

**Title:** Topological Complexity and Predictability in the Dynamics of a Tumor Growth Model with Shilnikov's Chaos

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**Abstract:** Dynamical systems modeling tumor growth have been investigated to determine the dynamics between tumor and healthy cells. Recent theoretical investigations indicate that these interactions may lead to different dynamical outcomes, in particular to homoclinic chaos. In the present study, we analyze both topological and dynamical properties of a recently characterized chaotic attractor governing the dynamics of tumor cells interacting with healthy tissue cells and effector cells of the immune system. By using the theory of symbolic dynamics, we first characterize the topological entropy and the parameter space ordering of kneading sequences from one-dimensional iterated maps identified in the dynamics, focusing on the effects of inactivation interactions between both effector and tumor cells. The previous analyses are complemented with the computation of the spectrum of Lyapunov exponents, the fractal dimension and the predictability of the chaotic attractors. Our results show that the inactivation rate of effector cells by the tumor cells has an important effect on the dynamics of the system. The increase of effector cells inactivation involves an inverse Feigenbaum (i.e. period-halving bifurcation) scenario, which results in the stabilization of the dynamics and in an increase of dynamics predictability. Our analyses also reveal that, at low inactivation rates of effector cells, tumor cells undergo strong, chaotic fluctuations, with the dynamics being highly unpredictable. Our findings are discussed in the context of tumor cells potential viability.

**Author Keywords:** Cancer; Tumor cell dynamics; Chaos; Complex systems; Topological entropy; Predictability

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