0.1}{1-0.769} = -0.433$; $\lambda_2 = 0.05 + \frac{0.1}{1-0.769} = 0.483$. The saddle trajectory, given by (1.12), is $x(t) = 1.069\,[1-b(t)]$; in this case, in the convergence towards (x^*, b^*) following the saddle path, as $b(t)$ increases one unit, $x(t)$ will fall 1.069 units.

1.6 Conclusion

The paper presented a dynamic optimization problem concerning the allocation of a budget from a capital owner to a manager or a team of managers. The problem is meaningful because it involves a trade-off: in this agency relation, the agent wants to maximize the value of the budget she / he can apply for, but this comes with a cost: to obtain additional funds, the manager will have to prepare new investment proposals that imply spending resources. On the principal's side, there is a maximum budget that the capital owner is willing to transfer to the manager, but the budget effectively transferred will depend on the capacity of the agent to present new projects; if these are not presented, the budget will be progressively cut over time at a constant rate.

Although simple, this theoretical structure is rich enough to deliver interesting results: an equation of motion for the control variable of the problem is derived and, evaluating such equation together with the rule that describes how the assigned budget evolves, one can address the stability of the long-run result. The respective dynamic system is saddle-path stable, what is sufficient to guarantee convergence towards the equilibrium, since one of the variables is a control variable and, thus, the corresponding value can be adjusted in order for the system to follow the saddle trajectory in the direction of the steady-state, where the values of the endogenous variables will end up by remaining constant.

In economics, as well as in other research fields, agency relations are common and subject to important discussion. The proposed model intends to contribute to this literature by furnishing a general framework of analysis. The framework is particularly suited to study the allocation of funds in situations where this allocation depends on proposals made by those who want to access the funds; for instance, the application to research grants by teams of scientists could be a relevant setting to explore further the possibilities of this setup.

References

1. Akerlof, G.A. (1970). "The Market for 'Lemons': Quality Uncertainty and the Market Mechanism." *Quarterly Journal of Economics,* vol. 84, pp. 488–500.
2. Brekelmans, R. (2000). *Stochastic Models in Risk Theory and Management Accounting*, PhD thesis, Tilburg University.
3. Fama, E.F. and M.C. Jensen (1983). "Agency Problems and Residual Claims." *Journal of Law & Economics,* vol. 26, pp. 327–349.
4. Gox, R.F. and A. Wagenhofer (2007). "Economic research on Management Accounting." in T. Hopper, D. Northcott and R. Scapens (eds.), *Issues in Management Accounting*, 3rd edition, Harlow, England: Pearson Education Limited. Chapter 19, pp. 399.424.
5. Jensen, M.C. and W.H. Meckling (1976). "Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure." *Journal of Financial Economics*, vol. 3, pp. 305–360.
6. Pissarides, C.A. (1979). "Job Matching with State Employment Agencies and Random Search." *Economic Journal*, vol. 89, pp. 818-833.
7. Spence, M. (1973). "Job Market Signaling." *Quarterly Journal of Economics,* vol. 87, pp. 355–374.
8. Stiglitz, J. (1974). "Incentives and Risk Sharing in Sharecropping." *Review of Economic Studies*, vol. 41, pp. 219–255.
9. Walde, K. (2011). *Applied Intertemporal Optimization*. Mainz, Germany: Mainz University Gutenberg Press.