

RECEIVED: December 22, 2020

REVISED: February 2, 2021

ACCEPTED: February 13, 2021

PUBLISHED: March 23, 2021

Vacuum structure of the \mathbb{Z}_2 symmetric Georgi-Machacek model

Duarte Azevedo,^a Pedro Ferreira,^{a,b} Heather E. Logan^c and Rui Santos^{a,b}

^a*Centro de Física Teórica e Computacional, Faculdade de Ciências, Universidade de Lisboa, Campo Grande, Edifício C8, 1749-016 Lisboa, Portugal*

^b*ISEL — Instituto Superior de Engenharia de Lisboa, Instituto Politécnico de Lisboa, 1959-007 Lisboa, Portugal*

^c*Ottawa-Carleton Institute for Physics, Carleton University, 1125 Colonel By Drive, Ottawa, Ontario K1S 5B6, Canada*

E-mail: drpazevedo@fc.ul.pt, pmmferreira@fc.ul.pt, logan@physics.carleton.ca, rasantos@fc.ul.pt

ABSTRACT: We discuss the vacuum structure of a version of the Georgi-Machacek model with an exact \mathbb{Z}_2 symmetry acting on the triplet fields. Besides the usual custodial-symmetric model, with $\rho = 1$ at tree-level, a model with a dark matter candidate is also viable. The other phases of the model lead to electric charge breaking, a wrong pattern of electroweak symmetry breaking or to $\rho \neq 1$ at tree-level. We derive conditions to have an absolute minimum in each of the two viable phases, the custodial and the dark matter phases.

KEYWORDS: Beyond Standard Model, Higgs Physics

ARXIV EPRINT: [2012.07758](https://arxiv.org/abs/2012.07758)

Contents

1	Introduction	1
2	Scalar potential and vacuum structure	3
3	Bilinear formalism and mass matrices for the GM model	5
4	Models with custodial symmetry after SSB	7
4.1	The custodial vacuum	7
4.2	The dark matter vacuum	9
5	Models where the custodial symmetry is broken after SSB	10
5.1	The real non-custodial vacuum	10
5.2	Charge breaking vacua	11
5.3	Wrong-electroweak vacuum	13
5.4	Tantamount vacua	13
6	Stability of the viable models	14
6.1	Stability of dark matter minima	14
6.2	Stability of the custodial vacuum	16
7	Conclusions	19
A	Vacua	20

1 Introduction

The Georgi-Machacek [1] (GM) model was proposed in 1985 in an attempt to understand if patterns of electroweak symmetry breaking different than the one proposed in the Standard Model (SM) were still viable. Besides the SM complex doublet, the model has one real triplet and one complex triplet. Clearly the model leads to different phenomenology at colliders but it is also able to reproduce all the available experimental results. It is of particular importance, because it is not trivial, that the SM ratio of charged to neutral currents can be obtained at tree-level. This ratio is translated into the relation $\rho = m_W^2/(m_Z^2 \cos^2 \theta_W) = 1$ at tree-level ($m_{W(Z)}$ is the $W(Z)$ boson mass) and is measured with great precision allowing only for small deviations from higher order corrections. So far experiments have neither confirmed nor excluded the spontaneous breaking pattern predicted by the SM. Hence, all possibilities which agree with the observed phenomenology should be considered.

In the same year a Higgs potential was written explicitly for the GM model by Chanowitz and Golden [2]. The potential has only terms with mass dimension of order 2 and order 4 and is invariant under a global $SU(2)_L \times SU(2)_R$ symmetry. For this

particular version of the model the vacuum expectation values of the neutral components of the triplet can be arranged such that a custodial $SU(2)$ symmetry survives after the spontaneous breaking of the global $SU(2)_L \times SU(2)_R$ symmetry. This way, the equality of the vacuum expectation values of the neutral fields which leads to $\rho = 1$ at tree-level is naturally preserved in the Higgs sector.

Several versions of GM model and its phenomenology have been discussed over the years [3–17] and several searches were proposed and performed at colliders and in particular at the Large Hadron Collider (LHC). The latest constraints on the GM model can be found in [18, 19]. There is also a public code available, GMCALC [20], which allows to calculate the particle spectrum of the model together with the couplings and decay widths. Checks of perturbative unitarity and boundedness from below are included in the code. Starting from a parameter point that constitutes a viable custodial-symmetric minimum of the scalar potential, the code also checks for deeper alternative minima by performing a numerical scan following the method proposed in ref. [12]. A full study of the possible vacua of the model has not been performed.

In this work we discuss the vacuum structure of the \mathbb{Z}_2 symmetric model as proposed in ref. [2] which is a simple yet viable version of the GM potential. The model with this potential and no further field additions has a Dark Matter (DM) phase when spontaneous symmetry breaking leaves the \mathbb{Z}_2 unbroken, that is, when the triplets have no vacuum expectation values (VEVs). This DM phase has not been studied before and may be phenomenologically viable, though a detailed phenomenological study of the DM vacuum is not the focus of this paper.¹ The GM model studied in the literature is the one where custodial $SU(2)$ is preserved and $\rho = 1$ at tree-level. The model has however other types of minima that do not preserve the custodial symmetry that range from electric charge breaking minima to models with just the wrong pattern of spontaneous symmetry breaking (SSB). Besides, these non-custodial vacua always give rise to extra Goldstone bosons. We will compare the depths of the potential calculated at the different vacua at tree-level to understand if either the DM or the Custodial vacuum are naturally stable and derive the conditions for absolute stability in case they are not. The expressions found can be used to ensure that the desired vacuum is the deepest one at tree-level.

In this first attempt to understand the vacuum structure of the model we make the analysis more tractable by considering only the real component of the (in general complex) VEVs. In that sense the conditions we obtain for the potential to be in an absolute minimum are necessary conditions.

The paper is organized as follows. In section 2 we briefly present the potential and the vacuum structure of the model and in section 3 we present the bilinear formalism.

¹Our interest in the DM phase of the GM model is as an alternative vacuum that could potentially destabilize the desired Custodial vacuum. We consider it to be not particularly well motivated as a model for dark matter because when the triplets have zero VEV, the custodial symmetry upon which the GM model is based is no longer necessary to ensure that $\rho = 1$. Models in which the SM is extended by a single scalar triplet (either real or complex) that does not get a VEV and that yields a dark matter candidate were first studied in detail in ref. [21]. Dark matter *extensions* of the Custodial phase of the GM model have been studied in refs. [22–24].

Models that retain the custodial symmetry after spontaneous symmetry breaking (SSB) are discussed in section 4 while the ones where said symmetry is broken are discussed in 5. In section 6 we discuss the stability of the two viable models. We summarise our findings in section 7.

2 Scalar potential and vacuum structure

The Georgi-Machacek model [1] consists of the usual complex doublet (ϕ^+, ϕ^0) with hypercharge, Y , equal to 1, a real triplet (ξ^+, ξ^0, ξ^-) with $Y = 0$, and a complex triplet $(\chi^{++}, \chi^+, \chi^0)$ with $Y = 2$. The fields can be written in the following matrix form

$$\Phi = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^{+*} & \phi^0 \end{pmatrix}, \quad (2.1)$$

$$\Xi = \begin{pmatrix} \chi^{0*} & \xi^+ & \chi^{++} \\ -\chi^{+*} & \xi^0 & \chi^+ \\ \chi^{++*} & -\xi^{+*} & \chi^0 \end{pmatrix}. \quad (2.2)$$

which transform under the global $SU(2)_L \times SU(2)_R$ symmetry according to

$$\exp(iT^a \theta_L^a)(\Phi \text{ or } \Xi) \exp(-iT^b \theta_R^b). \quad (2.3)$$

The most general scalar potential invariant under the global $SU(2)_L \times SU(2)_R$ symmetry is

$$\begin{aligned} V(\Phi, \Xi) = & \frac{\mu_2^2}{2} \text{Tr}(\Phi^\dagger \Phi) + \frac{\mu_3^2}{2} \text{Tr}(\Xi^\dagger \Xi) + \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 + \lambda_2 \text{Tr}(\Phi^\dagger \Phi) \text{Tr}(\Xi^\dagger \Xi) \\ & + \lambda_3 \text{Tr}(\Xi^\dagger \Xi \Xi^\dagger \Xi) + \lambda_4 [\text{Tr}(\Xi^\dagger \Xi)]^2 - \lambda_5 \text{Tr}(\Phi^\dagger \tau^a \Phi \tau^b) \text{Tr}(\Xi^\dagger t^a \Xi t^b) \end{aligned} \quad (2.4)$$

where we have further imposed a \mathbb{Z}_2 symmetry, $\Xi \rightarrow -\Xi$, which eliminates the cubic terms $\text{Tr}(\Phi^\dagger \tau^a \Phi \tau^b)(U \Xi U^\dagger)_{ab}$ and $\text{Tr}(\Xi^\dagger t^a \Xi t^b)(U \Xi U^\dagger)_{ab}$ (for the definition of U see ref. [6]). This is the potential originally proposed by Chanowitz and Golden [2] which not only preserves $\rho = 1$ at tree-level but also offers a new possibility that the triplets have the dominant contribution to the W boson mass. The potential must be bounded from below, and the necessary and sufficient conditions that the quartic couplings λ_i must obey for that to happen have been established [12, 20]:

$$\begin{aligned} \lambda_1 &> 0, \\ \lambda_4 &> \begin{cases} -\frac{1}{3}\lambda_3 & \text{for } \lambda_3 \geq 0, \\ -\lambda_3 & \text{for } \lambda_3 < 0, \end{cases} \\ \lambda_2 &> \begin{cases} \frac{1}{2}\lambda_5 - 2\sqrt{\lambda_1 \left(\frac{1}{3}\lambda_3 + \lambda_4\right)} & \text{for } \lambda_5 \geq 0 \text{ and } \lambda_3 \geq 0, \\ \omega_+(\zeta)\lambda_5 - 2\sqrt{\lambda_1(\zeta\lambda_3 + \lambda_4)} & \text{for } \lambda_5 \geq 0 \text{ and } \lambda_3 < 0, \\ \omega_-(\zeta)\lambda_5 - 2\sqrt{\lambda_1(\zeta\lambda_3 + \lambda_4)} & \text{for } \lambda_5 < 0, \end{cases} \end{aligned} \quad (2.5)$$

with

$$\omega_{\pm}(\zeta) = \frac{1}{6}(1 - B) \pm \frac{\sqrt{2}}{3} \left[(1 - B) \left(\frac{1}{2} + B \right) \right]^{1/2}, \quad (2.6)$$

and

$$B \equiv \sqrt{\frac{3}{2} \left(\zeta - \frac{1}{3} \right)} \in [0, 1]; \quad \zeta \in \left[\frac{1}{3}, 1 \right]. \quad (2.7)$$

Due to the hermiticity of the potential, and its field structure, all of the couplings in eq. (2.4) are necessarily real, and therefore the CP symmetry cannot be explicitly broken. As we will see later any attempt of spontaneous CP breaking would lead to the breaking of the custodial symmetry.

As we are interested in the vacuum structure of the GM model we start by writing the vacuum expectation values (VEVs) of the fields in the following generic form

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 - iv_2 & v_3 + iv_4 \\ -v_3 + iv_4 & v_1 + iv_2 \end{pmatrix}, \quad \Xi = \frac{1}{\sqrt{2}} \begin{pmatrix} v_8 - iv_9 & v_6 + iv_7 & v_{12} + iv_{13} \\ -v_{10} + iv_{11} & \sqrt{2}v_5 & v_{10} + iv_{11} \\ v_{12} - iv_{13} & -v_6 + iv_7 & v_8 + iv_9 \end{pmatrix}. \quad (2.8)$$

There are 6 symmetry transformations in $SU(2)_L \times SU(2)_R$ that can be used to eliminate the redundant VEVs.² We first use the three broken symmetry transformations, $\theta_R^a = -\theta_L^a$ (these are the ones orthogonal to the custodial $SU(2)$), to rotate away three of the four components of the doublet, leaving only v_1 . We can then use the three custodial symmetry transformations, $\theta_R^a = \theta_L^a$, to rotate away three of the VEVs in the triplet. There are several possibilities and we choose to eliminate both the real and imaginary components of χ^{++} and the imaginary component of ξ^+ , which leads to the following form of the vacuum configuration

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 & 0 \\ 0 & v_1 \end{pmatrix}, \quad \Xi = \frac{1}{\sqrt{2}} \begin{pmatrix} v_8 - iv_9 & v_6 & 0 \\ -v_{10} + iv_{11} & \sqrt{2}v_5 & v_{10} + iv_{11} \\ 0 & -v_6 & v_8 + iv_9 \end{pmatrix}. \quad (2.9)$$

We are therefore left with 7 non-redundant VEVs. The scalar potential in eq. (2.4) contains 4 distinct invariants under the $SU(2)_L \times SU(2)_R$ global symmetry. These were identified in ref. [12] and we write them as

$$\text{Tr}(\Phi^\dagger \Phi) = r^2 \cos^2 \gamma, \quad (2.10)$$

$$\text{Tr}(\Xi^\dagger \Xi) = r^2 \sin^2 \gamma, \quad (2.11)$$

$$\text{Tr}(\Xi^\dagger \Xi \Xi^\dagger \Xi) = \zeta \cdot (r^2 \sin^2 \gamma)^2, \quad (2.12)$$

$$\text{Tr}(\Phi^\dagger \tau^a \Phi \tau^b) \text{Tr}(\Xi^\dagger t^a \Xi t^b) = \omega \cdot (r^2 \cos^2 \gamma)(r^2 \sin^2 \gamma). \quad (2.13)$$

The fact that the potential can be expressed in a four-dimensional subspace spanned by these four invariants implies that there are three extra symmetries besides $SU(2)_L \times SU(2)_R$ that have not yet been identified.

²While these symmetry transformations can change the field configuration of an extremum of the potential, they cannot change the value of the potential at the extremum.

3 Bilinear formalism and mass matrices for the GM model

In this section we will outline how a bilinear formalism may be applied to the study of the vacuum structure of the GM model. In short, such a formalism expresses the potential, and its minimisation conditions, in terms of quadratic functions of its fields and/or VEVs, which has been proven to be an effective strategy to compare the values of scalar potentials at different stationary points, and ascertain their relative depths. In the context of the two-Higgs doublet model (2HDM), a first version of the bilinear formalism was first applied in [25–27], and then refined in [28–38], allowing for the study of the 2HDM’s vacuum structure and possible global symmetries. The bilinear formalism was also applied to models with different scalar content, to wit the 3HDM [39, 40], the complex singlet-doublet model (CXSM) [41], the 2HDM with a real or complex singlet (N2HDM) [42, 43], and the Higgs triplet model (HTM) [44]. For most models, the bilinears are simply constructed as the four gauge-invariant quantities allowed with the fields present; but in the case of the HTM, the bilinears were simply the most convenient quadratic products of VEVs to express the value of the potential at generic stationary points as a quadratic polynomial. As we will shortly see, the same holds for the GM model — the presence of a triplet, as in [44], yields such a structure of the potential that we have not found it possible to express it as a product of quadratic gauge invariant field products.

First however, in order to identify whether a given stationary point is a minimum, we will need to study the second derivatives of the potential. In order to obtain the mass matrices for all minima simultaneously we will express the doublet Φ and the triplet Ξ of eq. (2.2) in terms of the real components of the fields φ_i , according to

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_1 - i\varphi_2 & \varphi_3 + i\varphi_4 \\ -\varphi_3 + i\varphi_4 & \varphi_1 + i\varphi_2 \end{pmatrix}, \quad (3.1)$$

$$\Xi = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_8 - i\varphi_9 & \varphi_6 + i\varphi_7 & \varphi_{12} + i\varphi_{13} \\ -\varphi_{10} + i\varphi_{11} & \sqrt{2}\varphi_5 & \varphi_{10} + i\varphi_{11} \\ \varphi_{12} - i\varphi_{13} & -\varphi_6 + i\varphi_7 & \varphi_8 + i\varphi_9 \end{pmatrix}. \quad (3.2)$$

With these conventions, the matrix elements of the 13×13 mass matrix are

$$[M^2]_{ij} = \frac{\partial^2 V}{\partial \varphi_i \partial \varphi_j}, \quad (3.3)$$

with the potential given in eq. (2.4). In order to make the analysis tractable we will only consider the real part of the VEVs from eq. (2.9), that is,

$$\langle \varphi_1 \rangle = v_1, \quad \langle \varphi_5 \rangle = v_5, \quad \langle \varphi_6 \rangle = v_6, \quad \langle \varphi_8 \rangle = v_8, \quad \langle \varphi_{10} \rangle = v_{10}, \quad (3.4)$$

with v_9 and v_{11} set equal to zero. We leave a full analysis including these imaginary parts of the VEVs to future work.³ This leads to a potential (evaluated at a stationary point for

³The most important bottleneck in this work is determining whether each combination of nonzero VEVs simultaneously satisfies up to 13 independent minimization conditions. Including nonzero values of v_9 and v_{11} combinatorially increases the complexity of the analysis.

which these VEVs hold) equal to

$$\begin{aligned}
 V = & v_1^2 \left[\frac{\mu_2^2}{2} + \lambda_1 v_1^2 + \lambda_2 v_5^2 + \left(\lambda_2 - \frac{\lambda_5}{4} \right) v_8^2 + \lambda_2 v_6^2 + \lambda_2 c_{10}^2 - \frac{\lambda_5 v_5 v_8}{\sqrt{2}} - \frac{\lambda_5 v_{10} v_6}{2} \right] \\
 & + v_5^2 \left[\frac{\mu_3^2}{2} + (\lambda_3 + \lambda_4) v_5^2 + 2\lambda_4 v_8^2 + (2\lambda_3 + 2\lambda_4) v_6^2 + (2\lambda_3 + 2\lambda_4) v_{10}^2 \right] \\
 & + v_8^2 \left[\frac{\mu_3^2}{2} + \left(\frac{\lambda_3}{2} + \lambda_4 \right) v_8^2 + (\lambda_3 + 2\lambda_4) v_6^2 + (\lambda_3 + 2\lambda_4) v_{10}^2 \right] \\
 & + v_6^2 \left[\frac{\mu_3^2}{2} + (\lambda_3 + \lambda_4) v_6^2 + 2\lambda_4 v_{10}^2 \right] + v_{10}^2 \left[\frac{\mu_3^2}{2} + (\lambda_3 + \lambda_4) v_{10}^2 \right] - 2\sqrt{2}\lambda_3 v_{10} v_5 v_6 v_8.
 \end{aligned} \tag{3.5}$$

Let us now define the vector A and the symmetric matrix B as

$$A = \begin{pmatrix} \mu_2^2 \\ \mu_3^2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad B = \begin{pmatrix} 8\lambda_1 & 4\lambda_2 & -\lambda_5 & -\lambda_5 & -\lambda_5 & 0 \\ 4\lambda_2 & 8(\lambda_3 + \lambda_4) & 0 & 0 & 0 & 0 \\ -\lambda_5 & 0 & -2\lambda_3 & 0 & -2\lambda_3 & 0 \\ -\lambda_5 & 0 & 0 & -4\lambda_3 & 0 & -2\lambda_3 \\ -\lambda_5 & 0 & -2\lambda_3 & 0 & -4\lambda_3 & 0 \\ 0 & 0 & 0 & -2\lambda_3 & 0 & 0 \end{pmatrix}, \tag{3.6}$$

and the vector X , which depends on the VEVs as

$$X = \begin{pmatrix} v_1^2/2 \\ (v_{10}^2 + v_5^2 + v_6^2 + v_8^2)/2 \\ \sqrt{2}v_5 v_8 \\ v_8^2/2 \\ v_{10} v_6 \\ v_{10}^2 + v_6^2 \end{pmatrix}. \tag{3.7}$$

With these definitions the value of the potential at each stationary point (SP) with VEVs v_i , is given by

$$V_{\text{SP}} = A^T X_{\text{SP}} + \frac{1}{2} X_{\text{SP}}^T B X_{\text{SP}}. \tag{3.8}$$

Thus we see that it is possible to express the value of the potential at a given stationary point as a quadratic polynomial on X , that is, an order 2 polynomial expressed in terms of bilinears, quadratic combinations of VEVs.⁴ Further, since the potential is the sum of two homogeneous functions of order 2 and 4 (V_2 and V_4 , the quadratic and quartic terms in the VEVs — linear and quadratic in X), we have

$$\text{At any stationary point: } \frac{\partial V}{\partial \varphi_i} = 0 \implies \sum_i \varphi_i \frac{\partial V}{\partial \varphi_i} = 0 \implies 2V_2 + 4V_4 = 0. \tag{3.9}$$

The value of the potential at a given stationary point, V_{SP} , is simply

$$V_{\text{SP}} = \frac{1}{2} (V_2)_{\text{SP}} = - (V_4)_{\text{SP}} = \frac{1}{2} A^T X_{\text{SP}} = -X_{\text{SP}}^T B X_{\text{SP}}. \tag{3.10}$$

⁴Though it is not possible, due to the λ_3 and λ_5 terms in the potential, to express it as a quadratic polynomial of quadratic products of fields.

Finally, we also need the vector V' , defined as

$$V' = \frac{\partial V}{\partial X^T} = A + B X, \quad (3.11)$$

and since V' is the gradient of the potential along the vector X , the condition $X^T V' = 0$ has to hold. Let us now consider two distinct vacua with vectors X_1 and X_2 , respectively. Let V'_1 and V'_2 be the gradient of the potential considered at those stationary points. We can contract the vectors of the VEVs from one minimum with the gradient of the other to obtain

$$X_1^T V'_2 = X_1^T A + X_1^T B X_2. \quad (3.12)$$

Since B is symmetric, we have

$$X_1^T B X_2 = X_2^T B X_1, \quad (3.13)$$

which leads to

$$X_2^T V'_1 - X_2^T A = X_1^T V'_2 - X_1^T A. \quad (3.14)$$

The second term in each side of the equation is the quadratic part of the potential computed at the corresponding stationary point. Therefore using eq. (3.10) we can rewrite eq. (3.14) as

$$V_2 - V_1 = \frac{1}{2} (X_2^T V'_1 - X_1^T V'_2), \quad (3.15)$$

which relates the relative height between the two minima with the VEV vectors and gradients of the potential. We want to express the right hand side for each case as a function of physical parameters from one vacuum as much as possible. In this way, the stability of that vacuum is just a function of quantities computed at that stationary point. In the next section we will classify the different type of stationary points and write the corresponding X and V' vectors. The minimum conditions that led to the different phases to be presented are shown in appendix A. The vacua classified as charge breaking are the ones where the photon becomes massive and the mass matrix for the gauge bosons is also given in appendix A.

4 Models with custodial symmetry after SSB

4.1 The custodial vacuum

The GM model was first proposed due to the very nice property that there is a solution of the minimization conditions with no charged VEVs where the triplet neutral VEVs (those of the real fields φ_5 and φ_8) are proportional. This condition preserves the custodial symmetry and maintains the SM structure of the gauge bosons mass matrix, that is, the Weinberg angle is preserved at tree-level. As discussed in the introduction, the equality of vacuum expectation values is naturally preserved in the Higgs sector because the custodial $SU(2)$ survives the spontaneous breaking of the global $SU(2)_L \times SU(2)_R$ symmetry. In this scenario the VEVs are chosen as

$$\langle \varphi_1 \rangle = v_1, \quad \langle \varphi_5 \rangle = v_5, \quad \langle \varphi_6 \rangle = 0, \quad \langle \varphi_8 \rangle = \sqrt{2}v_5, \quad \langle \varphi_{10} \rangle = 0, \quad (4.1)$$

leading to the minimization conditions

$$\begin{aligned}\mu_2^2 &= -4v_1^2\lambda_1 + 3v_5^2(\lambda_5 - 2\lambda_2), \\ \mu_3^2 &= v_1^2(\lambda_5 - 2\lambda_2) - 4v_5^2(\lambda_3 + 3\lambda_4).\end{aligned}\tag{4.2}$$

The mass spectrum is the following: there are two custodial singlet states that mix, h and H , with masses given by

$$m_{h,H}^2 = 4\lambda_1 v_1^2 + 4v_5^2(\lambda_3 + 3\lambda_4) \mp 2\sqrt{4\lambda_1^2 v_1^4 + v_1^2 v_5^2 [3(\lambda_5 - 2\lambda_2)^2 - 8\lambda_1(\lambda_3 + 3\lambda_4)] + 4v_5^4(\lambda_3 + 3\lambda_4)^2},\tag{4.3}$$

a custodial triplet, H_3^0, H_3^\pm , with masses given by

$$m_3^2 = \frac{1}{2}\lambda_5 (v_1^2 + 8v_5^2),\tag{4.4}$$

and a custodial 5-plet, $H_5^0, H_5^\pm, H_5^{\pm\pm}$, with masses

$$m_5^2 = \frac{3\lambda_5 v_1^2}{2} + 8\lambda_3 v_5^2.\tag{4.5}$$

In order that the gauge bosons have the observed mass the VEVs have to obey

$$v_1^2 + 8v_5^2 = v^2 = \frac{1}{\sqrt{2}G_F} \approx (246 \text{ GeV})^2.\tag{4.6}$$

There are also two mixing angles usually taken as input parameters and that parametrize the mixing between the doublet and the triplet components [3].

Since the cubic terms are absent due to the \mathbb{Z}_2 symmetry, $\Xi \rightarrow -\Xi$, the mass scale of the scalar states is set by the electroweak scale and the model does not possess a decoupling limit. The upper bounds on the scalar masses from perturbative unitarity were first studied in ref. [6] which found the following constraints on the masses: $m_3 \lesssim 400 \text{ GeV}$, $m_5 \lesssim 700 \text{ GeV}$. Despite the absence of a decoupling limit, the model in this phase is still not excluded by data [45].

Finally, the vectors X and V' can be written as

$$X_C = \begin{pmatrix} v_1^2/2 \\ 3v_5^2/2 \\ 2v_5^2 \\ v_5^2 \\ 0 \\ 0 \end{pmatrix}, \quad V'_C = A + BX_C = \begin{pmatrix} 0 \\ 2\bar{m}^2 \\ -\bar{m}^2 \\ -\bar{m}^2 \\ -\bar{m}^2 \\ -2\lambda_3 v_5^2 \end{pmatrix},\tag{4.7}$$

with $\bar{m}^2 = \frac{1}{2}\lambda_5 v_1^2 + 4\lambda_3 v_5^2$.

4.2 The dark matter vacuum

If only the doublet Φ develops a VEV, the Lagrangian will still be exactly \mathbb{Z}_2 symmetric after SSB. This in turn means that all the particles from the triplet do not interact with fermions nor do they have triple vertices with two gauge bosons. The lightest of these particles is thus stable and is therefore a good dark matter candidate. Such a situation is indeed a possible solution of the minimization equations of the potential, corresponding to VEVs $v_1 \equiv v$ and all others zero, in eq. (3.4). The only minimization condition is

$$v = \sqrt{-\frac{\mu_2^2}{4\lambda_1}}. \quad (4.8)$$

In this scenario the custodial symmetry is still preserved and therefore the physical particles fall into the same custodial representations as in the custodial vacuum. The two custodial singlet states no longer mix; the 125 GeV Higgs boson is identified with $h = \varphi_1$ with mass given by

$$m_h^2 = 8\lambda_1 v^2. \quad (4.9)$$

The \mathbb{Z}_2 -odd states comprise the familiar custodial singlet, triplet, and fiveplet with masses given by

$$\begin{aligned} m_1^2 &= (\mu_3^2 + 2\lambda_2 v^2) - \lambda_5 v^2, \\ m_3^2 &= (\mu_3^2 + 2\lambda_2 v^2) - \frac{1}{2}\lambda_5 v^2, \\ m_5^2 &= (\mu_3^2 + 2\lambda_2 v^2) + \frac{1}{2}\lambda_5 v^2, \end{aligned} \quad (4.10)$$

respectively.

When λ_5 is positive the real custodial-singlet neutral scalar with mass m_1 is the lightest of the \mathbb{Z}_2 -odd states and is the dark matter candidate. When λ_5 is negative the custodial-fiveplet states, comprising one neutral, one charged, and one doubly-charged scalar, are the lightest. The question now is if we can have just one neutral state as the dark matter candidate. As shown in [46] the mass degeneracy between charged and neutral states can be lifted via quantum corrections, which tend to make the charged components slightly heavier than the neutral ones. Therefore one would expect that in this phase of the GM model quantum corrections also lift the degeneracy and that the neutral state would be the dark matter candidate. The model has a fully dark sector that only communicates with the visible sector via couplings to the SM Higgs and to the gauge bosons in quartic couplings. The visible sector is indistinguishable from the SM from the point of view of the (tree-level) Higgs couplings to fermions and gauge bosons, though the loop-induced coupling to photon pairs will be modified due to contributions from loops of the \mathbb{Z}_2 -odd states.

Because μ_3^2 remains a free parameter that can be taken arbitrarily large, this phase of the model possesses a decoupling limit, in which all of the \mathbb{Z}_2 -odd states can be taken heavy with masses of order $\sqrt{\mu_3^2}$.

We end this section writing the X and V' vectors for this stationary point, as defined in eqs. (3.7) and (3.11),

$$X_{\text{DM}} = \begin{pmatrix} m_h^2/(16\lambda_1) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad V'_{\text{DM}} = A + BX_{\text{DM}} = \begin{pmatrix} 0 \\ m_1^2 + \bar{k}^2 \\ \bar{k}^2 \\ \bar{k}^2 \\ \bar{k}^2 \\ 0 \end{pmatrix}, \quad (4.11)$$

with $\bar{k}^2 = -\frac{\lambda_5 m_h^2}{16\lambda_1}$.

We note that this DM vacuum is *not* the limit of the custodial vacuum with $v_5 \rightarrow 0$, but rather a disconnected phase of the model. Setting $v_5 = 0$ leads to one of the minimization conditions being trivially satisfied, which removes a constraint that otherwise applies in the Custodial vacuum. A major phenomenological consequence of this is that, in the DM vacuum, the masses of the \mathbb{Z}_2 -odd scalars can all be made arbitrarily large by taking μ_3^2 large, whereas in the Custodial vacuum of the \mathbb{Z}_2 -symmetric potential the masses of all the physical scalars are bounded from above by perturbative unitarity of the quartic couplings [6].

5 Models where the custodial symmetry is broken after SSB

5.1 The real non-custodial vacuum

Let us now consider the scenario where the solution of the minimization equations has $\langle \varphi_8 \rangle \neq \sqrt{2} \langle \varphi_5 \rangle$ (but still no charged VEVs) which means that the custodial symmetry is broken. The absence of the dimension 3 soft breaking terms will then lead to two massless scalars (the result of spontaneous breaking of continuous global symmetries of the model). Let the VEV choice be

$$\langle \varphi_1 \rangle = v_1, \quad \langle \varphi_5 \rangle = v_5, \quad \langle \varphi_6 \rangle = 0, \quad \langle \varphi_8 \rangle = v_8, \quad \langle \varphi_{10} \rangle = 0, \quad (5.1)$$

which leads to the minimization conditions

$$\begin{aligned} \mu_2^2 &= -4\lambda_1 v_1^2 - 2\lambda_2 (v_5^2 + v_8^2) + \frac{2\lambda_3 v_8^2 (2\sqrt{2}v_5 + v_8) (v_5 v_8^2 - 2v_5^3)}{v_1^2 (\sqrt{2}v_5^2 + v_5 v_8 - \sqrt{2}v_8^2)}, \\ \mu_3^2 &= -2\lambda_2 v_1^2 - 4v_5^2 (\lambda_3 + \lambda_4) - 4v_8^2 (\lambda_3 + \lambda_4) + \frac{4\lambda_3 v_8^3}{\sqrt{2}v_5 + 2v_8}, \\ \lambda_5 &= \frac{4\lambda_3 v_5 v_8 (v_8^2 - 2v_5^2)}{v_1^2 (\sqrt{2}v_5^2 + v_5 v_8 - \sqrt{2}v_8^2)}. \end{aligned} \quad (5.2)$$

The vectors X and V' are

$$X_{\text{NC}} = \begin{pmatrix} v_1^2/2 \\ 1/2(v_5^2 + v_8^2) \\ \sqrt{2}v_5v_8 \\ v_8^2/2 \\ 0 \\ 0 \end{pmatrix}, \quad V'_{\text{NC}} = \begin{pmatrix} 0 \\ \frac{4\lambda_3 v_8^3}{\sqrt{2}v_5 + 2v_8} \\ \frac{2\lambda_3 v_5 v_8^2}{v_5 + \sqrt{2}v_8} \\ 2\lambda_3 v_8 \left(\frac{2v_5^3 - v_5 v_8^2}{\sqrt{2}v_5^2 + v_5 v_8 - \sqrt{2}v_8^2} - v_8 \right) \\ -\frac{2\lambda_3 v_5 v_8^2}{v_5 + \sqrt{2}v_8} \\ \lambda_3 (-v_8^2) \end{pmatrix}. \quad (5.3)$$

5.2 Charge breaking vacua

We now consider the scenarios where charge breaking (CB) occurs spontaneously, and where therefore the photon becomes massive after SSB. *A priori*, it would seem that there would be a large number of possible CB vacua, but in fact — as will be explained in appendix A — the minimization equations of the GM model curtail most of those possibilities. Indeed, there are but five distinct vacua that lead to charge breaking,

1. $v_5 = v_8 = 0$ and $v_{10} = -\frac{\lambda_5}{\lambda_3} \frac{v_1^2}{8v_6}$.
2. $v_5 = v_8 = 0$ and $v_{10} = -v_6$.
3. $v_5 = v_8 = 0$ and $v_{10} = v_6$.
4. $v_1 = v_5 = v_8 = 0$ and $v_6 = -v_{10}$.
5. $v_1 = v_5 = v_8 = 0$ and $v_6 = v_{10}$.

To insist on this point, these are the *only* combinations of real VEVs originating CB which are possible solutions of the extremum equations of the GM model. The minimum conditions then lead to the following relations for vacuum number 1,

$$\begin{aligned} \mu_2^2 &= -\frac{\lambda_2 \lambda_5^2 v_1^4}{32 \lambda_3^2 v_6^2} - \frac{v_1^2 (32 \lambda_1 \lambda_3 + \lambda_5^2)}{8 \lambda_3} - 2 \lambda_2, \\ \mu_3^2 &= -\frac{\lambda_5^2 v_1^4 (\lambda_3 + \lambda_4)}{16 \lambda_3^2 v_6^2} - 2 \lambda_2 v_1^2 - 4 v_6^2 (\lambda_3 + \lambda_4). \end{aligned} \quad (5.4)$$

For vacuum number 2 we have

$$\begin{aligned} \mu_2^2 &= -4 \lambda_1 v_1^2 - v_6^2 (4 \lambda_2 + \lambda_5), \\ \mu_3^2 &= -\frac{1}{2} v_1^2 (4 \lambda_2 + \lambda_5) - 4 v_6^2 (\lambda_3 + 2 \lambda_4). \end{aligned} \quad (5.5)$$

For vacuum number 3 we obtain

$$\begin{aligned} \mu_2^2 &= -4 \lambda_1 v_1^2 + v_6^2 (-4 \lambda_2 + \lambda_5), \\ \mu_3^2 &= \frac{1}{2} v_1^2 (-4 \lambda_2 + \lambda_5) - 4 v_6^2 (\lambda_3 + 2 \lambda_4). \end{aligned} \quad (5.6)$$

Finally for vacua 4 and 5 the condition is

$$\mu_3^2 = -4 v_6^2 (\lambda_3 + 2 \lambda_4). \quad (5.7)$$

It should be clear to the reader that the values of the VEVs shown here is almost certainly different from vacuum to vacuum, though we are not labeling them differently. Following the bilinear formalism, we can then write X and V' vectors for each CB vacuum:

- Charge-breaking 1:

$$X_{CB1} = \begin{pmatrix} v_1^2/2 \\ \bar{l}^2/2 \\ 0 \\ 0 \\ -v_1^2\lambda_5/(8\lambda_3) \\ \bar{l}^2 \end{pmatrix}, \quad V'_{CB1} = A + BX_{CB1} = \begin{pmatrix} 0 \\ 0 \\ -v_1^2\lambda_5/4 \\ -v_1^2\lambda_5/2 - 2\lambda_3\bar{l}^2 \\ 0 \\ 0 \end{pmatrix}, \quad (5.8)$$

with $\bar{l}^2 = v_6^2 + \frac{v_1^4\lambda_5^2}{64v_6^2\lambda_3^2}$.

- Charge-breaking 2:

$$X_{CB2} = \begin{pmatrix} v_1^2/2 \\ v_6^2 \\ 0 \\ 0 \\ -v_6^2 \\ 2v_6^2 \end{pmatrix}, \quad V'_{CB2} = A + BX_{CB2} = \begin{pmatrix} 0 \\ 4v_6^2\lambda_3 - v_1^2\lambda_5/2 \\ 2v_6^2\lambda_3 - v_1^2\lambda_5/2 \\ -4v_6^2\lambda_3 - v_1^2\lambda_5/2 \\ 4v_6^2\lambda_3 - v_1^2\lambda_5/2 \\ 0 \end{pmatrix}. \quad (5.9)$$

- Charge-breaking 3:

$$X_{CB3} = \begin{pmatrix} v_1^2/2 \\ v_6^2 \\ 0 \\ 0 \\ v_6^2 \\ 2v_6^2 \end{pmatrix}, \quad V'_{CB3} = A + BX_{CB3} = \begin{pmatrix} 0 \\ 4v_6^2\lambda_3 + v_1^2\lambda_5/2 \\ -2v_6^2\lambda_3 - v_1^2\lambda_5/2 \\ -4v_6^2\lambda_3 - v_1^2\lambda_5/2 \\ -4v_6^2\lambda_3 - v_1^2\lambda_5/2 \\ 0 \end{pmatrix}. \quad (5.10)$$

- Charge-breaking 4:

$$X_{CB4} = \begin{pmatrix} 0 \\ v_6^2 \\ 0 \\ 0 \\ -v_6^2 \\ 2v_6^2 \end{pmatrix}, \quad V'_{CB4} = A + BX_{CB4} = \begin{pmatrix} \mu_2^2 + v_6^2(4\lambda_2 + \lambda_5) \\ 4v_6^2\lambda_3 \\ 2v_6^2\lambda_3 \\ -4v_6^2\lambda_3 \\ 4v_6^2\lambda_3 \\ 0 \end{pmatrix}. \quad (5.11)$$

- Charge-breaking 5:

$$X_{CB5} = \begin{pmatrix} 0 \\ v_6^2 \\ 0 \\ 0 \\ v_6^2 \\ 2v_6^2 \end{pmatrix}, \quad V'_{CB5} = A + BX_{CB5} = \begin{pmatrix} \mu_2^2 + v_6^2(4\lambda_2 - \lambda_5) \\ 4v_6^2\lambda_3 \\ -2v_6^2\lambda_3 \\ -4v_6^2\lambda_3 \\ -4v_6^2\lambda_3 \\ 0 \end{pmatrix}. \quad (5.12)$$

5.3 Wrong-electroweak vacuum

There is a set of unphysical solutions of the minimization equations that do not break the electroweak symmetry correctly, leading to a massless Z boson — a curious situation, only made possible by the possibility of having VEVs for the triplet fields with zero hypercharge. The two possible minima in this category are

- **W1:** $\langle\varphi_1\rangle = 0$, $\langle\varphi_5\rangle = 0$, $\langle\varphi_6\rangle = \frac{1}{2}\sqrt{-\frac{\mu_3^2}{\lambda_3+\lambda_4}}$, $\langle\varphi_8\rangle = 0$, $\langle\varphi_{10}\rangle = 0$,
 - **W2:** $\langle\varphi_1\rangle = 0$, $\langle\varphi_5\rangle = v_5$, $\langle\varphi_6\rangle = v_6$, $\langle\varphi_8\rangle = 0$, $\langle\varphi_{10}\rangle = 0$,
- with the further condition

$$\mu_3^2 = -4(\lambda_3 + \lambda_4) (v_5^2 + v_6^2). \quad (5.13)$$

Notice, in fact, how the above stationary points correspond to VEVs for the fields ξ^0 and ξ^+ from eq. (2.2), which carry no hypercharge, allowing the Z boson to remain massless.⁵ Indeed, the symmetry group left invariant after SSB for these vacua is $U(1) \times U(1)$ — one of these $U(1)$ is the unbroken hypercharge group, the other is “extracted” from $SU(2)$. The X and V' vectors in this scenario are

- W1:

$$X_{W1} = \begin{pmatrix} 0 \\ -\frac{\mu_3^2}{8(\lambda_3+\lambda_4)} \\ 0 \\ 0 \\ 0 \\ -\frac{\mu_3^2}{4(\lambda_3+\lambda_4)} \end{pmatrix}, \quad V'_{W1} = A + BX_{W1} = \begin{pmatrix} \mu_2^2 - \frac{\lambda_2\mu_3^2}{2(\lambda_3+\lambda_4)} \\ 0 \\ 0 \\ \frac{\lambda_3\mu_3^2}{2(\lambda_3+\lambda_4)} \\ 0 \\ 0 \end{pmatrix}. \quad (5.14)$$

- W2:

$$X_{W2} = \begin{pmatrix} 0 \\ \frac{1}{2}(v_5^2 + v_6^2) \\ 0 \\ 0 \\ 0 \\ v_6^2 \end{pmatrix}, \quad V'_{W2} = A + BX_{W2} = \begin{pmatrix} \mu_2^2 + 2\lambda_2(v_5^2 + v_6^2) \\ 0 \\ 0 \\ -2\lambda_3(v_5^2 + v_6^2) \\ 0 \\ 0 \end{pmatrix}. \quad (5.15)$$

5.4 Tantamount vacua

There is a set of vacua that produces three massive gauge bosons and thus is tantamount to the SM electroweak breaking vacuum. The difference lies in the different gauge bosons' mass structure. In the SM breaking we obtain $m_W = gv/2$ and $m_Z = \sqrt{g^2 + g'^2}v/2$ while in this

⁵It may seem peculiar to have a VEV for ξ^+ and still have a massless photon in these vacua, but in fact the definition of electric charge is different in this situation.

case the neutral component of the $SU(2)_L$ gauge bosons remains massless (“the photon”) and the “ Z boson” is just the (massive) hypercharge gauge boson. These vacua would therefore yield an unrealistic value for the ρ parameter, as well as unrealistic couplings of the Z boson to fermions, and are thus in disagreement with current experimental data.

The vacua that produce these results are

- **T1:** $\langle\varphi_1\rangle = 0$, $\langle\varphi_5\rangle = v_5$, $\langle\varphi_6\rangle = 0$, $\langle\varphi_8\rangle = 0$, $\langle\varphi_{10}\rangle = v_{10}$, with the condition

$$\mu_3^2 = -4(\lambda_3 + \lambda_4) (v_{10}^2 + v_5^2). \quad (5.16)$$

In this vacuum the gauge boson masses are $m_W = g\sqrt{v_{10}^2 + v_5^2}$, $m_Z = g'v_{10}$.

- **T2:** $\langle\varphi_1\rangle = 0$, $\langle\varphi_5\rangle = 0$, $\langle\varphi_6\rangle = 0$, $\langle\varphi_8\rangle = 0$, $\langle\varphi_{10}\rangle = v_{10}$, with the condition

$$\mu_3^2 = -4(\lambda_3 + \lambda_4)v_{10}^2. \quad (5.17)$$

In this vacuum the gauge boson masses are $m_W = gv_{10}$, $m_Z = g'v_{10}$.

The corresponding X and V' vectors are

- T1:

$$X_{T1} = \begin{pmatrix} 0 \\ -\frac{\mu_3^2}{8(\lambda_3 + \lambda_4)} \\ 0 \\ 0 \\ 0 \\ -\frac{\mu_3^2}{4(\lambda_3 + \lambda_4)} - v_5^2 \end{pmatrix}, \quad V'_{T1} = A + BX_{T1} = \begin{pmatrix} \mu_2^2 - \frac{\lambda_2\mu_3^2}{2(\lambda_3 + \lambda_4)} \\ 0 \\ 0 \\ \frac{\lambda_3\mu_3^2}{2(\lambda_3 + \lambda_4)} + 2\lambda_3v_5^2 \\ 0 \\ 0 \end{pmatrix}. \quad (5.18)$$

- T2:

$$X_{T2} = \begin{pmatrix} 0 \\ -\frac{\mu_3^2}{8(\lambda_3 + \lambda_4)} \\ 0 \\ 0 \\ 0 \\ -\frac{\mu_3^2}{4(\lambda_3 + \lambda_4)} \end{pmatrix}, \quad V'_{T2} = A + BX_{T2} = \begin{pmatrix} \mu_2^2 - \frac{\lambda_2\mu_3^2}{2(\lambda_3 + \lambda_4)} \\ 0 \\ 0 \\ \frac{\lambda_3\mu_3^2}{2(\lambda_3 + \lambda_4)} \\ 0 \\ 0 \end{pmatrix}. \quad (5.19)$$

6 Stability of the viable models

6.1 Stability of dark matter minima

Let us now consider the coexistence of a dark matter *minimum* with any of the other stationary points (minima or not) listed above. The starting point of this analysis is that the minimization equations of the potential admit solutions of two different types simultaneously. We force the dark matter solution to be a minimum while the stationary point to be compared has no constraints on the second derivatives. The bilinear formalism

provides an easy method to compute and compare the values of the potential at each of those two stationary points.

Let us start by comparing the DM vacuum with all the CB ones. We will analyse in detail two of the cases, the arguments will be the same for all the others.

- CB1 vs. DM:

- for $\lambda_5 > 0$

$$V_{\text{CB1}} - V_{\text{DM}} = \frac{1}{2} (v_6^2 + v_{10}^2) m_5^2 + \frac{1}{4} v_1^2 \lambda_5 (v_6 - v_{10})^2 > 0, \quad (6.1)$$

- for $\lambda_5 < 0$

$$V_{\text{CB1}} - V_{\text{DM}} = \frac{1}{2} (v_6^2 + v_{10}^2) m_1^2 - \frac{1}{4} v_1^2 \lambda_5 (v_6 + v_{10})^2 > 0, \quad (6.2)$$

where all explicit VEVs are from the CB1 stationary point, and all masses are from the DM minimum. Given our initial hypothesis of the DM phase being a minimum, the squared scalar mass m_5^2 is necessarily positive; likewise, we see that all VEV combinations figuring in the above expressions are perforce positive — and the signs affecting the λ_5 coupling render all terms where it appears also positive. Thus, the difference between the potential at the different SPs, $V_{\text{CB1}} - V_{\text{DM}}$ is always positive — this means that, if there is a DM minimum, any CB1 extremum will necessarily lie *above* it.

- CB2 vs. DM:

$$V_{\text{CB2}} - V_{\text{DM}} = \frac{m_5^2 v_6^2}{2}, \quad (6.3)$$

where all explicit VEVs are from CB2 and all masses are from the DM phase. This difference is clearly always positive when the DM phase is a minimum.

- CB3 vs. DM:

$$V_{\text{CB3}} - V_{\text{DM}} = \frac{m_1^2 v_6^2}{2}, \quad (6.4)$$

where again all explicit VEVs are from CB3 and all masses are from the DM phase. And this difference is again always positive.

- CB4 and CB5 vs. DM:

$$V_{\text{CB4/5}} - V_{\text{DM}} = \frac{m_h^4 + 32\lambda_1 \mu_3^2 v_6^2}{64\lambda_1}, \quad (6.5)$$

where all explicit VEVs are from CB4 or CB5 and all masses are from the DM phase. This difference can be either positive or negative — notice how the quadratic coupling μ_3^2 can be negative, and therefore the sign of the potential difference between the two stationary points is not determined. In fact, a numerical study of this case found regions of parameter space for which $V_{\text{CB4/5}} - V_{\text{DM}} > 0$, but also other regions where $V_{\text{CB4/5}} - V_{\text{DM}} < 0$.

- Custodial vacuum vs. DM:

$$V_C - V_{\text{DM}} = \frac{3m_1^2 v_5^2}{4}, \quad (6.6)$$

where all explicit VEVs are from the custodial vacuum and all masses are from the DM phase. This difference is thus always positive.

- Non-custodial vacuum vs. DM:

$$V_{\text{NC}} - V_{\text{DM}} = \frac{1}{12} \left[m_1^2 (v_5 + \sqrt{2}v_8)^2 + m_5^2 (\sqrt{2}v_5 - v_8)^2 \right], \quad (6.7)$$

where all explicit VEVs are from the non-custodial vacuum and the masses are from the DM minimum. This difference is thus always positive.

- W1, W2, T1, T2 vs. DM:

$$V_{W1, W2, T1, T2} - V_{\text{DM}} = \frac{1}{16} \left(\frac{\mu_2^4}{\lambda_1} - \frac{\mu_3^4}{\lambda_3 + \lambda_4} \right), \quad (6.8)$$

The difference in values of the potential can be positive or negative, depending on the magnitudes of both quadratic coefficients.⁶

Therefore, taking into account all the conditions above, in order to force the DM vacuum to be an absolute minimum at tree-level we just need to impose the two conditions:

$$\frac{8\lambda_1\mu_3^4}{\lambda_3 + 2\lambda_4} \leq m_h^4, \quad (6.9)$$

$$\frac{\mu_2^4}{\lambda_1} - \frac{\mu_3^4}{\lambda_3 + \lambda_4} \geq 0, \quad (6.10)$$

where, remember, in the DM minimum h is supposed to represent the SM-like Higgs boson and as such $m_h = 125$ GeV. Using eq. (4.8) and eq. (4.9), we can cast these two equations in a different form, as they become lower bounds on combinations of λ_3 and λ_4 :

$$\min \left(\frac{1}{2}\lambda_3 + \lambda_4, \lambda_3 + \lambda_4 \right) \geq \frac{\mu_3^4}{2v^2 m_h^2}. \quad (6.11)$$

Notice that the bounded from below conditions of eq. (2.5) force these two combinations of λ_3 and λ_4 to be positive, this constraint is therefore more restrictive.⁷

6.2 Stability of the custodial vacuum

As in the previous section for the DM phase, our starting point now will be to assume the existence of a custodial *minimum*, coexisting with other types of stationary points. Let us go through the several possibilities:

⁶Notice that, due to the bounded from below conditions of eq. (2.5), the combinations of λ coefficients in eq. (6.8) are necessarily positive.

⁷Though this constraint presupposes the existence of some Wrong-electroweak or Tantomount stationary point coexisting with the DM minimum.

- DM vs. Custodial:

$$V_{\text{DM}} - V_{\text{C}} = \frac{3v_1^2 v_5^2 (4\lambda_1 \mu_3^2 - 2\lambda_2 \mu_2^2 + \lambda_5 \mu_2^2)^2}{4\lambda_1 m_h^2 m_H^2}, \quad (6.12)$$

where please notice that the masses figuring in this expression are now those obtained in the custodial vacuum (see eq. (4.3), eq. (4.4) and eq. (4.5)). This difference in values of the potential at the two stationary points is clearly positive, which indicates that, whenever there is a custodial minimum, any existing DM stationary point will necessarily lie *above* it. Notice, though, that we had already concluded, in eq. (6.6), that a DM minimum implied that any custodial stationary point needed to lie above it. The two conclusions are not contradictory, rather they imply that if a DM (custodial) minimum exists, any coexisting custodial (DM) stationary point lies above it, and is necessarily a *saddle point*. In fact, comparing eq. (6.6) and eq. (6.12), we see that when the minimum corresponds to the DM phase, then $V_{\text{DM}} - V_{\text{C}} < 0$ which, from the equation above, implies that *one* of the squared scalar masses of the custodial stationary point, m_h^2 or m_H^2 , must perforce be negative, while the other is positive — thus we conclude that not only the custodial stationary point lies above the DM minimum, it is also necessarily a *saddle point*. Analogously, if we have a custodial minimum then $V_{\text{DM}} - V_{\text{C}} > 0$ and from eq. (6.6) we see that one must necessarily have one of the squared masses computed at the DM stationary point, m_2^2 , be *negative* — it is easy to then show that in that case the DM stationary point is also a saddle point, lying above the custodial minimum.

- Non-custodial vs. custodial:

$$V_{\text{NC}} - V_{\text{C}} = \frac{A^{\text{NC/C}}}{B^{\text{NC/C}}} \quad (6.13)$$

with

$$\begin{aligned} A^{\text{NC/C}} = & -\lambda_3 \left(2v_5^2 - v_8^2 \right) \left\{ -v_1^4 \left(-32v_5^6 v_8^2 - 4\sqrt{2}v_5^5 v_8^3 + 50v_5^4 v_8^4 - 11\sqrt{2}v_5^3 v_8^5 - 26v_5^2 v_8^6 \right. \right. \\ & \left. \left. + 4\sqrt{2}v_5^7 v_8 + 8v_5^8 + 14\sqrt{2}v_5 v_8^7 - 4v_8^8 \right) \right. \\ & \times \left\{ v_5^2 \left[\lambda_2^2 - 4\lambda_1(\lambda_3 + \lambda_4) \right] - 2v_8^2 \left[\lambda_2^2 - 2\lambda_1(\lambda_3 + 2\lambda_4) \right] \right\} \\ & \left. + 4\lambda_2 \lambda_3 v_1^2 v_5 v_8^2 \left(2v_5^2 - v_8^2 \right) \right. \\ & \times \left(-12v_5^5 v_8^2 - 8\sqrt{2}v_5^4 v_8^3 + 17v_5^3 v_8^4 + 3\sqrt{2}v_5^2 v_8^5 + 4\sqrt{2}v_5 v_8^6 + 4v_5^7 - 10v_5 v_8^6 + 2\sqrt{2}v_8^7 \right) \\ & + 2\lambda_3 v_5^2 v_8^2 \left(2v_5^2 - v_8^2 \right) \left[24v_5^8 (\lambda_3 + \lambda_4) - 16v_5^6 v_8^2 (5\lambda_3 + 6\lambda_4) - 12\sqrt{2}v_5^5 v_8^3 (\lambda_3 + \lambda_4) \right. \\ & + 6v_5^4 v_8^4 (17\lambda_3 + 25\lambda_4) - \sqrt{2}v_5^3 v_8^5 (17\lambda_3 + 33\lambda_4) \\ & - 6v_5^2 v_8^6 (7\lambda_3 + 13\lambda_4) + 12\sqrt{2}v_5^7 v_8 (\lambda_3 + \lambda_4) \\ & \left. \left. + 6\sqrt{2}v_5 v_8^7 (3\lambda_3 + 7\lambda_4) - 4v_8^8 (\lambda_3 + 3\lambda_4) \right] \right\}, \end{aligned} \quad (6.14)$$

$$\begin{aligned} B^{\text{NC/C}} = & 2\sqrt{2} \left(v_5 + \sqrt{2}v_8 \right)^2 \left(\sqrt{2}v_5^2 + v_5 v_8 - \sqrt{2}v_8^2 \right) \\ & \times \left\{ v_1^4 \left(-3v_5^2 v_8^2 + 2\sqrt{2}v_5^3 v_8 + 2v_5^4 - 2\sqrt{2}v_5 v_8^3 + 2v_8^4 \right) \left[3\lambda_2^2 - 4\lambda_1(\lambda_3 + 3\lambda_4) \right] \right. \\ & \left. + 12\lambda_2 \lambda_3 v_1^2 v_5 v_8 \left(2v_5^2 - v_8^2 \right) \left(\sqrt{2}v_5^2 + v_5 v_8 - \sqrt{2}v_8^2 \right) + 12\lambda_3^2 v_5^2 v_8^2 \left(v_8^2 - 2v_5^2 \right)^2 \right\}. \end{aligned} \quad (6.15)$$

The VEVs are from the NC vacuum. The difference can be positive or negative.

- CB1 vs. Custodial:

$$V_{\text{CB1}} - V_{\text{C}} = \frac{A^{\text{CB1/C}}}{B^{\text{CB1/C}}}, \quad (6.16)$$

with

$$\begin{aligned} A^{\text{CB1/C}} = & \mu_3^4 \left[1024\lambda_1^2\lambda_3^2 + 128\lambda_1\lambda_3\lambda_5(\lambda_5 - 3\lambda_2) + 3\lambda_5^2(\lambda_5 - 2\lambda_2)^2 \right] \\ & + 8\mu_2^2\mu_3^2 \{ 32\lambda_1\lambda_3 [3\lambda_5(\lambda_3 + \lambda_4) - 4\lambda_2\lambda_3] + 3\lambda_5(2\lambda_2 - \lambda_5) [4\lambda_2\lambda_3 - \lambda_5(\lambda_3 + \lambda_4)] \} \\ & + 16\mu_2^4 \left[16\lambda_2^2\lambda_3^2 - 24\lambda_2\lambda_3\lambda_5(\lambda_3 + \lambda_4) + \lambda_5^2(\lambda_3 + \lambda_4)(7\lambda_3 + 3\lambda_4) \right], \end{aligned} \quad (6.17)$$

$$B^{\text{CB1/C}} = 16 \left[3(\lambda_5 - 2\lambda_2)^2 - 16\lambda_1(\lambda_3 + 3\lambda_4) \right] \left[8\lambda_2^2\lambda_3 - (\lambda_3 + \lambda_4) (32\lambda_1\lambda_3 + \lambda_5^2) \right]. \quad (6.18)$$

Numerically, we found that a coexisting CB1 vacuum can have an energy above or below the Custodial vacuum.

- CB2 vs. Custodial:

$$V_{\text{CB2}} - V_{\text{C}} = \frac{1}{8} \left[\frac{9m_3^2 (v_{\text{C},1}^2 v_{\text{CB},6}^2 + 2v_{\text{CB},1}^2 v_{\text{C},5}^2)}{v_{\text{C},1}^2 + 8v_{\text{C},5}^2} + m_5^2 v_{\text{CB},6}^2 \right], \quad (6.19)$$

where VEVs are marked as either being charge-breaking or custodial and all masses are from the custodial vacuum. The difference is thus always positive.

- CB3 vs. Custodial:

$$V_{\text{CB3}} - V_{\text{C}} = \frac{1}{8} \left[\frac{m_3^2 (v_{\text{C},1}^2 v_{\text{CB},6}^2 + 6v_{\text{CB},1}^2 v_{\text{C},5}^2)}{v_{\text{C},1}^2 + 8v_{\text{C},5}^2} + m_5^2 v_{\text{CB},6}^2 \right], \quad (6.20)$$

where VEVs are marked as either being charge-breaking or custodial and all masses are from the custodial vacuum. The difference is always positive.

- CB4/5 vs. Custodial:

$$V_{\text{CB4/5}} - V_{\text{C}} = -\frac{v_1^2(-2\lambda_2\mu_3^2 + 2\lambda_3\mu_2^2 + 4\lambda_4\mu_2^2 + \lambda_5\mu_3^2) + 2\lambda_3\mu_3^2 v_5^2}{8(\lambda_3 + 2\lambda_4)}, \quad (6.21)$$

where VEVs are from the custodial vacuum. The difference can be positive or negative.

- W1, W2, T1 and T2 vs. Custodial. For all these scenarios the difference in height in the potential is given by the expression

$$V_{W1,W2,T1,T2} - V_{\text{C}} = \frac{v^2 (2\mu_3^2 v_5^2 (m_5^2 - 2\lambda_2 v_1^2) + \mu_2^2 v_1^2 (m_5^2 + 8\lambda_4 v_5^2)) - m_3^2 (3\mu_2^2 v_1^4 + 2\mu_3^2 v_1^2 v_5^2)}{12m_3^2 v_1^2 - 4v^2 (m_5^2 + 8\lambda_4 v_5^2)}, \quad (6.22)$$

where VEVs and masses are from the Custodial vacuum. The difference can be positive or negative.

Therefore, we can finally state the conditions to have the custodial extremum as the absolute minimum. Besides having all masses positive to guarantee that we are indeed in a minimum, we need to force the parameters of the potential to comply with the following conditions

$$V_{\text{NC}} - V_{\text{C}} \geq 0, \quad (6.23)$$

$$V_{\text{CB1}} - V_{\text{C}} \geq 0, \quad (6.24)$$

$$-\frac{v_1^2(-2\lambda_2\mu_3^2 + 2\lambda_3\mu_2^2 + 4\lambda_4\mu_2^2 + \lambda_5\mu_3^2) + 2\lambda_3\mu_3^2v_5^2}{8(\lambda_3 + 2\lambda_4)} \geq 0, \quad (6.25)$$

$$\frac{v^2(2\mu_3^2v_5^2(m_5^2 - 2\lambda_2v_1^2) + \mu_2^2v_1^2(m_5^2 + 8\lambda_4v_5^2)) - m_3^2(3\mu_2^2v_1^4 + 2\mu_3^2v_1^2v_5^2)}{12m_3^2v_1^2 - 4v^2(m_5^2 + 8\lambda_4v_5^2)} \geq 0. \quad (6.26)$$

The first equation, eq. (6.23), is written using eq. (6.13), eq. (6.14) and eq. (6.15). They are written as a function of the non-custodial VEVs that obey eqs. (5.2). The remaining three equations eq. (6.24), eq. (6.25), eq. (6.26) are written as a function of the parameters of the potential, the masses in the custodial phase and the custodial VEVs. In particular, eq. (6.24) needs eq. (6.16), eq. (6.17) and eq. (6.18).

7 Conclusions

We have analysed the vacuum structure of a simple version of the GM model where the scalar potential is not only invariant under $\text{SU}(2)_L \times \text{SU}(2)_R$ but is also invariant under the \mathbb{Z}_2 symmetry of the triplet fields, $\Xi \rightarrow -\Xi$. We have then used the invariance freedom to reduce the number of possible VEVs to 7. In this first attempt to understand the vacuum structure of the model we have further reduced the number to 5 by considering only the real component of the VEVs. In that sense the conditions obtained for the potential to be in an absolute minimum are necessary conditions.

We have classified the phases of the model in viable phases, the Custodial and DM phases, and the non-viable. The non-viable are the non-custodial vacuum where 5 Goldstone bosons are generated, the charge-breaking vacua (CB1,2 with 6 Goldstone bosons and CB3,4,5 with 5 Goldstone bosons), and the wrong-electroweak and tantamount vacua with 4 Goldstone bosons. We have then derived a set of analytical formulae that allows to compare the difference in the depths of two stationary points from two distinct phases. If we want to be certain that we have an absolute minimum in the DM phase, we just need that all masses be positive in that phase and to impose the conditions given in equation (6.11). These are the two conditions for which we have checked numerically that the difference in depths could be either positive or negative. Hence, we need to force the DM minimum to be below the other phases' stationary point. If in turn we want the custodial vacuum to be a minimum we impose positivity of the masses in that phase together with the conditions expressed by equations (6.23), (6.24), (6.25) and (6.26). Note that there are many cases where the minimum is always below the stationary point of the other phase at tree-level. This is for instance the case where the DM minimum is compared with the Custodial stationary point or when the Custodial minimum is compared with the DM stationary point.

The Georgi-Machacek model has therefore an elaborate vacuum structure — vacua with Dark Matter candidates; vacua which preserve or not custodial symmetry; and patterns of electroweak breaking with remaining symmetry group $U(1) \times U(1)$, with massless photon and Z , or a wrong value predicted for the Weinberg angle. Crucially, though, this foray into the many possible vacua of the model revealed that neither of the vacua of interest — the Dark Matter minimum or the minimum which preserves custodial symmetry — are automatically stable, as occurs for instance in the 2HDM [26, 27, 30, 34], a model for which it was proven that minima which break different symmetries cannot coexist. For the GM model, though, we have shown that, though the Dark Matter minimum cannot coexist with deeper Charge Breaking, Custodial or Non-Custodial vacua, it can nonetheless have deeper stationary points with incorrect electroweak symmetry breaking (yielding massless Z 's or the wrong value of θ_W). Further, the Custodial minimum is not safe from eventual tunnelling to a deeper charge breaking vacuum of the type CB1, or indeed to a non-custodial vacuum (this last conclusion is known, see [12, 20]). The situation will of course become even more complex when one starts analysing the GM model with soft breaking terms of the \mathbb{Z}_2 considered in this paper — the more common form of the GM model, studied for instance in [12]. Such terms will obviously change the possible vacua and, being cubic in the fields, will necessarily complicate the algebra. Many of the stranger vacua discussed here — the Wrong-Electroweak and Tantamount ones — may well no longer be possible when soft breaking terms appear in the potential. On the other hand, taking the 2HDM as an example, new types minima may become possible with those soft breaking terms (such as for instance spontaneous CP breaking vacua), and coexistence of minima impossible with exact symmetries may be possible when they are softly broken [31, 34].

A Vacua

We have presented the simplest and most general vacuum structure in eq. (2.9). We will now show the possible types of vacua in the \mathbb{Z}_2 symmetric model. The most general vacuum configuration is written as

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 & 0 \\ 0 & v_1 \end{pmatrix}, \quad X = \frac{1}{\sqrt{2}} \begin{pmatrix} v_8 & v_6 & 0 \\ -v_{10} & \sqrt{2}v_5 & v_{10} \\ 0 & -v_6 & v_8 \end{pmatrix}, \quad (\text{A.1})$$

and leads to the following minimum conditions:

$$\left. \frac{\partial V}{\partial \varphi_1} \right|_0 = v_1 \left[\mu_2^2 + 4\lambda_1 v_1^2 + 2\lambda_2(v_5^2 + v_6^2 + v_8^2 + v_{10}^2) - \lambda_5(v_{10}v_6 + \sqrt{2}v_5)v_8 + \frac{v_8^2}{2} \right], \quad (\text{A.2})$$

$$\left. \frac{\partial V}{\partial \varphi_3} \right|_0 = -\frac{\lambda_5}{\sqrt{2}} v_1 v_8 (v_6 + v_{10}), \quad (\text{A.3})$$

$$\begin{aligned} \left. \frac{\partial V}{\partial \varphi_5} \right|_0 = v_5 \left[\mu_3^2 + 4\lambda_4 v_8^2 + 4(v_5^2 + v_6^2 + v_{10}^2)(\lambda_3 + \lambda_4) - 2\sqrt{2}\lambda_3 \frac{v_6 v_8 v_{10}}{v_5} \right. \\ \left. + v_1^2 \left(2\lambda_2 - \frac{\lambda_5 v_8}{\sqrt{2}v_5} \right) \right], \quad (\text{A.4}) \end{aligned}$$

$$\left. \frac{\partial V}{\partial \varphi_6} \right|_0 = v_6 \left[\mu_3^2 + 4\lambda_4 v_{10}^2 + 4(v_5^2 + v_6^2)(\lambda_3 + \lambda_4) + 2v_8^2(\lambda_3 + 2\lambda_4) - 2\sqrt{2}\lambda_3 \frac{v_5 v_8 v_{10}}{v_6} + v_1^2 \left(2\lambda_2 - \frac{\lambda_5 v_{10}}{2v_6} \right) \right], \quad (\text{A.5})$$

$$\left. \frac{\partial V}{\partial \varphi_8} \right|_0 = v_8 \left\{ \mu_3^2 + 4\lambda_4 v_5^2 + 2(v_6^2 + v_8^2 + v_{10}^2)(\lambda_3 + 2\lambda_4) - 2\sqrt{2}\lambda_3 \frac{v_5 v_6 v_{10}}{v_8} + v_1^2 \left[2\lambda_2 - \left(1 + \sqrt{2} \frac{v_5}{v_8} \right) \frac{\lambda_5}{2} \right] \right\}, \quad (\text{A.6})$$

$$\left. \frac{\partial V}{\partial \varphi_{10}} \right|_0 = v_{10} \left[\mu_3^2 + 4\lambda_4 v_6^2 + 4(v_5^2 + v_{10}^2)(\lambda_3 + \lambda_4) + 2v_8^2(\lambda_3 + 2\lambda_4) - 2\sqrt{2}\lambda_3 \frac{v_5 v_6 v_8}{v_{10}} + v_1^2 \left(2\lambda_2 - \frac{\lambda_5 v_6}{2v_{10}} \right) \right], \quad (\text{A.7})$$

$$\left. \frac{\partial V}{\partial \varphi_{11}} \right|_0 = 2\lambda_3 \left(\sqrt{2}v_5 v_6 v_{10} - v_8 v_{10}^2 - v_6^2 v_8 \right), \quad (\text{A.8})$$

and the conditions for the remaining fields are identically zero.

The minimum conditions for φ_3 can be used as a starting point of three main subgroups with either $v_1 = 0$, $v_8 = 0$ or $v_6 = -v_{10}$. We then found step by step all configurations that led to stationary points of the potential. All charge breaking minima have in common the following conditions

$$v_5 = v_8 = 0, \quad v_6 \neq 0, \quad \text{and } v_{10} \neq 0, \quad (\text{A.9})$$

that is, the neutral VEVs from the triplet have to vanish while the charged ones have to be non-zero. There are also Charge-Breaking minima that obey all the above conditions plus $v_1 = 0$. In principle there could be other vacuum configurations that would break electric charge but we found that they are never stationary points. By taking $v_5 = v_8 = 0$ only three stationarity equations survive, the ones related to φ_1 , φ_6 and φ_{10} . This imposes conditions on three (arbitrary) Lagrangian parameters, and we choose

$$\begin{aligned} \mu_2^2 &= -4\lambda_1 v_1^2 - \frac{8\lambda_3 v_{10}^2 v_6^2}{v_1^2} - 2\lambda_2 (v_{10}^2 + v_6^2), \\ \mu_3^2 &= -2 \left(\lambda_2 v_1^2 + 2(\lambda_3 + \lambda_4) (v_{10}^2 + v_6^2) \right), \\ \lambda_5 &= -\frac{8\lambda_3 v_{10} v_6}{v_1^2}. \end{aligned} \quad (\text{A.10})$$

The condition $v_5 = v_8 = 0$ together with $v_6 \neq 0$ and $v_{10} \neq 0$ leads to the following mass matrix for the gauge bosons in the W^1, W^2, W^3, B basis.

$$\begin{pmatrix} \frac{1}{4}g^2 (v_1^2 + 4v_{10}^2) & 0 & 0 & 0 \\ 0 & \frac{1}{4}g^2 (v_1^2 + 4(v_6^2 + v_{10}^2)) & 0 & 0 \\ 0 & 0 & \frac{1}{4}g^2 (v_1^2 + 4v_6^2) & -\frac{1}{4}v_1^2 g g' \\ 0 & 0 & -\frac{1}{4}v_1^2 g g' & \frac{1}{4}g'^2 (v_1^2 + 4v_{10}^2) \end{pmatrix} \quad (\text{A.11})$$

Clearly there is no zero eigenstate and thus the photon becomes massive. There is another Charge-Breaking stationary point with the conditions $v_1 = v_5 = v_8 = 0$ together with $v_6 \neq 0$ and $v_{10} \neq 0$. Again for this case we see that no zero mass eigenstate emerges.

Acknowledgments

DA, PF and RS are supported by FCT, Contracts UIDB/00618/2020, UIDP/00618/2020, PTDC/FIS-PAR/31000/2017, CERN/FISPAR/0002/2017, CERN/FIS-PAR/0014/2019, and by the HARMONIA project, contract UMO-2015/18/M/ST2/0518. HEL is supported by the Natural Sciences and Engineering Research Council of Canada (NSERC). This work was also supported by the grant H2020-MSCA-RISE-2014 No. 645722 (NonMinimalHiggs).

Open Access. This article is distributed under the terms of the Creative Commons Attribution License ([CC-BY 4.0](https://creativecommons.org/licenses/by/4.0/)), which permits any use, distribution and reproduction in any medium, provided the original author(s) and source are credited.

References

- [1] H. Georgi and M. Machacek, *Doubly charged Higgs Bosons*, *Nucl. Phys. B* **262** (1985) 463 [[INSPIRE](#)].
- [2] M.S. Chanowitz and M. Golden, *Higgs Boson Triplets With $M_W = M_Z \cos \theta_W$* , *Phys. Lett. B* **165** (1985) 105 [[INSPIRE](#)].
- [3] J.F. Gunion, R. Vega and J. Wudka, *Higgs triplets in the standard model*, *Phys. Rev. D* **42** (1990) 1673 [[INSPIRE](#)].
- [4] J.F. Gunion, R. Vega and J. Wudka, *Naturalness problems for $\rho = 1$ and other large one loop effects for a standard model Higgs sector containing triplet fields*, *Phys. Rev. D* **43** (1991) 2322 [[INSPIRE](#)].
- [5] K.-m. Cheung, R.J.N. Phillips and A. Pilaftsis, *Signatures of Higgs triplet representations at TeV e^+e^- colliders*, *Phys. Rev. D* **51** (1995) 4731 [[hep-ph/9411333](#)] [[INSPIRE](#)].
- [6] M. Aoki and S. Kanemura, *Unitarity bounds in the Higgs model including triplet fields with custodial symmetry*, *Phys. Rev. D* **77** (2008) 095009 [Erratum *ibid.* **89** (2014) 059902] [[arXiv:0712.4053](#)] [[INSPIRE](#)].
- [7] H.E. Logan and M.-A. Roy, *Higgs couplings in a model with triplets*, *Phys. Rev. D* **82** (2010) 115011 [[arXiv:1008.4869](#)] [[INSPIRE](#)].
- [8] C.-W. Chiang and K. Yagyu, *Testing the custodial symmetry in the Higgs sector of the Georgi-Machacek model*, *JHEP* **01** (2013) 026 [[arXiv:1211.2658](#)] [[INSPIRE](#)].
- [9] C. Englert, E. Re and M. Spannowsky, *Pinning down Higgs triplets at the LHC*, *Phys. Rev. D* **88** (2013) 035024 [[arXiv:1306.6228](#)] [[INSPIRE](#)].
- [10] C.-W. Chiang, A.-L. Kuo and K. Yagyu, *Enhancements of weak gauge boson scattering processes at the CERN LHC*, *JHEP* **10** (2013) 072 [[arXiv:1307.7526](#)] [[INSPIRE](#)].
- [11] S. Kanemura, K. Tsumura, K. Yagyu and H. Yokoya, *Fingerprinting nonminimal Higgs sectors*, *Phys. Rev. D* **90** (2014) 075001 [[arXiv:1406.3294](#)] [[INSPIRE](#)].

- [12] K. Hartling, K. Kumar and H.E. Logan, *The decoupling limit in the Georgi-Machacek model*, *Phys. Rev. D* **90** (2014) 015007 [[arXiv:1404.2640](#)] [[INSPIRE](#)].
- [13] C.-W. Chiang, S. Kanemura and K. Yagyu, *Novel constraint on the parameter space of the Georgi-Machacek model with current LHC data*, *Phys. Rev. D* **90** (2014) 115025 [[arXiv:1407.5053](#)] [[INSPIRE](#)].
- [14] C. Degrande, K. Hartling, H.E. Logan, A.D. Peterson and M. Zaro, *Automatic predictions in the Georgi-Machacek model at next-to-leading order accuracy*, *Phys. Rev. D* **93** (2016) 035004 [[arXiv:1512.01243](#)] [[INSPIRE](#)].
- [15] J. Chang, C.-R. Chen and C.-W. Chiang, *Higgs boson pair productions in the Georgi-Machacek model at the LHC*, *JHEP* **03** (2017) 137 [[arXiv:1701.06291](#)] [[INSPIRE](#)].
- [16] B. Keeshan, H.E. Logan and T. Pilkington, *Custodial symmetry violation in the Georgi-Machacek model*, *Phys. Rev. D* **102** (2020) 015001 [[arXiv:1807.11511](#)] [[INSPIRE](#)].
- [17] N. Ghosh, S. Ghosh and I. Saha, *Charged Higgs boson searches in the Georgi-Machacek model at the LHC*, *Phys. Rev. D* **101** (2020) 015029 [[arXiv:1908.00396](#)] [[INSPIRE](#)].
- [18] C.-W. Chiang, G. Cottin and O. Eberhardt, *Global fits in the Georgi-Machacek model*, *Phys. Rev. D* **99** (2019) 015001 [[arXiv:1807.10660](#)] [[INSPIRE](#)].
- [19] A. Ismail, H.E. Logan and Y. Wu, *Updated constraints on the Georgi-Machacek model from LHC Run 2*, [arXiv:2003.02272](#) [[INSPIRE](#)].
- [20] K. Hartling, K. Kumar and H.E. Logan, *GMCALC: a calculator for the Georgi-Machacek model*, [arXiv:1412.7387](#) [[INSPIRE](#)].
- [21] T. Araki, C.Q. Geng and K.I. Nagao, *Dark Matter in Inert Triplet Models*, *Phys. Rev. D* **83** (2011) 075014 [[arXiv:1102.4906](#)] [[INSPIRE](#)].
- [22] R. Campbell, S. Godfrey, H.E. Logan and A. Poulin, *Real singlet scalar dark matter extension of the Georgi-Machacek model*, *Phys. Rev. D* **95** (2017) 016005 [[arXiv:1610.08097](#)] [[INSPIRE](#)].
- [23] T. Pilkington, *Dark matter in the Georgi-Machacek model with an additional inert doublet*, [arXiv:1711.04378](#) [[INSPIRE](#)].
- [24] S.-L. Chen, A. Dutta Banik and Z.-K. Liu, *Common origin of radiative neutrino mass, dark matter and leptogenesis in scotogenic Georgi-Machacek model*, [arXiv:2011.13551](#) [[INSPIRE](#)].
- [25] J. Velhinho, R. Santos and A. Barroso, *Tree level vacuum stability in two Higgs doublet models*, *Phys. Lett. B* **322** (1994) 213 [[INSPIRE](#)].
- [26] P.M. Ferreira, R. Santos and A. Barroso, *Stability of the tree-level vacuum in two Higgs doublet models against charge or CP spontaneous violation*, *Phys. Lett. B* **603** (2004) 219 [*Erratum ibid.* **629** (2005) 114] [[hep-ph/0406231](#)] [[INSPIRE](#)].
- [27] A. Barroso, P.M. Ferreira and R. Santos, *Charge and CP symmetry breaking in two Higgs doublet models*, *Phys. Lett. B* **632** (2006) 684 [[hep-ph/0507224](#)] [[INSPIRE](#)].
- [28] C.C. Nishi, *CP violation conditions in N-Higgs-doublet potentials*, *Phys. Rev. D* **74** (2006) 036003 [*Erratum ibid.* **76** (2007) 119901] [[hep-ph/0605153](#)] [[INSPIRE](#)].
- [29] M. Maniatis, A. von Manteuffel, O. Nachtmann and F. Nagel, *Stability and symmetry breaking in the general two-Higgs-doublet model*, *Eur. Phys. J. C* **48** (2006) 805 [[hep-ph/0605184](#)] [[INSPIRE](#)].

- [30] I.P. Ivanov, *Minkowski space structure of the Higgs potential in 2HDM*, *Phys. Rev. D* **75** (2007) 035001 [[Erratum ibid.](#) **76** (2007) 039902] [[hep-ph/0609018](#)] [[INSPIRE](#)].
- [31] A. Barroso, P.M. Ferreira and R. Santos, *Neutral minima in two-Higgs doublet models*, *Phys. Lett. B* **652** (2007) 181 [[hep-ph/0702098](#)] [[INSPIRE](#)].
- [32] C.C. Nishi, *The Structure of potentials with N Higgs doublets*, *Phys. Rev. D* **76** (2007) 055013 [[arXiv:0706.2685](#)] [[INSPIRE](#)].
- [33] M. Maniatis, A. von Manteuffel and O. Nachtmann, *CP violation in the general two-Higgs-doublet model: A Geometric view*, *Eur. Phys. J. C* **57** (2008) 719 [[arXiv:0707.3344](#)] [[INSPIRE](#)].
- [34] I.P. Ivanov, *Minkowski space structure of the Higgs potential in 2HDM. II. Minima, symmetries, and topology*, *Phys. Rev. D* **77** (2008) 015017 [[arXiv:0710.3490](#)] [[INSPIRE](#)].
- [35] M. Maniatis, A. von Manteuffel and O. Nachtmann, *A New type of CP symmetry, family replication and fermion mass hierarchies*, *Eur. Phys. J. C* **57** (2008) 739 [[arXiv:0711.3760](#)] [[INSPIRE](#)].
- [36] C.C. Nishi, *Physical parameters and basis transformations in the Two-Higgs-Doublet model*, *Phys. Rev. D* **77** (2008) 055009 [[arXiv:0712.4260](#)] [[INSPIRE](#)].
- [37] M. Maniatis and O. Nachtmann, *On the phenomenology of a two-Higgs-doublet model with maximal CP symmetry at the LHC*, *JHEP* **05** (2009) 028 [[arXiv:0901.4341](#)] [[INSPIRE](#)].
- [38] P.M. Ferreira, M. Maniatis, O. Nachtmann and J.P. Silva, *CP properties of symmetry-constrained two-Higgs-doublet models*, *JHEP* **08** (2010) 125 [[arXiv:1004.3207](#)] [[INSPIRE](#)].
- [39] I.P. Ivanov and C.C. Nishi, *Properties of the general NHDM. I. The Orbit space*, *Phys. Rev. D* **82** (2010) 015014 [[arXiv:1004.1799](#)] [[INSPIRE](#)].
- [40] I.P. Ivanov and C.C. Nishi, *Symmetry breaking patterns in 3HDM*, *JHEP* **01** (2015) 021 [[arXiv:1410.6139](#)] [[INSPIRE](#)].
- [41] P.M. Ferreira, *The vacuum structure of the Higgs complex singlet-doublet model*, *Phys. Rev. D* **94** (2016) 096011 [[arXiv:1607.06101](#)] [[INSPIRE](#)].
- [42] P.M. Ferreira, M. Mühlleitner, R. Santos, G. Weiglein and J. Wittbrodt, *Vacuum Instabilities in the $N2HDM$* , *JHEP* **09** (2019) 006 [[arXiv:1905.10234](#)] [[INSPIRE](#)].
- [43] I. Engeln, P. Ferreira, M.M. Mühlleitner, R. Santos and J. Wittbrodt, *The Dark Phases of the $N2HDM$* , *JHEP* **08** (2020) 085 [[arXiv:2004.05382](#)] [[INSPIRE](#)].
- [44] P.M. Ferreira and B.L. Gonçalves, *Stability of neutral minima against charge breaking in the Higgs triplet model*, *JHEP* **02** (2020) 182 [[arXiv:1911.09746](#)] [[INSPIRE](#)].
- [45] D. Das and I. Saha, *Cornering variants of the Georgi-Machacek model using Higgs precision data*, *Phys. Rev. D* **98** (2018) 095010 [[arXiv:1811.00979](#)] [[INSPIRE](#)].
- [46] M. Cirelli, N. Fornengo and A. Strumia, *Minimal dark matter*, *Nucl. Phys. B* **753** (2006) 178 [[hep-ph/0512090](#)] [[INSPIRE](#)].