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Learning Rational Numbers through an Emergent Modeling Process enhanced by Percentage

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In this paper, a discussion about how elementary students' conceptual understanding of rational numbers is fostered, along with number sense development, in a sociocultural learning perspective, is carried out. We highlight how an emergent modeling process, enhanced by percentage and supported by multiple representations, emphasizing numerical values of magnitude contexts, may contribute to that understanding. A classroom experiment was developed, following the methodological procedures of a Design Research. We analyze student's interactions and their work, as representations are used and transformed as models. Findings suggest that percentage, associated to students' real world experiences and previous knowledge, is intuitive and seems to be helpful in students' numerical development from whole numbers to rational numbers. In a classroom community that supports a co-participated learning, where multiple representations serve and become models as dynamic process, can be a way to reach the meaning of rational number's concepts and to strength interrelationships between those concepts.

Keywords: Elementary school mathematics, co-participated learning, rational numbers, models, percentage.

In mathematics education research, learning and understanding rational numbers has been studied widely up until today and are commonly considered very important, but also of increased complexity (Behr, Lesh, Post, & Silver, 1983). In a perspective that emphasizes a crucial continuity between learning whole numbers and rational numbers, a numerical development approach based on magnitude representations and supported on the development of students' conceptual understandings can show improvements in the rational numbers' learning process (Moss & Case, 1999; Siegler, Thompson, & Schneider, 2011). This idea of continuity is consistent with number sense, a process of progressively broadening development, since early ages, of useful, flexible and efficient numerical strategies and relations (McIntosh, Reys, & Reys, 1992). Percentage, as a representation common in everyday life, which emphasizes measurements of quantities through comparative relations, can be a significant stepping-stone in rational numbers' learning process.

Within a broader study that seeks to understand how elementary school students construct their understandings of rational numbers[1], in the social context of the classroom, a classroom experiment has been conducted, regarding number sense and based on a sociocultural learning perspective. In this paper, our central question is to provide useful insights onto how an emergent modeling process supported by multiple representations and enhanced by percentage in a measurement context, may be an anchor to foster students' conceptual understandings of rational numbers.

Learning Rational Numbers with Understanding in Elementary Grades

In elementary grades, rational numbers emerge as a natural extension of the way that students use numbers, allowing to solve problems that are not possible to solve with just whole numbers (NCTM, 2010). Conceptual understanding of rational numbers can be trigged by coordinating students' intuitive and existing understandings of numbers with the development of a qualitative structure for proportional evaluation, supported by percentage (Moss & Case, 1999). By focusing on percentage, a network of concepts, schemes, and symbols is constructed out of the old ones. It presumes an integrated understanding of rational numbers, regarding the meaning of numbers and the contexts in which they are important, as stressed by Moss and Case (1999).

Deepening Conceptual Understandings of Rational Numbers

Rational numbers can be difficult to teach due to the complexities with the concepts they involve. Literature in this domain points out several explanations for the difficulties that students have to deal with, suggesting that attention should be paid to the development of students' conceptual understanding, instead of emphasizing procedural skills and computational algorithms (Fosnot & Dolk, 2002; Monteiro & Pinto, 2007; Moss & Case, 1999; NCTM, 2010). Thus, conceptual understanding is one of the strands of mathematics learning that some authors believe necessary for effective learning. It means being able to relate mathematical ideas, understanding their relationships in an integrated way into a coherent structure of concepts, operations and relations (Kilpatrick, Swafford, & Findell, 2001). It draws from Skemp's (1976) conception of relational understanding as a meaningful kind of learning that concerns the learning process, with its links and relationships, which is useful, as the author stresses, to explore ideas further.

This conceptual understanding is closely related to number sense, as it refers to general understanding of number and operations and to the ability of using it in flexible ways, in order to make mathematical judgments and develop effective strategies in dealing with numbers, and rational numbers as well, in different situations (Abrantes, Serrazina, & Oliveira, 1999; Mcintosh, Reys, & Reys, 1992). It is a process that students should gradually develop from early age. It

[1] The term *rational numbers* refers to the set of non-negative rational numbers.

written work, and teacher´s research journal. We analyzed students' interactions and written work when exploring tasks looking for evidences of students' mathematical learning development, as they participate in classroom community, by inferring the taken-as-shared intentions and meanings and interpreting the social use of representations.

Data Analysis

The analysis discussed in this paper focuses students' activity with some representations during the classroom experiment. This analysis has been carried out, according to a content analysis' procedure, which is still ongoing. We considered three interrelated categories as indicators of students' conceptual understandings of rational numbers as they handle percentage to: (1) encourage an emergent modeling process, (2) foster numerical relations and (3) support reasoning strategies. These analytical categories came from theoretical ideas crossed with elements that emerge from data, in a permanent interaction process, grounded in the details of the specific episodes.

Results

Preliminary Study

In this study, which took place at the beginning of third grade, students were presented to some contextualized tasks, which involved iconic representations related to percentage from daily life situations, as using smartphones, tablets or laptops. Our intention was to perceive how familiar students were with those representations and how significant they were establishing the relation between students' everyday life intuitive understandings of percentage and proportionality and coordinating it with students' strategies for manipulating whole numbers (Figure 2).

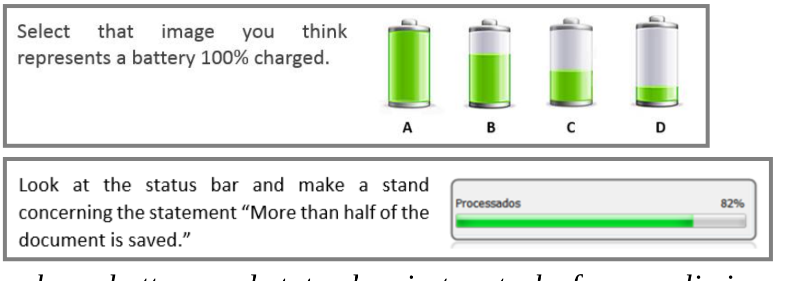


Figure 2. *Mobile phone battery and status bar in two tasks from preliminary study.*

Although percentage has not yet been covered in school, findings of this preliminary study support the idea that students are familiar with percentage terminology and appeared to have a good intuition about percentage from their everyday experiences. All students recognized the expression 100% and associate it, in the context of a mobile phone battery, to image A (Figure 2), the

battery, which is completely charged. The majority of students involved were also able to identify 50% as a half and justify using valid written explanations such as "... if it was 50% it would be a half, but as it is 82% you have more than a half saved" or "... because half is 50% and it is more than 50% saved".
In this sense, the classroom experiment was designed prioritizing the use of different representations associated to percentage from students' real-world situations. These representations were followed by others chosen to extend the understanding of whole numbers to the acquisition of knowledge about rational numbers, as an emergent and continuous modeling process of understanding numerical magnitude values' (Siegler et al., 2011).

The Emergent Modeling Process enhanced by Percentage

Understanding percentages. One of the representations used in the first stage of the classroom experiment was an iconic representation of a mobile phone battery (Figure 3) and students were asked to estimate the percentage represented.

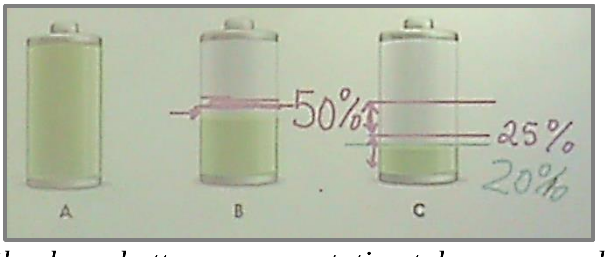


Figure 3. *Mobile phone battery representation taken as a model in a whole-class discussion.*

In small groups, some students came to the conclusion that the shaded part in battery C would represent 25% of fully charged and others said that it would be 20%. In the whole group discussion, students shared their arguments in order to justify their reasoning strategy, considering battery C. Dina's group calls for 25% and argues:

Dina: I saw 100 and made a half, which is 50 then it looked like a half again.

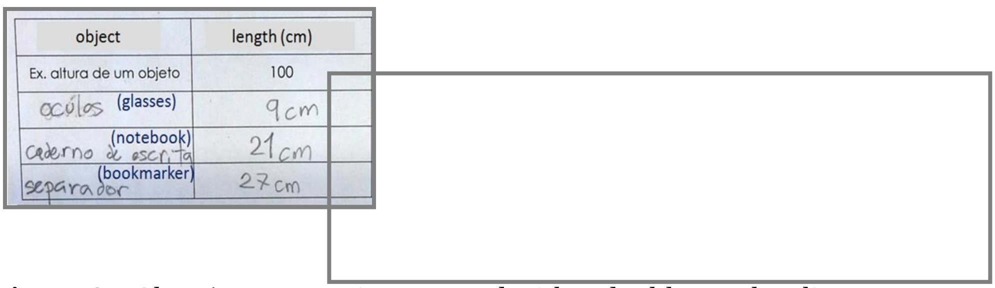
Dina used the successive halving strategy, a very common and useful multiplicative strategy, but inappropriate to this situation. Other students suggested a more precise measure:

Simão: That is not 25 percent.
Teacher: […] So, what do you think it is?
Students: 20 percent.
Teacher: Why? Marco.
Marco: Because it fits 5 times.

Students that stand for the 20% suggested that if the shaded part were iterated, it would fit the unit five times, so it should be 20% and not 25%, as it is shown in Ana's group written record, in Figure 4.

Through this representation, students were able to establish numerical relations and to start seeing numbers as magnitude values'. In this task, the minutes taken to save the whole program were related to the amount of program saved shown in percentage, as Mafalda's group explains "The whole program took forty minutes to be saved because if a half is twenty, the double is forty". The explanation of this reasoning strategy seemed to make sense to the others as it expressed the multiplicative relation between half and double, keeping the ratio constant. Although not all students have clearly perceived it yet as a ratio, they saw that saving half of the program takes as much time as it takes to save the remaining half of the program, in the same proportion as 50% is to 100%. Thus status bar encouraged an intuitive ratio understanding, involving quantities of a standard unit, which can also be achieved when shifting to the double number line.

Understanding decimals. The double number line was crucial in this number system expanding idea with decimal's representation. In one of the tasks where double number line was used, students were invited to represent the value of the length measure of some classroom objects (Figure 8).



object	length (cm)
Ex. altura de um objeto	100
ocúlos (glasses)	9cm
caderno de escrita (notebook)	21cm
separador (bookmarker)	27 cm

Figure 8. *Clara's group written record with a double number line.*

Students had to establish comparison relations between equivalent ratios, like percentage and the measurement of the length that represents a number in the number line, where identifying the unit was key. Number line seems to contribute to a growing understanding of the unit concept and its importance, as we can perceive in the passage of the dialogue amongst students during the whole-classroom discussion.

Bruna: The whole number line goes from zero to one hundred in percentage, but also in centimeters. When we think ninety-one centimeters we have ninety one percent.

Simão: Each one percent is a centimeter.

Teacher: In this specific situation because our unit is one hundred centimeters, that we can see as one hundred percent.

The double number line became a model as students use it to support their reasoning strategies. In another task, when changing the reference unit, decimal numbers emerged in the double number line (Figure 9) as students use it to explain the percentage of "fullness" of the unit, converting from one symbolic

Most of the students could locate it correctly on the number line, partitioning the unit through a halving strategy, halving and then halving again, as Dinis's group written record (Figure 10) shows. They used percentage benchmarks to identify measurements as points in the number line which represented where each relay baton change, considering 100% as 200 meters.
In the same task, students had to determine if the given sentence "Each of the team players runs $\frac{1}{5}$ of the sprint relay" was true or false and justify using the number line. Most of the students agreed that the sentence was false and argued, during the whole-class discussion, saying:

Dinis: There were only four people in the race.
Ivo: Each one runs fifty meters.
Teacher: And?
Ana: If it were four people it must be from fifty to fifty and not from forty to forty. That would be five persons.
Teacher: So, looking into the number line, is the claim true?
Heitor: No, it´s one fourth. Each one runs twenty-five percent, not twenty percent.

According to students' argument, they can relate the number of players with the parts into which the unit must be partitioned, relying on the idea of fair sharing and associating with a percentage value. Some students, as Dina's group written record in Figure 11, represented each stage of the sprint relay considering both situations, if each player had to run one fourth or if each player had to run one fifth, in order to compare both, and trying to justify why the sentence is false.

Figure 11. *Dina's group written record with the double number line*

Dina´s group, as other students, used the number line to locate fractions and compare the length they represent, reasoning about their sizes. Number line in this task was taken as model as it supported numerical relations and compare relative sizes of fractions, as they arise while making sense of a situation embedded in a reference context.

fractions. This seems to contribute to the emergence of a co-participated learning (Pontecorvo et al., 2005) on efficient reasoning strategies and a developing awareness of the rational numbers' relationships, among the classroom community.

References

Abrantes, P., Serrazina, L., & Oliveira, I. (1999). *A matemática na educação básica*. Lisboa: Ministério da Educação/Departamento da Educação Básica (ME/DEB).

Behr, M., Lesh, R., Post, T., & Silver, E. (1983). Rational number concepts. In R. Lesh & M. Landau (Eds.), *Acquisition of mathematics concepts and processes* (pp. 91-126). New York, NY: Academic Press.

Bruner, J. (1962). *The process of education*. Cambridge, MA: Harvard University Press.

Cobb, P., Stephan, M., McClain, K., & Gravemeijer, K. (2001). Participating in Classroom Mathematical Practices. In E. Yackel, K. Gravemeijer & A. Sfard (Eds.), *A journey in mathematics education research: Insights from the work of Paul Cobb* (pp. 117–163). New York, NY: Springer.

Confrey, J., & Lachance, A. (2000). Transformative teaching experiments through conjecture-driven research design. In A. Kelly, & R. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 231-266). Mahwah, NJ: Erlbaum.

Dole, S., Cooper, T. J., Baturo, A., & Conoplia, Z. (1997) Year 8, 9 and 10 students' understanding and access of percent knowledge. People in mathematics education: *Proceedings of the 20th Annual Conference of the Mathematics Education Research Group of Australasia* (pp.147–154). New Zealand, Rotorua.

Fosnot, C. T., & Dolk, M. (2002). *Young mathematicians at work: Constructing fractions, decimals, and percents*. Portsmouth, NH: Heinemann.

Gravemeijer, K. (1999). How emergent models may foster the constitution of formal mathematics. *Mathematical thinking and learning*, *1*(2), 155-177.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: The National Academies Press. Retrieved from https://www.nap.edu/read/9822.

Lamon, S. (2007) Rational numbers and proportional reasoning. In F. Lester (Ed), *Second handbook of mathematics teaching and learning* (pp 629-667). Greenwich, CT: Information Age.

McIntosh, A., Reys, J., & Reys, E. (1992). A proposed framework for examining basic number sense. *For the Learning of Mathematics, 12* (3), 2-8.

Monteiro, C., & Pinto, H. (2007). *Desenvolvendo o sentido de número racional*. Lisboa, Portugal: APM.