A Model of Animal Spirits via Sentiment Spreading

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Abstract: In order to incorporate animal spirits in a scientifically rigorous inquiry about the causes of aggregate business cycles, one needs to explore the foundations of human behavior, namely concerning the process through which sentiment switching occurs. Which factors drive human sentiments? In what conditions a pessimistic individual becomes an optimist, or the other way around? Is it possible to justify persistent waves of optimism and pessimism under reasonable assumptions concerning social behavior? This article proposes a framework to address the posed questions. The setup is based on rumor propagation theory and it explains how social interaction may lead individuals to change from one sentiment state to the other, eventually triggering a rotation between periods of dominant optimism and periods of dominant pessimism.

Keywords: Animal spirits; Waves of optimism and pessimism; Sentiment propagation; Steady-state analysis; Transitional dynamics; Endogenous fluctuations; Interactive Markov chain.

JEL classification: C61; D83; D85.

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Introduction

The Keynesian notion of animal spirits is part of the lexicon of any economist. While many agree that the economic analysis should concentrate on explaining the actions of goal oriented rational agents that take decisions on a cost-benefit basis, others believe that the performance of the economy as a whole, namely in what relates short-term fluctuations, is inseparable from the psychological factors that recurrently influence human deliberation.

In order to rehabilitate the notion of animal spirits and bring it back to the mainstream of economic research, three steps are required: First, one must look at the social process through which waves of optimism and pessimism might be triggered; second, sentiment cyclicality must be associated to economic behavior, for instance through its impact on agents' expectations. Third, macroeconomic models must be modified in order to incorporate the new behavioral or expectational setting.

This article concentrates on the first of the above steps, i.e., on how animal spirits arise in the context of a given social interaction environment. A social network is considered. In this network, at each time period t, every individual belongs to one of two classes, optimists and pessimists, and each class contains three categories. Both optimists and pessimists may be exuberant spreaders of the respective sentiment, susceptible individuals that may switch sentiment when interacting with an exuberant in the other class, and non-susceptible agents, who are in one of the sentiment classes, do not spread such sentiment but are not subject, as well, to the eventuality of being "infected" by the opposite sentiment.

The proposed setup involves a continuous closed loop flow in which individuals in one sentiment category may switch to another category as they interact with other individuals in the network. The transition rules to be adopted are simple and stylized. However, as discussed later, with the presentation of the model, they are adequate to implement a framework where the divergence to a state of universal optimism or universal pessimism is excluded. This is a convenient outcome, because it is empirically supported: No society or economy displays an aptitude to evolve to a long-term locus where all individuals share the same sentiment and where the possibility of changing from one sentiment to the other is excluded for all the eternity.

When analyzing the characterized process from an aggregate point of view, one will be able to draw some meaningful conclusions about the evolution in time of overall market sentiments. A first, simpler version, of the model points to the existence of a fixed-point steady-state in which the densities of optimists and pessimists in the population remain constant. However, it is straightforward to change this setting in order to introduce the possibility of perpetual waves of optimism and pessimism. In the specific framework that is offered, this occurs by imposing the assumption that the transition from the non-susceptibility state to the susceptibility state will be dependent on the rate at which the shares of optimistic and pessimistic agents evolve.

Note that even the fixed-point steady-state outcome does not signify, under the proposed setup, a state of individual sentiment immutability. Individuals will, systematically, change from one sentiment category to another but, in this case, in such a way that the global percentage of individuals in each category remains constant. Thus, even the fixed-point equilibrium involves creating micro-dynamics. In any circumstance, this framework is, then, a step forward relative to the neoclassical economics notion of static equilibria that underlies most of the mainstream economic thought. At this respect, the reasoning in this article follows and supports the view of Dore and Rosser (2007), according to whom nonlinearities, regime shifts, structural breaks, irreversibilities and lagged dependencies are pervasive to economic phenomena.

The work mentioned in the above paragraph interprets the economic science as being on the verge of a paradigm shift that ought to lead to a re-examination of the reality re-centered on the notions of nonlinear dynamics and endogenous fluctuations. This also implies a more interdisciplinary approach to economic problems, an approach that should make use of the powerful instruments already available in other sciences, namely in what concerns chaos theory, agent-based modeling and complexity analysis. With the intention of modestly contributing to the referred paradigm shift, this article offers a model that works as a device to explain and make operational sentiment switching in an economic or social network. Heterogeneity, decentralized interaction and out-of-equilibrium dynamics, all features that reflect economic complexity, are basic components of the framework, which serves the purpose of firmly advocating that to introduce animal spirits in mainstream economics and, as a consequence, to give a consistent explanation for observable business fluctuations, one needs to definitively depart from the neoclassical paradigm of full efficiency, hyper-rationality and complete markets.

The framework to explore is based on rumor spreading theory, as built upon the contributions of Daley and Kendall (1964, 1965) and Maki and Thompson (1973). Rumor theory considers three categories of agents, ignorants, spreaders and stiflers, but involves a steady-state with a nature that is different from the one underlying the analysis in this article, since individuals in the spreader category tend to vanish over time and only two types of agents subsist in the long-run: Those who never heard the rumor and those who are acquainted with it but are not motivated to continue to spread it. The literature on rumor propagation is vast, with meaningful contributions being produced in the last few years. The extensions to the original framework go essentially in two directions: On one hand, additional features, as trust, skepticism, forgetting and remembering mechanisms, and additional players, as incubators or hibernators, are added in order to complete and extend the benchmark environment (see, e.g., Chen, Shen, Ye, Chen & Kerr, 2013; Cheng, Liu, Shen & Yuan, 2013; Huang, 2011; Huo, Huang & Guo, 2012; Thompson, Estrada, Daugherty & Cintron-Arias, 2003; Wang, Yang, Han & Wang, 2013; Zhao, Qiu, Wang & Wang, 2013; Zhao, Xie, Gao, Qiu, Wang & Zhang, 2013); on the other hand, attention is paid to the structure of interaction. Homogeneous networks, with each node possessing the exact same number of links to other nodes are relatively straightforward to analyze; the analysis becomes harder in the presence of inhomogeneous networks of several types (see, e.g., Borge-Holthoefer & Moreno, 2012; Li & Zhao, 2011; Nekovee, Moreno, Bianconi & Marsili, 2007; Zanette, 2002).

The only recent attempt in the literature to adapt the rumor spreading framework to the context of sentiment contagion has been conducted by Zhao, Wang, Huang, Cui, Qiu and Wang (2014), who define transition probabilities for the binary emotional state optimism and pessimism, in parallel with the transition probabilities usually taken in the case of the rumor spreading framework. Nevertheless, the current study goes deeper, since it maintains the ignorant - spreader - stifler structure of interaction, adapting it to a susceptible - exuberant- non-susceptible setup, in order to explain waves of optimism and pessimism that may constitute the foundations of a macro theory of animal spirits. A relevant conclusion will be that, in the same way a spreader is needed to start the propagation of a rumor, at least an exuberant individual is required to exist in some initial state in order for the process of sentiment switching to be ignited and, from the initial point forward, perpetuated over time.

The remainder of the article is organized as follows. The second section makes an assessment of the existing literature on animal spirits in macroeconomics, in order to support the opportunity of this study. The third section lays out the social structure in which agents interact. The fourth section addresses sentiment dynamics, and the following two sections characterize, respectively, the steady-state and the local dynamics of the model. The seventh section explains in what conditions the framework is adaptable to a setting of perpetual waves of optimism and pessimism. The eighth section presents a more compact version of the model, not so complete and informative but more straightforward to analyze from a general point of view. A final section concludes.

Literature on Macroeconomics and Animal Spirits

Among the many contributions that John Maynard Keynes has offered with his General Theory of Employment, Interest and Money, published in 1936, one of the most outstanding ideas relates to the concept of animal spirits. Keynes understood that sentiments, instincts and emotions necessarily drive human action, even when strict economic decisions are subject to deliberation. Despite the evident relevance of the mentioned behavioral features, economists since then preferred to focus the attention on rational choices; idiosyncratic individual behavior naturally exists, but it is interpreted as being irrelevant to determine aggregate outcomes, since individuals avoid incurring in systematic mistakes and evaluation errors are strongly penalized by market relations.

The erratic trajectories followed over time by some of the most relevant macroeconomic variables contradict the strict rationality view. Episodes as the great depression of the 1930s or the great recession of the end of the first decade of the 21st century are not compatible with an interpretation of reality where agents' confidence and sentiments are absent. Apparently, the economic science has deviated from its true purpose: To explain how decision-making at the individual and collective levels generate unexpected and unpredictable outcomes that decisively depart from what a rational agent model would predict.

With the great recession, economic theory has explicitly recovered the notion of animal spirits in order to build a more consistent explanation for aggregate phenomena in the sphere of economics. The book by Akerlof and Shiller (2009) launches this discussion and offers a guide on how to recover such notion and turn it into a relevant ingredient of economic analysis. In the words of these authors, the causes of economic events are largely mental in nature; agents are often guided by noneconomic motivations and their choices are frequently misguided. Moreover, the concept of animal spirits, as interpreted by the mentioned authors, also involves an idea of cyclicality: Economies go through booms and recessions because the individuals' willingness to consume, invest or participate in the economic activity in any other way oscillates over time, generating periods of overconfidence and pessimism that are, then, reflected in the performance of the economy.

On an animal spirits interpretation about economic decisions, business cycles are inseparable from the mood swings or the waves of optimism and pessimism that are often observed and that many times do not have a direct causality link with the observed performance of the economy. To be more specific on what they mean by animal spirits, Akerlof and Shiller (2009) identify five different behavioral features that may explain departures from full rationality and produce the non-equilibrium economic outcome that is often observed in reality; they are: Confidence, fairness, corruption and antisocial behavior, money illusion and stories. The approach in this article will concentrate attention in the first of these items; agents interact in a given social network and sentiments of optimism and pessimism propagate across the economy, as the interaction process takes place, generating peaks and troughs of confidence.

The mentioned book is not a scientific treatise. Nevertheless, it launched important interrogations and has inspired a revival of the economic research on the Keynesian notion of animal spirits. The most recent effort on bringing back animal spirits to the scientific debate on business cycles has been conducted, among others, by De Grauwe (2011, 2012), Milani (2011), Bidder and Smith (2012), Franke (2012) and Angeletos and La'O (2013). In general terms, these authors agree that market sentiments are one of the probable causes of short-run fluctuations, and recognize that this idea has been left out from mainstream macroeconomic research since the rational expectations revolution took place.

Although resorting to different approaches, the above mentioned studies share the same desire of incorporating in standard general equilibrium models some sort of deviation from sentiment neutrality. Individual decisions are not independent from the influence others exert upon our own behavior, what leads to well documented phenomena as herd behavior or peaks and troughs on the confidence of economic agents.

Waves of optimism and pessimism might be the outcome of a multiplicity of factors, such as communication constraints, misinterpretation of market signals, adoption of perceived laws of motion that differ from the actual laws, inefficient learning, costs in acquiring and processing information or cognitive constraints that lead individuals to decide based on heuristics. These sentiment fluctuations will, then, impact the economy e.g. by provoking biases in the formation of expectations or by changing the nature of market relations, what can even lead to a redefinition of the morphology of the markets.

As highlighted by De Grauwe (2011), the introduction of animal spirits in macroeconomic theory may open the door to a richer and more substantive analysis of observable phenomena and to the discovery of solutions to puzzles that challenge the economic knowledge. Under animal spirits, the representative agent paradigm is no longer an acceptable benchmark to analyze market relations. Heterogeneity becomes a necessary part of the discussion and the coexistence of different sentiments allows for an insightful investigation of the relations that take place in a decentralized economy. Nevertheless, some care is required: The rational expectations paradigm is founded on solid logical principles; a theory of animal spirits can only aspire to compete with the rationality paradigm if it is equally supported on the same strong foundations of rigorous scientific analysis.

Although the mentioned studies constitute the most recent attempt to integrate animal spirits into the mainstream economic analysis, there were earlier contributions that are worth citing. A relevant strand of literature is the one pursued by a group of prominent economists that intended to preserve the integrity of the benchmark neoclassical dynamic general equilibrium model, while allowing for decisions based on discretionary agents' choices. This work, that offered the so called “sunspots” interpretation of the macro economy, was developed in the 1980s and 1990s by, among others, Azariadis (1981), Cass and Shell (1983), Spear (1984), Farmer and Guo (1994) and Benhabib and Farmer (1994). This strand of thought is still active currently (Farmer, 2011). The notion of “sunspots” is associated to the idea of extrinsic variables determining economic outcomes in a way that is unrelated to economic fundamentals.

The “sunspots” literature explicitly maintains intact the rational expectations assumption but, simultaneously, it also allows for the emergence of fluctuations as a result of self-fulfilling prophecies. Specifically, a formal neoclassical model of a frictionless economy with a continuum of steady-states (multiple equilibria) is taken, and animal spirits are associated to the idea that, under this indeterminacy setting, agents' beliefs constitute the force that is responsible for selecting one of the equilibrium points from those that are available. It is the decision of the agents, that goes beyond the strict evaluation of economic fundamentals, that will determine the solution of the model, i.e., the trajectories followed by the main economic aggregates. In this case, a path is followed because it is believed that such a path is the one to follow, implying a self-fulfilling prophesy by economic agents.

Animal spirits arise also, with considerable frequency, in the scientific literature on finance. Amongst the most prominent attempts to associate financial market outcomes to sentiment contagion and sentiment heterogeneity, one finds the work of Banerjee (1992) and Shiller (1995), on herd behavior, and the contributions of Brock and Hommes (1997, 1998), on rational routes to randomness (i.e., chaotic dynamics) emerging from discrete choice interaction between heterogeneous agents (namely, agents classified as fundamentalists and chartists).

The new literature on animal spirits, which has gained vitality in the aftermath of the great recession, necessarily builds on the mentioned previous contributions. Some of the work in progress emphasizes the interaction across heterogeneous agents, while others focus on the compatibility of sentiment formation with the representative rational agent paradigm. This literature needs to go further and deeper; it must explore the foundations of interaction processes, as suggested by many theorists in the fields of complexity and agent-based models (see, concerning this point, among the most influential contributions, Dosi, Fagiolo & Roventini, 2008; Gaffeo, Delli Gatti, Desiderio & Gallegati, 2008; Dawid & Neugart, 2011; Ashraf, Gershman, & Howitt, 2012; Lengnick, 2013).

Building on the mentioned contributions, the fundamental purpose of this article consists in conceiving a network of social relations, where through direct interaction each agent may change the respective sentiment. Because agents are in systematic contact with each other, the masses of individuals possessing optimistic and pessimistic feelings will oscillate over time. Furthermore, under a couple of meaningful interaction rules, such sentiment transitions will possibly imply waves of overconfidence and lack of confidence that exert influence over the performance of the aggregate economy. In the setting to explore in the following sections, one offers an explanation on how animal spirits emerge from the contact between individuals who are susceptible to acquire other “views of the world”. This should be interpreted as a foundation on which the analysis of the macro economy can be built upon: If sentiment spreading through human contact is capable of generating self-sustained waves of optimism and pessimism, then business cycles will have a credible extrinsic reason to occur and to be amplified. This reason is the set of instincts and emotions that were central in Keynes' reasoning.

A Simple Social Network

Consider a homogeneous social network composed by a large but finite number of nodes. Each node is the location of an individual in the population and, as a result, a direct correspondence exists between the number of nodes and the dimension of the assumed population. Homogeneity signifies that every node has an identical number of links to other nodes. The constant number of links will be denoted by the positive integer k.

The homogeneity assumption is convenient to allow for a straightforward analytical study of the model, however one should note that real-world networks tend to be complex structures of interaction, where the number of links varies significantly across nodes. Some regularities on the structure of spontaneously formed social networks have been identified in the literature; the most relevant of these regularities is the one pointing to the existence of scale-free networks, as documented by Barabási and Albert (1999) and Barabási (2009). Scale-free networks are networks where a small number of nodes possess many links and acquire the quality of hubs, and a large number of nodes is scarcely connected to other nodes. Therefore, one might interpret the assumption of link homogeneity just as a theoretical construction that helps in analytically approaching the interaction among heterogeneous agents. This assumption is justifiable in scenarios in which the structure of the network is not of primary importance to qualitatively determine the interaction outcome. For instance, the rumor propagation theory conducts to a same type of long-term equilibrium regardless of the network structure (spreaders become extinct and the population is split into ignorants and stiflers).

In the rumor setting, rates of convergence towards the long-term locus might vary under homogeneous and inhomogeneous networks, and across different degrees of inhomogeneity; however, the qualitative nature of the long-term equilibrium does not change. This is the reason why some of the rumor spreading literature mentioned in the introduction undertakes the dynamic analysis under the homogeneity framework. The same arguments apply to the sentiment propagation framework under discussion in this article, which may, conveniently, be addressed under a setting of link homogeneity. Moreover, part of the analysis will take the simplest case, $k=1$. Basically, this case means that, at each period in time, an agent enters in contact with just one other individual, randomly drawn from the entire population; this is an admissible and plausible assumption given the limited capacity human beings have in establishing multiple interactions simultaneously (of course, this will depend on the time span one is taking as relevant). Note that scenario $k=1$ does not imply that the contact in different time periods occurs among the same individuals, i.e., degree-one homogeneous networks do not mean that individuals exist in isolated pairs; they just mean that a single interaction across any two individuals takes place at each date.

At a given date t, the elements of the population, who interact having has scenario the mentioned homogeneous network, are allocated to two possible sentiment classes: optimists and pessimists. Individuals might change from one class to the other, over time, given the specificity of the interaction process. This process will require classifying individuals within each sentiment class in three categories: susceptible, exuberant and non-susceptible.

The interaction process unfolds as follows: A susceptible optimist is an optimistic individual who might change from her sentiment class to the other when initiating contact with an exuberant pessimist. The exuberant pessimist is a spreader of negative sentiments, but she can also change her status when she interacts with another exuberant pessimist or with a non-susceptible pessimist; in this case, the first individual becomes a non-susceptible pessimist. A non-susceptible pessimistic agent is a stifler, i.e., someone who has been exuberant about the pessimistic sentiment in the past, continues to share this sentiment but loses the motivation to spread it. The same kind of process occurs for the other categories: a susceptible pessimist might turn into an exuberant optimist, when interacting with an agent in this category; the exuberant optimist will, sooner or later, through the contact with other exuberant or non-susceptible optimists, become a non-susceptible optimist.

The above characterization explains two independent processes: How an initially susceptible optimist eventually ends as a non-susceptible pessimist, and how an initially susceptible pessimist might end up as a non-susceptible optimist. This scenario is a possibility in the framework to be explored, but it will be treated as a particular limit case. The general setup also assumes that there will be a given probability with which a non-susceptible agent makes the transition, from one period to the next, to the susceptible category, within the same sentiment class.

The mentioned transition rules define a specific behavior pattern that must be carefully evaluated in terms of its adherence to reality. A first corollary of the characterized interaction is that every agent goes through every possible animal spirits category as the process of social contact unfolds. This eliminates any pre-disposition of individuals to be optimistic or pessimistic by nature. Evidently, in practice, there are genetic, cultural, social and economic factors that might impose insurmountable barriers to sentiment shifting (e.g., someone that is exuberantly optimistic by nature may never fall in the class of those catalogued as pessimists). In the specific environment considered in this article, one does not impose any artificial distinction on the sentiments of agents. They are all alike, in the sense that once in a given animal spirits stage they will potentially change to another state, just by following the conceived rules. In other words, individuals are, at date t, different from each other not because of their intrinsic (initial) features but only as the direct consequence of the sentiment position they currently occupy. The possibility of a systematic flow of individuals changing from one category to another as they establish contact with those that might influence them originates a mechanism of perpetual motion, as referred in the introduction, that eliminates the possibility of the individual agent to remain in a fixed-sentiment steady-state position even when the aggregate outcome converges to a long-run immutability state.

The described rules of transition across states are, as pointed out earlier, adapted from rumor spreading theory and they have the merit of allowing for a cyclical process of sentiment changing that avoids unrealistic long-term outcomes where all individuals become and remain completely concentrated in one of the sentiment classes. Individuals often change their “view of the world” and a model of animal spirits evolution must capture this inherent perpetual motion. Furthermore, economic outcomes tend to reflect systematic and persistent shifts on consumers and investors confidence; recurrent business cycles are necessarily the outcome of cyclical sentiments, a result that is not compatible with the eventual convergence (or, in this case, probably a divergence) of animal spirits dynamics towards a corner solution where one hundred percent of the individuals concur in adopting a same optimistic or pessimistic "view of the world". This would lead to an interpretation of a static and linear economy, that exists in neoclassical economics textbooks but that departs from what reality effectively shows us (Dore & Rosser, 2007).

As in the rumor propagation theory, the three categories of agents, in our case in each sentiment class, are vital for the mechanics of the interaction to work (although in the section immediately before the conclusions we show that the setup can be reduced to a minimal structure of interaction that preserves the possibility of fluctuations). Animal spirits have to do with exuberance; the exuberance category is the one that serves as a catalyst provoking changes from one sentiment class to the other: When an exuberant individual meets a susceptible agent in the other class, the first is able to convince the second of her own interpretation of reality and to convert this second individual to her specific category.

As in the rumor propagation setting, exuberance (in the form of the will to convert others to the respective sentiment) does not tend to be perpetuated over time. If the next contact of the exuberant is with another exuberant or with a non-exuberant individual in the same sentiment class, exuberance fades out. She will no longer feel the need to continue to act exuberantly because she is engaging in contact with someone that already shares the same understanding on how the economy will evolve and, consequently, the individual maintains her conviction but under the form of a more moderate attitude, i.e., she becomes a non-exuberant and non-susceptible supporter of the sentiment.

Finally, the last assumption is one of erosion of non-susceptibility. If an agent remains too long as a non-susceptible optimist or as a non-susceptible pessimist, she will probably forget why she became optimistic or pessimistic in the first place and she will shift to the category of individuals who are susceptible of being infected with the opposite sentiment. Although this last process could occur, as the previous ones, as the outcome of the contact with an agent in another category (namely, the susceptibles), here we adopt the view that there is a sort of depreciation or obsolescence of non-susceptibility: When remaining for a period of time in this category, there is a probability to make a transition to the state of susceptibility regardless of whom the agent under appreciation contacts. This assumption is also useful as a benchmark to introduce endogenous fluctuations later, in the section concerning waves of optimism and pessimism. There, the rate at which the transition between non-susceptibility and susceptibility takes place will no longer be constant; instead, it will respond to the overall flow of agents switching from one of the animal spirits class to the other.

In synthesis, sentiment propagation rules are adopted with two concerns in mind: (*i*) Sentiments spread under a process similar to rumors; (*ii*) The propagation of animal spirits has an inherent cyclical nature that avoids complete and definitive concentration in one of the sentiment classes.

In order to proceed with a formal presentation of the above ideas, let us begin by defining the relevant variables. Share $\omega_t \in [0,1]$ relates to the density of optimists in the population at time *t*; obviously, $1-\omega_t$ represents the density of pessimists. In each sentiment class, individuals may belong to one of three categories. The established classification is the following:

(*a*) x_t^{ω}: Susceptible optimists;

(*b*) y_t^{ω}: Exuberant optimists;

(*c*) z_t^{ω}: Non-susceptible optimists;

(*d*) $x_t^{1-\omega}$: Susceptible pessimists;

(*e*) $y_t^{1-\omega}$: Exuberant pessimists;

(*f*) $z_t^{1-\omega}$: Non-susceptible pessimists.

The presented list of variables relates to densities of individuals that obey conditions $\omega_t = x_t^{\omega} + y_t^{\omega} + z_t^{\omega}$ and $1-\omega_t = x_t^{1-\omega} + y_t^{1-\omega} + z_t^{1-\omega}$.

The rates at which the transition between categories of individuals occur are as follows:

1. $\lambda \in (0,1)$: Rate of transition from the susceptible optimist or susceptible pessimist state to the exuberant pessimist or exuberant optimist state. This transition might occur only when the susceptible meets an exuberant individual in the opposite sentiment class;

2. $\sigma \in (0,1)$: Rate of transition from the exuberant optimist or exuberant pessimist state to the non-susceptible optimist or non-susceptible pessimist state. This transition might occur only when the exuberant meets another exuberant individual or non-susceptible individual belonging to the same sentiment class;

3. $\theta \in (0,1)$: Rate of transition from the non-susceptible optimist or non-susceptible pessimist state to the susceptible optimist or susceptible pessimist state. The transition occurs at rate θ independently of who non-susceptible agents interact with.

Observe that the above rates serve both sentiment class transitions. This does not have to be always the case; we hold to this assumption in order to simplify computation and discussion.

Sentiment Dynamics

To study the dynamics underlying the described structure of social interaction, we follow Nekovee *et al.* (2007), who resort to interacting Markov chains in order to explain rumor spreading. In an interacting Markov chain, the internal transition probabilities, that make the state of each node to evolve according to an internal Markov chain, will depend on the current state of the node and on the states of the nodes that are connected to it. The internal Markov chains are associated to each of the six categories of individuals considered above. The dynamics of the interacting Markov chain are addressed below, on a mean-field level.

Start by assuming a node *j* in the network that, at date *t*, hosts a susceptible optimist. Let $p_t^{\omega,x}$ be the probability of the individual at node *j* to continue in the susceptibility state from period *t* to period $t+1$; $1-p_t^{\omega,x}$ is, then, the probability that node *j* changes state, i.e., it is the probability of the individual in the mentioned node to make a transition to the state of exuberant pessimism.

The relation between $p_t^{\omega,x}$ and rate λ, defined in the previous section, is the following: $p_t^{\omega,x}=(1-\lambda)^{\xi_t}$, where ξ_t is the number of neighbors of node *j* that are in the exuberant pessimistic state at period *t*. Assume that ξ_t is a stochastic variable with an underlying binomial distribution; the corresponding probability mass function is

$$f(\xi_t;k,\tilde{p}_t^{\omega,x})=\binom{k}{\xi_t}\left(\tilde{p}_t^{\omega,x}\right)^{\xi_t}\left(1-\tilde{p}_t^{\omega,x}\right)^{k-\xi_t} \tag{1}$$

Observe that the binomial distribution is adequate in this setting because the interaction among individuals implies either the permanence in the same state or the transition to the state of the neighboring node. Therefore, contact relations are reduced to a sequence of yes-no experiments that are best described by this discrete probability distribution.

In Eq. (1), variable $\tilde{p}_t^{\omega,x}$ relates to the probability of a susceptible optimist to establish contact with an exuberant pessimist, when the first is located in node *j*, a node that has *k* links to other nodes. In a homogeneous network, this probability has correspondence on the density of nodes containing exuberant pessimists, $\tilde{p}_t^{\omega,x}=y_t^{1-\omega}$.

The average probability of nodes with *k* links to continue in the susceptibility state is, then,

$$\bar{p}_t^{\omega,x}=\sum_{\xi_t=0}^{k} f(\xi_t;k,\tilde{p}_t^{\omega,x})p_t^{\omega,x} \tag{2}$$

an expression that is equivalent to (see apendix),

$$\bar{p}_t^{\omega,x} = \left(1 - \lambda y_t^{1-\omega}\right)^k \quad (3)$$

The same reasoning can be repeated for the transition from susceptible pessimist to exuberant optimist, what allows to present the following average probability, concerning the permanence of susceptible pessimists in nodes with *k* links in the same state, rather than shifting to exuberant optimism,

$$\bar{p}_t^{1-\omega,x} = \left(1 - \lambda y_t^{\omega}\right)^k \quad (4)$$

To characterize the movement of exuberant individuals towards the non-susceptible category, we proceed in a similar way. The difference is that exuberants will become non-susceptible through the contact with either other exuberants or with non-susceptible individuals, given the motives explained in the previous section. Taking the pessimists class, the following condition holds, $p_t^{1-\omega,y} = (1-\sigma)^{\xi_t'}$, with $p_t^{1-\omega,y}$ the probability of an exuberant to continue in the same category from period *t* to period $t+1$ and ξ_t' being the number of neighbors of the considered node that are exuberant pessimists or non-susceptible pessimists at date *t*. Taking, again, the binomial distribution,

$$f(\xi_t'; k, \tilde{p}_t^{1-\omega,y}) = \binom{k}{\xi_t'} \left(\tilde{p}_t^{1-\omega,y}\right)^{\xi_t'} \left(1 - \tilde{p}_t^{1-\omega,y}\right)^{k-\xi_t'} \quad (5)$$

with $\tilde{p}_t^{1-\omega,y} = y_t^{1-\omega} + z_t^{1-\omega}$. The average probability of maintaining the original state is

$$\bar{p}_t^{1-\omega,y} = \left[1 - \sigma(y_t^{1-\omega} + z_t^{1-\omega})\right]^k \quad (6)$$

For nodes containing optimistic agents,

$$\bar{p}_t^{\omega,y} = \left[1 - \sigma(y_t^{\omega} + z_t^{\omega})\right]^k \quad (7)$$

One has also assumed that non-susceptible agents in one sentiment class will turn into susceptible individuals in the same class, from period *t* to period $t+1$, with probability θ, following the respective arguments in the previous section. This assumption does not give place to the computation of an average transition probability, since it is considered that such change of category occurs independently of who the non-susceptibles are contacting with. Later, we will elaborate on this by establishing a general rule according to which the mentioned parameter depends on the overall evolution of the ratio between optimists and pessimists in the population.

Average transition probabilities, namely $1-\overline{p}_t^{\omega,x}$, $1-\overline{p}_t^{1-\omega,x}$, $1-\overline{p}_t^{\omega,y}$ and $1-\overline{p}_t^{1-\omega,y}$, are now used to construct dynamic equations for the motion of each of the six variables that represent densities of individuals in the assumed network. Considering that transitions occur under the binomial distribution, the following laws of motion hold true,

$$x_{t+1}^{\omega}-x_t^{\omega}=-x_t^{\omega}(1-\overline{p}_t^{\omega,x})+\theta z_t^{\omega} \tag{8}$$

$$y_{t+1}^{\omega}-y_t^{\omega}=x_t^{1-\omega}(1-\overline{p}_t^{1-\omega,x})-y_t^{\omega}(1-\overline{p}_t^{\omega,y}) \tag{9}$$

$$z_{t+1}^{\omega}-z_t^{\omega}=y_t^{\omega}(1-\overline{p}_t^{\omega,y})-\theta z_t^{\omega} \tag{10}$$

$$x_{t+1}^{1-\omega}-x_t^{1-\omega}=-x_t^{1-\omega}(1-\overline{p}_t^{1-\omega,x})+\theta z_t^{1-\omega} \tag{11}$$

$$y_{t+1}^{1-\omega}-y_t^{1-\omega}=x_t^{\omega}(1-\overline{p}_t^{\omega,x})-y_t^{1-\omega}(1-\overline{p}_t^{1-\omega,y}) \tag{12}$$

$$z_{t+1}^{1-\omega}-z_t^{1-\omega}=y_t^{1-\omega}(1-\overline{p}_t^{1-\omega,y})-\theta z_t^{1-\omega} \tag{13}$$

Replacing in the above list of equations the average transition probabilities, the system to be subject to analysis will contemplate the following set of expressions,

$$x_{t+1}^{\omega}-x_t^{\omega}=-x_t^{\omega}\left[1-\left(1-\lambda y_t^{1-\omega}\right)^k\right]+\theta z_t^{\omega} \tag{14}$$

$$y_{t+1}^{\omega}-y_t^{\omega}=x_t^{1-\omega}\left[1-\left(1-\lambda y_t^{\omega}\right)^k\right]-y_t^{\omega}\left(1-\left[1-\sigma(y_t^{\omega}+z_t^{\omega})\right]^k\right) \tag{15}$$

$$z_{t+1}^{\omega} - z_t^{\omega} = y_t^{\omega}\left(1 - \left[1 - \sigma(y_t^{\omega} + z_t^{\omega})\right]^k\right) - \theta z_t^{\omega} \quad (16)$$

$$x_{t+1}^{1-\omega} - x_t^{1-\omega} = -x_t^{1-\omega}\left[1 - \left(1 - \lambda y_t^{\omega}\right)^k\right] + \theta z_t^{1-\omega} \quad (17)$$

$$y_{t+1}^{1-\omega} - y_t^{1-\omega} = x_t^{\omega}\left[1 - \left(1 - \lambda y_t^{1-\omega}\right)^k\right] - y_t^{1-\omega}\left(1 - \left[1 - \sigma(y_t^{1-\omega} + z_t^{1-\omega})\right]^k\right) \quad (18)$$

$$z_{t+1}^{1-\omega} - z_t^{1-\omega} = y_t^{1-\omega}\left(1 - \left[1 - \sigma(y_t^{1-\omega} + z_t^{1-\omega})\right]^k\right) - \theta z_t^{1-\omega} \quad (19)$$

In Nekovee *et al.* (2007), similar dynamic rules are derived for the rumor propagation setup. There, once determined the discrete-time equations of motion built upon the transition rules, the system is then transformed into a continuous-time setting by taking the limit case $\Delta t \to 0$, and the dynamic behavior is approached by evaluating the differential equations system that is generated. In our framework, we choose to maintain a discrete notion of time. The reason for this relates to the perception that the contact among individuals leading to eventual sentiment changes occurs intermittently in time, rather than under the form of a continuous-time flow. Moreover, if one wants to empirically test the validity of the theory, a discrete-time environment is better equipped to proceed with such task, since the potentially available data is necessarily discrete.

Steady-state

The analysis of the dynamics underlying the system containing Eqs. (14) to (19) requires making a couple of prior considerations. The first remark relates to the initial state of the system. One considers an initial state where almost all the population is in the susceptibility category, i.e., $x_0^{\omega} + x_0^{1-\omega} = 1 - \varepsilon$, with $\varepsilon \to 0$ a positive value; one also considers that there are no non-susceptibles at $t = 0$, $z_0^{\omega} = 0$, $z_0^{1-\omega} = 0$, and that $y_0^{\omega} + y_0^{1-\omega} = \varepsilon$.

The second consideration relates to the structure of the network. Explicit analytical steady-state results can only be found for degree 1 networks, i.e., for *k*=1, and therefore this constraint is taken to describe the steady-state. Recall the discussion in the section in which the interaction setup was presented, where case *k*=1 was classified as the one reflecting the simplest network structure, although this does not signify an unrealistic setting. It just indicates that at each time period there is only one link connecting an individual to another agent, randomly selected from the neighborhood, becoming, at that date, her single adjacent node.

Proposition 1. *For k=1 and an initial state with the previously defined features, three possible steady-state points exist,*

a) Sentiment-endemic steady-state ($y_0^{\omega} \neq 0 \wedge y_0^{1-\omega} \neq 0$ *),*

$$\begin{bmatrix} (x^{\omega})^* \\ (y^{\omega})^* \\ (z^{\omega})^* \end{bmatrix} = \begin{bmatrix} (x^{1-\omega})^* \\ (y^{1-\omega})^* \\ (z^{1-\omega})^* \end{bmatrix} = \begin{bmatrix} \frac{\sigma}{2(\lambda+\sigma)} \\ \frac{\lambda\theta}{2\theta(\lambda+\sigma)+\lambda\sigma} \\ \frac{\lambda^2\sigma}{2(\lambda+\sigma)[2\theta(\lambda+\sigma)+\lambda\sigma]} \end{bmatrix}$$

b) Exuberant optimism-free steady-state ($y_0^{\omega} = 0$ *),*

$$(x^{1-\omega})^* = 1$$
$$(x^{\omega})^* = (y^{\omega})^* = (z^{\omega})^* = (y^{1-\omega})^* = (z^{1-\omega})^* = 0$$

c) Exuberant pessimism-free steady-state ($y_0^{1-\omega} = 0$ *),*

$$(x^{\omega})^* = 1$$
$$(y^{\omega})^* = (z^{\omega})^* = (x^{1-\omega})^* = (y^{1-\omega})^* = (z^{1-\omega})^* = 0$$

Proof. To determine steady-state results, start by defining $(x^{\omega})^* \equiv x_t^{\omega} = x_{t+1}^{\omega}$; $(y^{\omega})^* \equiv y_t^{\omega} = y_{t+1}^{\omega}$; $(z^{\omega})^* \equiv z_t^{\omega} = z_{t+1}^{\omega}$; $(x^{1-\omega})^* \equiv x_t^{1-\omega} = x_{t+1}^{1-\omega}$; $(y^{1-\omega})^* \equiv y_t^{1-\omega} = y_{t+1}^{1-\omega}$; $(z^{1-\omega})^* \equiv z_t^{1-\omega} = z_{t+1}^{1-\omega}$. Then apply these definitions to Eqs. (14) to (19) under constraint k=1. This allows to establish a chain of long-term conditions

$$\begin{aligned} \theta(z^{\omega})^* &= \lambda(x^{\omega})^*(y^{1-\omega})^* = \sigma(y^{1-\omega})^*\left[(y^{1-\omega})^* + (z^{1-\omega})^*\right] \\ &= \theta(z^{1-\omega})^* = \lambda(x^{1-\omega})^*(y^{\omega})^* = \sigma(y^{\omega})^*\left[(y^{\omega})^* + (z^{\omega})^*\right] \end{aligned}$$

Taking into account the boundary values of parameters, it will be true, with no exception, that $(y^{\omega})^* = (y^{1-\omega})^*$ and $(z^{\omega})^* = (z^{1-\omega})^*$. The equality $(x^{\omega})^* = (x^{1-\omega})^*$ will hold only under $y_0^{\omega} \neq 0 \wedge y_0^{1-\omega} \neq 0$. If any of the exuberant shares is initially equal to zero, then the circular flow underlying the dynamic process is broken, i.e., when $y_0^{\omega} = 0$ there are no exuberant optimists to spread the optimistic sentiment and, sooner or later, all agents end up in the susceptible pessimists category. A similar but symmetric reasoning can be applied for $y_0^{1-\omega} = 0$.

The remaining situation, the one in case *a*) of the proposition, necessarily implies $\left(x^{\omega}\right)^* = \left(x^{1-\omega}\right)^*$. We now define $x^* \equiv \left(x^{\omega}\right)^* = \left(x^{1-\omega}\right)^*$; $y^* \equiv \left(y^{\omega}\right)^* = \left(y^{1-\omega}\right)^*$; $z^* \equiv \left(z^{\omega}\right)^* = \left(z^{1-\omega}\right)^*$, and present the previous chain of equalities under the form $\theta z^* = \lambda x^* y^* = \sigma y^*\left(y^* + z^*\right)$. In this case, we also know that $2x^* + 2y^* + 2z^* = 1$. Combining both conditions and maintaining $y_0 \neq 0$, straightforward algebra leads to the result in the proposition ■

A direct corollary of the proposition is that in the sentiment-endemic steady-state (the general case), $\omega^* = 1 - \omega^* = 1/2$, i.e., in the long-run equilibrium the number of optimists and the number of pessimists will coincide, independently of the initial conditions, safeguarding that a non-zero number of exuberants initially exists.

The equality between the number of optimists and the number of pessimists in the steady-state implies that, although individual agents continue to change sentiment category as time unfolds, the aggregate result is one of stability. The sentiment shares are identical and, therefore, optimistic and pessimistic feelings offset each other. If one attaches these feelings to the confidence of investors and consumers, on average the level of confidence remains constant and neutral in the steady-state and the performance of the economy can be associated solely with the fundamentals, forcing psychological factors or animal spirits to play no role, as predicted by neoclassical economics.

An inspection of the sentiment-endemic steady-state allows for the identification of the impact that changes on parameter values have over the long-term shares of each individuals' category, namely the following derivatives are computed,

$$\frac{\partial x^*}{\partial \lambda} = -\frac{\sigma}{2(\lambda+\sigma)^2} < 0$$

$$\frac{\partial x^*}{\partial \sigma} = \frac{\lambda}{2(\lambda+\sigma)^2} > 0$$

$$\frac{\partial x^*}{\partial \theta} = 0$$

$$\frac{\partial y^*}{\partial \lambda} = \frac{2\sigma\theta^2}{[2(\lambda+\sigma)\theta+\lambda\sigma]^2} > 0$$

$$\frac{\partial y^*}{\partial \sigma} = -\frac{\lambda\theta(2\theta+\lambda)}{[2(\lambda+\sigma)\theta+\lambda\sigma]^2} < 0$$

$$\frac{\partial y^*}{\partial \theta} = \frac{\lambda^2\sigma}{[2(\lambda+\sigma)\theta+\lambda\sigma]^2} > 0$$

$$\frac{\partial z^*}{\partial \lambda} = \frac{\lambda\sigma^2[4(\lambda+\sigma)\theta+\lambda\sigma]}{2(\lambda+\sigma)^2[2(\lambda+\sigma)\theta+\lambda\sigma]^2} > 0$$

$$\frac{\partial z^*}{\partial \sigma} = \lambda^2 \frac{2(\lambda^2-\sigma^2)\theta-\lambda\sigma^2}{2(\lambda+\sigma)^2[2(\lambda+\sigma)\theta+\lambda\sigma]^2}$$

$$\frac{\partial z^*}{\partial \theta} = -\frac{\lambda^2\sigma}{[2(\lambda+\sigma)\theta+\lambda\sigma]^2} < 0$$

The above results indicate that an increase in the rate at which susceptible individuals become exuberants of the other class will imply a rise on the shares of exuberants and non-susceptibles and a fall on the percentage of susceptible agents. An increase in the rate that transforms exuberants into non-susceptibles leads to an upward movement in the share of susceptibles, a downward movement in the share of exuberants and an effect over non-susceptibles that will depend on the specific values of the parameters (in particular, if $\theta > \lambda\sigma^2/[2(\lambda^2-\sigma^2)]$, then $\partial z^*/\partial\sigma > 0$). Finally, parameter θ has no effect over the long-term value of the susceptibles, and its change, if positive, makes the proportion of exuberants to increase and the proportion of non-susceptibles to decrease. Note, for the computed derivatives, that the following equalities necessarily hold: $\frac{\partial x^*}{\partial\lambda}+\frac{\partial y^*}{\partial\lambda}+\frac{\partial z^*}{\partial\lambda}=0$, $\frac{\partial x^*}{\partial\sigma}+\frac{\partial y^*}{\partial\sigma}+\frac{\partial z^*}{\partial\sigma}=0$, $\frac{\partial x^*}{\partial\theta}+\frac{\partial y^*}{\partial\theta}+\frac{\partial z^*}{\partial\theta}=0$.

The reader might inquire whether the computed partial derivative signs hold outside the steady-state. In such case, the variables that represent sentiment shares will not assume constant values and, therefore, one cannot present any of the shares as a combination of parameter values, in opposition with what occurs in the long-run equilibrium. As a result, it is not feasible to compute such derivatives and one should interpret the motion of the system as involving two distinct states: In the transitional dynamics phase, the dynamics are governed by the motion generated by the difference equations. In the steady-state, sentiment shares tend to remain constant, but they are subject to hypothetical disturbances on the probability parameter values.

A particular case of the general model is addressable, namely the one for which constraint $\lambda=\sigma=\theta$ is taken. In this case,

Proposition 2. *Let* $k=1$, $y_0^{\omega}\neq 0$, $y_0^{1-\omega}\neq 0$ *and* $\lambda=\sigma=\theta$. *The sentiment-endemic steady-state is, in this scenario,*

$$\begin{bmatrix}(x^{\omega})^*\\(y^{\omega})^*\\(z^{\omega})^*\end{bmatrix}=\begin{bmatrix}(x^{1-\omega})^*\\(y^{1-\omega})^*\\(z^{1-\omega})^*\end{bmatrix}=\begin{bmatrix}1/4\\1/5\\1/20\end{bmatrix}$$

Proof. Use the result in proposition 1*a*) and apply to it constraint $\lambda=\sigma=\theta$ ■

Although there is no specific intuition to justify the imposition of constraint $\lambda=\sigma=\theta$, we will observe, in the next section, that considering it allows to approach in a straightforward and generic form the local transitional dynamics of the underlying dynamic system.

Changing the structure of the network through the increase in the number of links across nodes, i.e., increasing *k*, will surely change the steady-state outcome. Given the impossibility of deriving explicit steady-state results for $k>1$, we present an illustrative numerical example. Let $\lambda = 0.25$, $\sigma = 1/3$, $\theta = 0.05$. Table 1 presents steady-state results for each of the individuals' categories for every value of *k* from 1 to 5. These values are the solutions compatible with $y_0^{\omega} \in (0,1]$ and $y_0^{1-\omega} \in (0,1]$.

Table 1 - Steady-state values for different network configurations.

	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$
x^*	0.2857	0.2821	0.2778	0.2733	0.2686
y^*	0.0882	0.0573	0.0434	0.0355	0.0304
z^*	0.1261	0.1606	0.1788	0.1912	0.2010

Values in table 1 display a clear tendency: The number of non-susceptibles increases with *k*, while the shares of the other two categories fall. When *k* tends to infinity (what implies an infinitely large network where each node is connected to all the others), it becomes, again, possible to obtain explicit steady-state values. In this case, $x^* = y^* = \theta z^*$; given that condition $2x^* + 2y^* + 2z^* = 1$ still holds, such a network implies the long-term outcome $x^* = y^* = \theta/(4\theta + 2)$ and $z^* = 1/(4\theta + 2)$. In this specific example, $x^* = y^* = 0.0227$ and $z^* = 0.4545$. These are the values to which population shares converge as the network becomes a highly interconnected network in which everybody is in contact with everybody.

Local Transitional Dynamics

In order to investigate whether the values of endogenous variables converge to the sentiment-endemic steady-state given an admissible initial state, one needs to linearize system (14)-(19) in the vicinity of (x^*, y^*, z^*). We restrict the general analysis to case $k=1$, although in a second stage one also addresses examples for $k>1$. In every approached case, stability will hold.

For $k=1$, the linearization of (14)-(19) in the vicinity of the steady-state yields

$$\begin{bmatrix} x_{t+1}^{\omega} - x^* \\ y_{t+1}^{\omega} - y^* \\ z_{t+1}^{\omega} - z^* \\ x_{t+1}^{1-\omega} - x^* \\ y_{t+1}^{1-\omega} - y^* \end{bmatrix} = J \times \begin{bmatrix} x_t^{\omega} - x^* \\ y_t^{\omega} - y^* \\ z_t^{\omega} - z^* \\ x_t^{1-\omega} - x^* \\ y_t^{1-\omega} - y^* \end{bmatrix} \qquad (20)$$

with

$$J=\begin{bmatrix} 1-\frac{\lambda^2\theta}{\chi} & 0 & \theta & 0 & -\frac{\lambda\sigma}{2(\lambda+\sigma)} \\ 0 & 1-\frac{\lambda\sigma\theta}{\chi} & -\frac{\lambda\sigma\theta}{\chi} & \frac{\lambda^2\theta}{\chi} & 0 \\ 0 & \lambda\sigma\frac{2(\lambda+\sigma)\theta+\chi}{2(\lambda+\sigma)\chi} & 1-\theta+\frac{\lambda\sigma\theta}{\chi} & 0 & 0 \\ -\theta & -\frac{\lambda\sigma}{2(\lambda+\sigma)}-\theta & -\theta & 1-\theta-\frac{\lambda^2\theta}{\chi} & -\theta \\ \frac{\lambda(\lambda+\sigma)\theta}{\chi} & \frac{\lambda\sigma\theta}{\chi} & \frac{\lambda\sigma\theta}{\chi} & \frac{\lambda\sigma\theta}{\chi} & 1 \end{bmatrix}$$

and $\chi \equiv 2(\lambda+\sigma)\theta+\lambda\sigma$. The presented system is five-dimensional since we have suppressed the equation relating the motion of $z_t^{1-\omega}$, because $z_t^{1-\omega}=1-(x_t^{\omega}+y_t^{\omega}+z_t^{\omega}+x_t^{1-\omega}+y_t^{1-\omega})$.

Variable $z_t^{1-\omega}$ was chosen to be removed from the system, as any other of the assumed six variables could be selected for this purpose. The removal of this variable implies its replacement in each equation where it appeared by one minus the sum of the other density variables. After such replacement, one arrives to the Jacobian matrix in system (20). Independently of the variable that is selected for elimination, the stability outcome will require the remaining 5-dimensional system to generate a Jacobian matrix with five eigenvalues lying inside the unit circle. If, instead of eliminating the redundant dimension, one solved the system with six equations, the respective local linearization would generate a 6×6 matrix J with five stable dimensions (five eigenvalues larger than -1 and lower than 1) and a bifurcation dimension (for which one of the eigenvalues must be equal to -1 or 1). The withdrawal of one of the equations, under the referred logic, allows for a more straightforward evaluation of stability.

Local stability is guaranteed if the real part of each of the eigenvalues of the Jacobian matrix lies inside the unit circle. As presented, it is not possible to derive explicit expressions for the eigenvalues or to determine any conditions that allow the evaluation, in generic terms, of the position of the eigenvalues relatively to the unit circle. Therefore, only through numerical exemplification, we can arrive to meaningful results. Taking any possible parameterization, i.e., $\lambda,\sigma,\theta \in (0,1)$, and computing the eigenvalues, one finds without exception a stability result: Some of the eigenvalues are real, while others are imaginary roots; despite this, in any possible parameterization, the evidence shows that the real parts of all eigenvalues lie inside the unit circle.

There is a specific case that leads directly to the computation of explicit generic eigenvalues. It is the case in which the constraint $\lambda=\sigma=\theta$ holds. In this circumstance, the Jacobian matrix is presentable as

$$J=\begin{bmatrix} 1-\frac{\lambda}{5} & 0 & \lambda & 0 & -\frac{\lambda}{4} \\ 0 & 1-\frac{\lambda}{5} & -\frac{\lambda}{5} & \frac{\lambda}{5} & 0 \\ 0 & \frac{9}{20}\lambda & 1-\frac{4}{5}\lambda & 0 & 0 \\ -\lambda & -\frac{5}{4}\lambda & -\lambda & 1-\frac{6}{5}\lambda & -\lambda \\ \frac{2}{5}\lambda & \frac{\lambda}{5} & \frac{\lambda}{5} & \frac{\lambda}{5} & 1 \end{bmatrix}$$

and the respective eigenvalues are: $\lambda_{1,2} = (1-0.6\lambda) \pm 0.3742\lambda i$; $\lambda_{3,4} = (1-0.1689\lambda) \pm 0.4246\lambda i$; $\lambda_5 = 1-0.8622\lambda$. In this scenario, stability will surely prevail $\forall \lambda \in (0,1)$ because the real parts of the eigenvalues are all positive values lower than 1.

Given the high-dimensionality of the studied system, generic analytical results are scarce. A numerical investigation allows to gain further insights into the dynamics of the model. Let us consider, as a benchmark example, $\lambda = 0.25$, $\sigma = 1/3$, $\theta = 0.05$. Take, as well, $x_0^{\omega} = 0.5999$, $x_0^{1-\omega} = 0.3999$, $y_0^{\omega} = y_0^{1-\omega} = 0.0001$, $z_0^{\omega} = z_0^{1-\omega} = 0$. For this setting, the trajectories of the endogenous variables are depicted in Figs. 1 to 4. Convergence towards the sentiment-endemic steady-state takes place, for any of the assumed variables. The first three figures directly compare the time trajectories of the same sentiment category for each of the two classes. The fourth graph captures the evolution of the aggregate percentages of optimists and pessimists. As one observes, after a transient phase where the dominant sentiment alternates, the densities converge to 1/2, i.e., those with a positive feeling and those with a negative feeling will populate the network in a same amount once the transient phase is overcome.

**** Fig.1 - Densities of susceptible optimists and susceptible pessimists*

*($\lambda = 0.25$, $\sigma = 1/3$, $\theta = 0.05$, $k = 1$) ****

**** Fig.2 - Densities of exuberant optimists and exuberant pessimists*

*($\lambda = 0.25$, $\sigma = 1/3$, $\theta = 0.05$, $k = 1$) ****

**** Fig.3 - Densities of non-susceptible optimists and non-susceptible pessimists*

*($\lambda = 0.25$, $\sigma = 1/3$, $\theta = 0.05$, $k = 1$) ****

**** Fig.4 - Densities of optimists and pessimists*

*($\lambda = 0.25$, $\sigma = 1/3$, $\theta = 0.05$, $k = 1$) ****

Changes on parameter values can be considered in order to analyze the impact over the presented trajectories. A second set of graphs is displayed in Figs. 5 to 8, for a different network structure ($k = 3$).

**** Fig.5 - Densities of susceptible optimists and susceptible pessimists*

*($\lambda = 0.25$, $\sigma = 1/3$, $\theta = 0.05$, $k = 3$) ****

**** Fig.6 - Densities of exuberant optimists and exuberant pessimists*

($\lambda = 0.25$, $\sigma = 1/3$, $\theta = 0.05$, $k = 3$ *) ****

**** Fig.7 - Densities of non-susceptible optimists and non-susceptible pessimists*

($\lambda = 0.25$, $\sigma = 1/3$, $\theta = 0.05$, $k = 3$ *) ****

**** Fig.8 - Densities of optimists and pessimists*

($\lambda = 0.25$, $\sigma = 1/3$, $\theta = 0.05$, $k = 3$ *) ****

Differences between trajectories in Figs. 1-4 and Figs. 5-8 are essentially the translation of the notion that additional links promote a faster convergence to the steady-state, being the steady-state different from one case to the other, according to what table 1 shows. The shares of optimists and pessimists continue to converge to 1/2 because there is a symmetry imposed by identical values of the parameters across the sentiment classes.

The main characteristic of the displayed results is that although individual agents are frequently changing status, from optimists to pessimists and the opposite, on the aggregate there is a tendency to converge to an unchangeable steady-state where the number of optimists and pessimists remain constant and equal to each other, i.e., the population will be separated into two sentiment groups with identical densities. Next section introduces an additional assumption that changes the nature of the long-term outcome.

Waves of Optimism and Pessimism

So far, sentiment dynamics culminated on a steady-state where, on the aggregate, shares of optimists and pessimists are constant values, with the population divided precisely in half between the two sentiment classes. Thus, in a long-run perspective there are no periods of dominance of one or the other type of animal spirit; individual idiosyncrasies are not reflected at the macro level, what makes animal spirits innocuous in an equilibrium position. To circumvent this fact, and turn animal spirits meaningful at a macro level, one may relax the assumption initially imposed that non-susceptible agents become susceptible of being infected with the opposite sentiment at a given arbitrary rate θ. Now, we transform this into an endogenous rate and, by doing so, we allow for the possibility of long-term waves of optimism and pessimism.

The assumption to take about the value of θ is intuitive. Imagine that, at time *t*, non-susceptible individuals are aware of how the share of optimists is changing, i.e., they know $\Delta\omega_t \equiv \omega_t - \omega_{t-1}$, and that each non-susceptible agent will change to susceptible under a probability that depends on $\Delta\omega_t$. The rules are as follows:

- A non-susceptible optimist is more likely to become susceptible when the share of pessimists is increasing, i.e., when the share of optimists is declining. The relevant probability is, in this case, $\theta^{\omega}(\Delta\omega_t)$, with $d\theta^{\omega}/d\Delta\omega_t < 0$. We define

$$\theta^{\omega}(\Delta\omega_t) = \begin{cases} \theta_0 + (1-\theta_0)(-\Delta\omega_t)^{\phi}, & \Delta\omega_t < 0 \\ \theta_0 - \theta_0(\Delta\omega_t)^{\phi}, & \Delta\omega_t \geq 0 \end{cases} \tag{21}$$

with $0 < \phi < 1$. Note that $\theta^{\omega}(0) = \theta_0$, $\theta^{\omega}(-1) = 1$ and $\theta^{\omega}(1) = 0$. Variable θ^{ω} replaces constant θ in Eqs. (14) and (16),

$$x_{t+1}^{\omega} - x_t^{\omega} = -x_t^{\omega}\left[1 - \left(1 - \lambda y_t^{1-\omega}\right)^k\right] + \theta^{\omega}(\Delta\omega_t) z_t^{\omega} \tag{22}$$

$$z_{t+1}^{\omega} - z_t^{\omega} = y_t^{\omega}\left(1 - \left[1 - \sigma(y_t^{\omega} + z_t^{\omega})\right]^k\right) - \theta^{\omega}(\Delta\omega_t) z_t^{\omega} \tag{23}$$

- A non-susceptible pessimist is more likely to become susceptible when the share of optimists is increasing. The probability of changing status, from non-susceptible to susceptible is, in this case, $\theta^{1-\omega}(\Delta\omega_t)$, with $d\theta^{1-\omega}/d\Delta\omega_t > 0$. The probability variable comes:

$$\theta^{1-\omega}(\Delta\omega_t) = \begin{cases} \theta_0 + (1-\theta_0)(\Delta\omega_t)^{\phi}, & \Delta\omega_t \geq 0 \\ \theta_0 - \theta_0(-\Delta\omega_t)^{\phi}, & \Delta\omega_t < 0 \end{cases} \tag{24}$$

Now, $\theta^{1-\omega}(0) = \theta_0$, $\theta^{1-\omega}(-1) = 0$ and $\theta^{1-\omega}(1) = 1$. Variable $\theta^{1-\omega}$ replaces constant θ in Eqs. (17) and (19),

$$x_{t+1}^{1-\omega} - x_t^{1-\omega} = -x_t^{1-\omega}\left[1 - \left(1 - \lambda y_t^{\omega}\right)^k\right] + \theta^{1-\omega}(\Delta\omega_t) z_t^{1-\omega} \tag{25}$$

$$z_{t+1}^{1-\omega} - z_t^{1-\omega} = y_t^{1-\omega}\left(1 - \left[1 - \sigma(y_t^{1-\omega} + z_t^{1-\omega})\right]^k\right) - \theta^{1-\omega}(\Delta\omega_t) z_t^{1-\omega} \tag{26}$$

Since in the steady-state, as characterized in previous sections, ω_t is constant, the steady-state properties already discussed are maintained intact for $\theta=\theta_0$. However, under the new specification in which the transition from non-susceptibility to the susceptibility state is not automatic but, instead, depends on which sentiment is recruiting a positive net number of elements, the system will not remain in a long-term fixed-point position. Waves of optimism and pessimism will subsist in the long-run, as the graphical examples in Figs. 9-12 show.

**** Fig.9 - Densities of susceptible optimists and susceptible pessimists*
($\lambda=0.25$, $\sigma=1/3$, $\theta_0=0.05$, $\phi=0.2$, $k=1$ *) ****

**** Fig.10 - Densities of exuberant optimists and exuberant pessimists*
($\lambda=0.25$, $\sigma=1/3$, $\theta_0=0.05$, $\phi=0.2$, $k=1$ *) ****

**** Fig.11 - Densities of non-susceptible optimists and non-susceptible pessimists*
($\lambda=0.25$, $\sigma=1/3$, $\theta_0=0.05$, $\phi=0.2$, $k=1$ *) ****

**** Fig.12 - Densities of optimists and pessimists*
($\lambda=0.25$, $\sigma=1/3$, $\theta_0=0.05$, $\phi=0.2$, $k=1$ *) ****

The cycles that emerge from the designed mechanism are not chaotic, since no sensitive dependence on initial conditions exists, neither there is a long-term irregular behavior of the time trajectories of the densities of optimists and pessimists. Independently of the initial state, the steady-state will possess the features that are patent in Figs. 9-12, i.e., a completely predictable limit cycle is formed. Transient dynamics depend on the characteristics of the initial state, but the long-term evolution of the densities is determined solely by the values of parameters and by the changes on θ provoked by the rules in (21) and (24). Note that the 1/2 steady-state result (equal number of optimists and pessimists in the long-run) holds on average, but it fails to hold in concrete at almost every period, as the shares of each sentiment class perpetually oscillate around such steady-state reference value.

The economic intuition for the obtained persistent waves of optimism and pessimism comes directly from how rules (21) and (24) were conceived. Individuals that are non-exuberant about one of the two sentiments may become susceptible of infection by the opposite sentiment. This occurs as the result of how this class of individuals interprets available information. They will have the possibility of reacting to the change on the ratio between optimists and pessimists. If they observe that the relative number of individuals associated with the other class is rising, this makes them more likely to become susceptible of turning to the opposite side as well. There is a kind of self-fulfilling prophecy associated with this reasoning: The probability of individuals becoming more susceptible of adopting the opposite sentiment rises if the economy as a whole is shifting faster towards that sentiment.

As the number of individuals associated with one sentiment class becomes large and the respective share approaches 1, the respective change begins to slow down what will imply a fall in the value of probability θ, i.e., a fall in the rate of conversion to the susceptibility state. This reverses the process and the sentiment in which the agent is begins to gain increased relevance, leading to the cyclical paths one observes in Figs. 9-12. Therefore, cycles on animal spirits become endogenous to the interaction process, allowing for the realistic result that "mood swings" are perpetuated in time and no polarization to a single state of full optimism or pessimism will take place.

The waves of optimism and pessimism displayed in the graphics constitute a behavioral basis over which one can analyze economic performance. Animal spirits, interpreted as the systematic oscillation between aggregate states of low and high confidence, will be a source of endogenous volatility that alongside with shocks to fundamentals (technology, public policies or preferences) will determine the pace of the main macroeconomic variables over time.

An Alternative Specification (A Minimal Structure)

The discussed framework is adaptable in various ways, e.g. by introducing novel categories of agents or by suppressing some of the existing categories of agents. One important concern relates the analytical tractability of the model; to increase it, one must reduce the dimensionality of the dynamic system. In this section, we present a minimal sentiment-switching structure that is analytically straightforward to address since it can be presented under the form of a two-equations dynamic system.

The structure of analysis taken in the previous section is probably the most adequate to study the proposed interaction environment. This is true essentially because agents in the two sentiment classes were treated symmetrically, i.e., susceptible, exuberant and non-susceptible individuals integrated both the optimism class and the pessimism class. However, a more compact setting, despite of certainly being more artificial, has the advantage of allowing for a more rigorous and detailed analytical treatment of the model. The minimal structure defines the existence of at least three categories of agents with one of them relating to an exuberant behavior.

Assume the following three categories of agents: Susceptible optimists, exuberant optimists and a single homogeneous category of pessimists. Under k=1, and following rules similar to the ones presented in previous versions of the model, we get

$$\begin{cases} x_{t+1}^{\omega} - x_t^{\omega} = -\lambda x_t^{\omega}(1-\omega_t) + \theta y_t^{\omega} \\ y_{t+1}^{\omega} - y_t^{\omega} = \sigma(1-\omega_t) y_t^{\omega} - \theta y_t^{\omega} \\ (1-\omega_{t+1}) - (1-\omega_t) = \lambda x_t^{\omega}(1-\omega_t) - \sigma(1-\omega_t) y_t^{\omega} \end{cases} \quad (27)$$

Because, in this case, $\omega_t = x_t^{\omega} + y_t^{\omega}$, the above system is equivalent to

$$\begin{cases} x_{t+1}^{\omega} - x_t^{\omega} = -\lambda x_t^{\omega}\left(1 - x_t^{\omega} - y_t^{\omega}\right) + \theta y_t^{\omega} \\ y_{t+1}^{\omega} - y_t^{\omega} = \sigma\left(1 - x_t^{\omega} - y_t^{\omega}\right) y_t^{\omega} - \theta y_t^{\omega} \end{cases} \quad (28)$$

Ignoring the exuberant-free steady-state, it is straightforward to compute the system's long-term locus,

$$(x^*, y^*) = \left(\frac{\sigma-\theta}{\lambda+\sigma}; \frac{\lambda(\sigma-\theta)}{\sigma(\lambda+\sigma)}\right)$$

The shares of optimists and pessimists will be, in the long-run equilibrium,

$$(\omega^*, 1-\omega^*) = \left(1 - \frac{\theta}{\sigma}; \frac{\theta}{\sigma}\right)$$

A constraint on the parameters is required for this version of the model to deliver feasible results, namely $\sigma \geq \theta$. Note that the higher the value of θ and the lower the value of σ, the larger is the density of pessimistic agents relatively to the density of optimists. In opposition to the more complete specification of previous sections, the result $\omega^* = 1 - \omega^*$ is not universal; it will hold only if $\sigma = 2\theta$.

To address local stability, dynamic system (28) is linearized in the vicinity of the steady-state. The result is

$$\begin{bmatrix} x_{t+1}^{\omega} - x^* \\ y_{t+1}^{\omega} - y^* \end{bmatrix} = \begin{bmatrix} 1 - \frac{\lambda}{\sigma}\frac{\sigma(2\theta-\sigma)+\lambda\theta}{\lambda+\sigma} & \frac{\sigma(\lambda+\theta)}{\lambda+\sigma} \\ -\frac{\lambda(\sigma-\theta)}{\lambda+\sigma} & 1 - \frac{\lambda(\sigma-\theta)}{\lambda+\sigma} \end{bmatrix} \cdot \begin{bmatrix} x_t^{\omega} - x^* \\ y_t^{\omega} - y^* \end{bmatrix} \quad (29)$$

The stability of system (29) can be approached, in generic form, through the evaluation of the stability conditions that involve the values of the trace and of the determinant of the Jacobian matrix. The eigenvalues will be located inside the unit circle if the following three conditions are simultaneously satisfied: $1 - Det > 0; 1 - Tr + Det > 0; 1 + Tr + Det > 0$. The values of the trace and of the determinant are, respectively, $Tr = 2 - \lambda\theta / \sigma$ and $Det = 1 - \lambda\theta[1 - (\sigma - \theta)] / \sigma$. For these values:

- $1 - Det = \lambda\theta[1-(\sigma-\theta)]/\sigma;$
- $1 - Tr + Det = \lambda\theta/\sigma - \lambda\theta(\sigma-\theta)/\sigma;$
- $1 + Tr + Det = 4 - \lambda\theta[2-(\sigma-\theta)]/\sigma.$

As long as $\sigma > \theta$, the above conditions are satisfied and one guarantees that the stability result holds. Fig. 13 depicts graphically the location of the dynamics of the system in a trace-determinant diagram; the stability area lies inside the inverted triangle, and therefore one can visually confirm the analytical result. The region in which the system is dynamically located, for admissible values of parameters, is the dark one. Analytically, it corresponds to

$$U = \{\lambda, \sigma, \theta \ : \ 1 < Tr < 2 \wedge Det > Tr - 1 \wedge Det < 1 - (2 - Tr)^2\}$$

**** Fig. 13 - Stability in the Trace - Determinant diagram ****

As in the precedent section, one may introduce cyclical motion by allowing for a function θ^{ω} that responds to the change on the relative number of optimistic individuals. Consider the exact same rule as (21). Fig. 14 addresses the motion of the system in this case, for the values of parameters assumed in previous occasions. Now, we observe stronger fluctuations; these are the result of taking less types of agents and, therefore, less steps in changing from one sentiment class to the other. As a consequence, the switching process naturally becomes faster.

**** Fig.14 - Densities of optimists and pessimists in the minimal structure model*

*($\lambda = 0.25$, $\sigma = 1/3$, $\theta_0 = 0.05$, $\phi = 0.2$, $k = 1$) ****

Conclusion

Waves of optimism and pessimism are a probable cause of observable fluctuations on macro variables. What exactly triggers animal spirits and how these impact on the performance of the economic system is yet an open question in the theoretical literature. This article contributed to the understanding of how animal spirits propagate.

Animal spirits are associated with a process of social interaction, where agents assume different positions in different stages of their contact with others. Individuals are not classified just as optimists or pessimists. Within each class they are susceptible individuals that might change from one sentiment class to the other, they can be exuberant spreaders of the sentiment they believe in, or they can possess a sentiment without sharing it and without being susceptible of “infection” by the opposite sentiment. All agents pass through each of the possible states at different periods, originating an aggregate result of stability, according to which the population can be grouped in two classes with an identical share of individuals in the long-run.

However, if one assumes that the rate of transition between non-susceptibility to the susceptibility state depends on the overall evolution of the number of individuals in each sentiment class, then endogenous fluctuations might be generated and, therefore, a consistent explanation for the occurrence of waves of optimism and pessimism emerges. Various versions of the model were discussed, but a common denominator was found: There must be some kind of exuberance of a possibly small part of the population in order to guarantee that the sentiment spreading process takes place, a result that gives true meaning to the notion of animal spirits.

A model of animal spirits as the one presented in this article is instrumental for studying the implications of changing sentiments over the macro economy. For sentiments to have impact on the aggregate economy, it is necessary to consider some kind of departure relatively to rational expectations; some efforts in this direction have been made, as it is the case of the work of Abel (2002) concerning expectations formulated by pessimistic agents.

Appendix – Derivation of Eq. (3)

The equivalence between Eqs. (2) and (3) is straightforward to obtain. Given the binomial distribution and the equality between $\tilde{p}_t^{\omega,x}$ and $y_t^{1-\omega}$, Eq. (2) is presentable under the form

$$\overline{p}_t^{\omega,x} = \binom{k}{0}\left(1-y_t^{1-\omega}\right)^k + \binom{k}{1} y_t^{1-\omega}\left(1-y_t^{1-\omega}\right)^{k-1}(1-\lambda) + \binom{k}{2}\left(y_t^{1-\omega}\right)^2\left(1-y_t^{1-\omega}\right)^{k-2}(1-\lambda)^2 + \ldots$$
$$+ \binom{k}{k-1}\left(y_t^{1-\omega}\right)^{k-1}\left(1-y_t^{1-\omega}\right)(1-\lambda)^{k-1} + \binom{k}{k}\left(y_t^{1-\omega}\right)^k(1-\lambda)^k$$

This expression can be simplified and presented as: $\overline{p}_t^{\omega,x} = \left[\left(1-y_t^{1-\omega}\right) + y_t^{1-\omega}(1-\lambda)\right]^k$. Simplifying further, one gets Eq. (3).

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