

**Title:** An Extension of Gompertzian Growth Dynamics Weibull and Frechet Models

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**Source:** Mathematical Biosciences and Engineering **Volume:** 10 **Issue:** 2

**Pages:** 379-398 **DOI:** 10.3934/mbe.2013.10.379 **Published:** Apr 2013

**Document Type:** Article

**Language:** English

**Abstract:** In this work a new probabilistic and dynamical approach to an extension of the Gompertz law is proposed. A generalized family of probability density functions, designated by Beta\* (p, q), which is proportional to the right hand side of the Tsoularis-Wallace model, is studied. In particular, for p = 2, the investigation is extended to the extreme value models of Weibull and Frechet type. These models, described by differential equations, are proportional to the hyper-Gompertz growth model. It is proved that the Beta\* (2, q) densities are a power of betas mixture, and that its dynamics are determined by a non-linear coupling of probabilities. The dynamical analysis is performed using techniques of symbolic dynamics and the system complexity is measured using topological entropy. Generally, the natural history of a malignant tumour is reflected through bifurcation diagrams, in which are identified regions of regression, stability, bifurcation, chaos and terminus.

**Author Keywords:** Growth models; Extreme value laws; Beta\* (p, q) densities; Bifurcations and chaos; Symbolic dynamics; Topological entropy; Tumour dynamics

**Keywords Plus:** Logistic Model; Tumor-Growth; Immunotherapy

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**Funding:**

Funding Agency	Grant Number
National Funds through FCT - Fundacao para a Ciencia e a Tecnologia	PEst-OE/MAT/UI0006/2011
PTDC/FEDER	

**Publisher:** Amer Inst Mathematical Sciences

**Publisher Address:** Po Box 2604, Springfield, MO 65801-2604 USA

**ISSN:** 1547-1063

**Citation:** ROCHA, J. Leonel; ALEIXO, Sandra M. - An Extension of Gompertzian Growth Dynamics Weibull and Frechet Models. Mathematical Biosciences and Engineering. ISSN 1547-1063. Vol. 10, nr 2 (2013), p. 379-398.