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This study is a research on teaching practice, developed in the context of an elective course on Dynamic Geometry for prospective kindergarten and elementary school teachers taught by the first author of this paper. We aim to analyse the role of GeoGeobra in the development of geometric reasoning, particularly the way individuals geometrically structure figures. The participants are a class of six future teachers. Data was gathered from the participants' portfolios and classroom observation while working on an exploratory task, which focuses on constructing draggable figures. The results show that this type of activity promotes spatial and geometric structuring, beginning with the perception of elements and relationships that enable the dynamic construction, and moving on to the description of the construction using formal concepts associated to the tools of the DGE.

Keywords: GeoGeobra, geometry, structuring, visualization, teacher education.

Introduction

Research has shown the interest in engaging students of different ages in activities using dynamic geometry environments (DGE), for the improvement of concept learning and the development of reasoning (Sinclair & Yerushalmy, 2016). In particular, Hanna and Sidoli (2007) indicate that the DGE are a promising way "in enhancing the students' ability to notice details, conjecture, reflect on and interpret relationships and to offer tentative explanations and proofs" (p. 77). Also, in education programs, relevant research confirms the effectiveness of the use of technological tools, including DGE, to improve the knowledge of teachers and future teachers (Jones & Tzekaki, 2016). In this study, we aim to analyse the role of GeoGeobra in the development of geometric reasoning, in a context of a geometry course for future kindergarten and elementary school teachers, based on exploratory tasks. In particular, we which to understand how the construction of draggable figures in the DGE contributes to geometric structuring (Battista, 2008)?

Theoretical framework

Prospective elementary teacher education in geometry

The knowledge necessary for teaching includes mastery of mathematical reasoning, ways to solve problems and communicate mathematics effectively, understanding of concepts, procedures and the process of doing mathematics (Albuquerque et al., 2005; NCTM, 1991). Concerning geometry, kindergarten and elementary school teachers should understand how it is used to describe the world; analyse two and three dimensional figures; use synthetic geometry, coordinates and transformations; improve skills in producing arguments, justifications and in visualization.

Some researchers have claimed that there are few studies about teachers and future teachers' knowledge in geometry (Chapman, 2013; Clements & Sarama, 2011; Steele, 2013). However, the existing literature provides reasons to believe this is a problematic area. As Clements and Sarama (2011) state, in many countries teachers from every level are not always provided with adequate preparation in geometry and lack of knowledge and confidence in this area. Concerning kindergarten and elementary prospective teachers, many only recognise and categorize shapes by their overall similarity to prototypes, instead of charactering them by their properties (Clements & Sarama, 2011; Fujita & Jones, 2006) a problem we also identify in Portugal (Menezes, Serrazina & Fonseca, 2014). Overall, as Jones & Tzekaki (2016) recently stressed, studies on teachers' geometric knowledge and teacher education programs indicate that we still need to give attention on how prospective teachers build their understanding of geometrical objects. Also, we should take into account the effectiveness of approaches such as the use DGE.

Developing geometric reasoning and the use of DGE

King e Shattschneider (2003) present eight reasons for a teacher to use a DGE: (i) to take advantage of the accuracy of geometric constructions and measurements, leading to confident results; (ii) to promote visualization; (iii) to encourage exploration, investigation and discovery leading to the formulation of questions, conjectures, and their test; (iv) to encourage demonstration because the experimental evidence offers the necessary conviction for such enterprise, and may provide clues; (v) to support the understanding of geometric transformations; (vi) to support the understanding of loci; (vii) to provide simulation opportunities for a wide variety of situations; and (viii) to allow the creation of microworlds, using new tools and allowing exploitation of non-Euclidean geometry.

A major emphasis on using DGE concerns the constructing of figures. Laborde (2001) compares this type of activity when performed using a DGE versus using paper and pencil. In her view, when we draw a figure using paper and pencil, the activity is often controlled by perception rather than being driven by the properties of the figure. Instead, in a DGE is not possible to construct a square in a similar way ("led by eye") and it requires more knowledge about the figure. But if the students are able to apply the properties correctly, we can ask ourselves, as does Battista (2007), what have they learned from the activity? For this researcher, "perhaps no *new* knowledge was acquired, but instead, the students' knowledge and reasoning were deepened and enriched . . . Or perhaps connections between properties were newly constructed or extended" (p. 878).

In order to analyse this reasoning we draw on the framework developed by Battista (2008). This researcher established a categorization of reasoning using three levels, corresponding to increasing degrees of sophistication: spatial structuring, geometric structuring and logical/axiomatic structuring. Spatial structuring is a special type of abstraction corresponding to the mental act of constructing an organization or form for an object or set of objects by identifying its components, combining them into spatial composites, and identifying the way they combine and relate. Spatial structuring enables a person to imagine manipulating an object, reflect, analyse and understand it. Geometric structuring describes spatial structuring using formal concepts such as congruence, parallelism, angle, transformations or coordinate systems. Geometric structuring is based on spatial structuring, that is, to be able to structure geometrically an object, it is necessary that one has interiorized the corresponding spatial structure. Logical/axiomatic structuring formally organizes geometric concepts in a system so that their relationship can be established through logical deduction. To operate at this level, it is necessary that verbal or symbolic statements can replace mental models. The research of Battista (2008) in a DGE (the *Shape Makers* microworld) with fifth graders showed that the manipulation of shapes and the reflection on that manipulation may enable the pupils to move from thinking holistically to thinking about the geometric properties of the figure, that is, to progress from spatial structuring to geometric structuring. However, he also points that there is a need for guidance, reflection and experimentation in order to construct formal geometric conceptualizations of the DGE constraints.

Methodology

The first author of this paper designed and taught an elective course on Dynamic Geometry in 2015/26, as a new offer in the teacher education program for prospective elementary school teachers in her institution, in Portugal. The course was divided into two phases: (i) 10 lessons dedicated to solving geometry tasks organized into four topics – problem solving, constructing, investigating and creating; and (ii) 5 lessons dedicated to didactics of geometry, projecting the work of the DGE with children from kindergarten to 6th grade. In the classroom, there was one computer for each participant, but they were encouraged to discuss with their colleagues. Regarding the assessment, each participant built a portfolio containing a task from each topic, detailed solution and a reflection on the activity, and also constructed a GeoGeobraBook with the files used to solve the tasks.

This study is a research on teaching practice based on the observation of the activity of the participants and their solutions of a task and aims contributing to their professional and organizational development, "as well as to generate important knowledge about educational processes, useful for other teachers, for academic educators and the community in general" (Ponte, 2002, p. 13). Data was gathered mainly from the portfolios and GeoGeobraBooks of the participants, complemented by the field notes taken by the first author, while observing the participants and supporting their work. There were only six participants: five females who were in the 2nd year of the program (also attending a compulsory course of Geometry) and a male in the 3rd year, the only one who had some experience with DGE. Since the Dynamic Geometry course is elective, the choice the participants may be considered an indicator that they like geometry and do not feel strong difficulties in this area, which was confirmed in this group. The task (Figure 1) was proposed in the 5th lesson, within the topic *Constructing*. It was intended that the participants would reproduce draggable figures, or families of figures, in GeoGeobra from the properties visually identified, thus corresponding to one of the major emphases reported by Battista (2007).

1. Construct both stars. Describe briefly the process.
2. For each of the stars, find another building process and describe it.
3. Construct other stars of this family with a larger number of points. Generalize one of the construction processes you used.
4. Establish relationship between the number of star points and other elements.

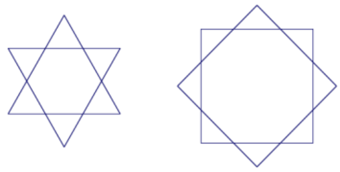


Figure 1 – Task *Constructing stars* (adapted from Johnston-Wilder and Mason, 2005)

The data was analysed using a framework (Table 1) built by the first author of the paper (Brunheira, 2016), based on the concepts of spatial and geometric structuring (Battista, 2008). The table does not include the logical/axiomatic level, since it means that one operates at a symbolic level, which is not the purpose in this task. We use the framework to analyse the solutions, looking for the evidence of the descriptors in order to characterize the level of structuring of the participants. However, we stress that despite the attribution of a level to a solution, this does not mean that we can characterize the level of structuring for an individual solely based on a solution of a task, so this must be seen as an indicator. Also, we cannot consider that solving a single task is enough to improve significantly, but this analysis may enable us to recognise it's potential.

<table>
<tr><th rowspan="2">Levels</th><th colspan="2">Geometric structuring</th></tr>
<tr><th>Spatial structuring</th><th>Knowledge of concepts</th></tr>
<tr><td>N0</td><td>Does not establish geometrical relationships between figures and their elements, or does not provide most of the times.</td><td>Does not know most of the basic concepts and the language is very limited in terms of geometric vocabulary.</td></tr>
<tr><td>N1</td><td>Perceives geometric relationships involving visible elements of figures, but it may depend on the position of the figures, their elements or the context.</td><td>Knows the concepts of side and angle, congruence, perpendicularity and parallelism in the plan; in space, knows the concept vertex, edge, and face.</td></tr>
<tr><td>N2</td><td>Perceives geometric relationships involving visible elements of figures in any positions or context.

Perceives geometric relationships involving invisible elements of figures, but it may depend on the position of the figures, their elements or the context.</td><td rowspan="2">Knows the concepts as axe of symmetry, diagonal, bisector, midpoint and the geometric transformations in the plan; in space, knows the concept of congruence, parallelism and perpendicularity.</td></tr>
<tr><td>N3</td><td>Perceives geometric relationships involving visible or invisible elements of figures in any positions or context.

Produces generalizations of geometric relations for a family of figures.</td></tr>
</table>

Table 1 – Descriptors of the levels of spatial and geometric structuring

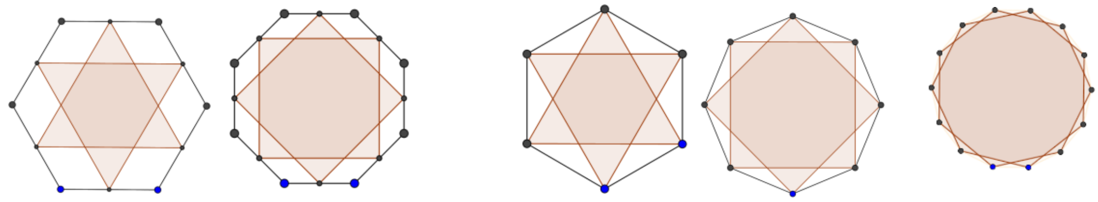
Results

Next we present an analysis of task solutions from prospective teachers Maria, Carla and Louise taken from their portfolios, which we consider to be representative of all the solutions presented.

Maria's solution of the task

In figure 2 we present an excerpt containing two processes presented by Maria. Process A was used to build the two initial stars and process B was used for the same purpose, as well as to generalize. Both constructions begin with the image of the star as a whole figure and a regular hexagon where the star is inscribed in two different ways.

Process A Process B



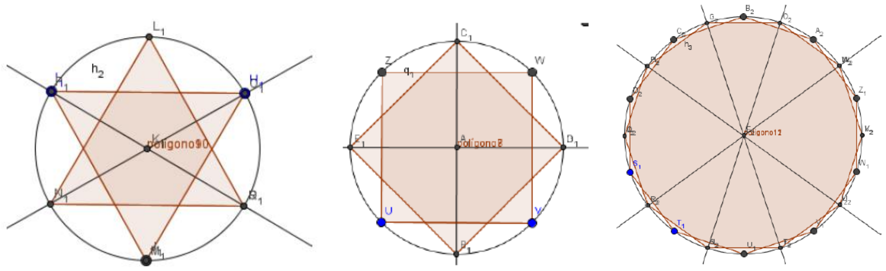
Construct a polygon with a certain number of sides and then two polygons from the union of non-consecutive vertices. The number of the star points corresponds to the number of the vertices of the polygon used for its construction. It is not possible to do this based on regular polygons with an odd number of sides, since there are not two sets of non-consecutive points to be connected.

Figure 2 – Excerpt of Maria's solution of the task

Maria looks at the star as a whole figure inscribed in a regular hexagon in two different ways. She draws on invisible elements that were created to assist the construction. Regarding the generalization, Maria presents a process which can be applied to any star and establishes a relationship between the initial polygon and the number of points of the star. Finally, she identifies that this polygon cannot have an odd number of sides and justifies her finding. Thus, Maria's solution shows a very good geometric structuring for this family of figures, corresponding to Level 3 of the framework.

Carla's solution of the task

Carla uses a procedure similar to Maria' process B and another process, shown in Figure 3.



1. Construct an initial figure in accordance with the number of points of the star (this polygon should be a regular polygon in which the number of vertices is half the number of points of the star). 2. Trace the perpendicular bisectors for each side of the polygon to find its center. 3. Draw a circle centered at the intersection point of the bisectors and a radius to reach a vertice of the figure. 4. The intersection points between the bisectors and the circumference will be the vertices of the second figure that makes up the star.

The number of points of the star is twice the number of sides of the inicial figure.

Figure 3 – Excerpt of Carla's solution of the task

She looks at the star decomposing it into two congruent regular polygons, one of which constitutes the starting point for construction. The determination of the second polygon involves visualizing the star inscribed in a circumference, and the vertices of the second polygon on the perpendicular bisectors (a concept that she did not know). Thus, she identifies that the consecutive vertices of the star are equidistant from each other and also equidistant from the centre of the star. Regarding the relations established, Carla identifies that the number of vertices of the initial polygon is twice the number of points of the star, but does not justify this. Therefore, Carla identifies various relationships between their elements, using visible and invisible elements and adequate concepts, such as the circle and the perpendicular bisector, thus showing a very good geometric structuring of the family of figures, which also corresponds to level 3.

Louise's solution of the task

Louise builds the stars initially as Carla (draws the first polygon, traces the bisectors and finds the point of intersection). However, while Carla seems to look at the star in a static point of view, Louise visualizes the "movement" of the first polygon to obtain the second. The participant had an intuitive idea that rotating the initial triangle in a certain way, it would be possible to obtain the second triangle and form the star, although she did not know the formal concept of rotation and that we should define the rotation by a centre and an angle. She asked for help to find out if the GeoGeobra could run this "movement" and the teacher explained how the "Rotation" tool worked. Next, Louise presented the following relationships: "For regular polygons with even number of sides, amplitude = 180°/(number of sides); For regular polygons with odd number of sides, amplitude = 180º". Thus, we consider that her solution also reflects level 3.

Discussion

All participants were successful in the task. They presented different and valid constructions mobilizing a variety of elements of the figures (visible and non-visible), relations between them, transformations and properties, some of them were unknown to them. So, the main conclusion we want to emphasize is that the construction of figures using GeoGeobra significantly enhances the geometric structuring by promoting the identification of properties and relations between elements, as Battista (2008) reported in his study. This improvement stems from different features and strengths that we recognize in the DGE, some of them indicated by King e Shattschneider (2003). We start with two features – easiness of use and accuracy of the constructions – which we associate the two strengths – promoting intuition and exploration. In fact, sometimes participants started the construction from an insight of the properties and elements of the figure (or auxiliary figures) that could be useful, but they were not sure. The possibility to easily test the conjectures through a quick and accurate construction was a key aspect, as Maria explains:

> With GeoGeobra it was possible to explore different forms of construction of the stars using polygons, lines, midpoints, parallel lines, among others, easily, simply and accurately. If we didn't have this software this would be a long and relatively difficult process, especially the construction of regular polygons used as a basis for the construction of stars. (Portfolio)

Another potential of GeoGeobra that emerged was the promotion of justification, which we did not ask for in the task. In fact, the ability to test the construction validity, as in a trial and error process, does not mean that participants do not reflect on their actions, as we note in Louise's comment:

> I had to stop and think why the rotation angle depends on the number of sides, as well as to find a mathematical answer to for the correct value. (Portfolio)

In this case, we see a need to reflect on the value of the angle, which led to the justification of the chosen value and the understanding the generalization. So, although the DGE played an important role in the user's belief that a relationship is valid, did not lead to underestimate justification, instead promoted the search for it (Hanna & Sidoli, 2007).

Another feature of GeoGeobra is that it leads the user to work with the formal concepts associated with its tools. In this way, we may think that we can only take advantage of the DGE when operating at the level of geometric structuring. In fact, as Battista (2007) suggests, we cannot make geometric constructions without reaching some level of "conceptual and representational explicitness". However, this investigation shows that GeoGeobra can facilitate the transition from spatial to geometric structuring. An example that supports this conclusion is the use of new concepts, like perpendicular bisector or rotation, that participants had just a vague memory from middle and high school, but were correctly applied as the DGE promoted their appropriation.

Finally, in connection to the nature of the task which favours different solutions, GeoGeobra supports this diversity through a set of tools available, which also stimulates creativity. As Peter says:

> The choice of this task reflects on the freedom it gives us to construct the figures using different processes . . . [which] depend on our ability to imagine overlapping figures, guidelines for the construction and other key points of the figure . . . improves the ability to find relationships between figures and their elements and encourages creativity. (Portfolio)

Conclusion

This research was based on a construction task for which we recognize the potential mentioned by Laborde (2001). Besides, we corroborate the claims of King and Shattschneider (2003) regarding the reasons that support the use of the DGE, particularly the use of rigorous constructions and the promotion of visualization, exploration, investigation, discovery and demonstration, to which we would like to add creativity and intuition. However, the data also shows that constructing draggable figures in GeoGeobra contributes to spatial and geometric structuring. The main contribution of this study concerns the importance of this work in prospective teacher education. From a mathematical point of view, the data shows the relevance of the exploratory work involving geometric constructions using a DGE, promoting the evolution in the way they structure the geometrical figures by identifying relationships and properties. Apart from this perspective, the comments of participants also show the relevance of reflecting on mathematical activity itself. This reflection – here enhanced by the portfolio – enables prospective teachers to become aware of their own learning in relation to the task, which can be an important contribution to their didactical knowledge.

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