Title: Scaling law in saddle-node bifurcations for one-dimensional maps: a complex variable approach

Author(s): **Duarte, Jorge**[1,2]; **Januário, Cristina**[1]; Martins, Nuno[2]; Sardanyes, Josep[3]

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Abstract: The study of transient dynamical phenomena near bifurcation thresholds has attracted the interest of many researchers due to the relevance of bifurcations in different physical or biological systems. In the context of saddle-node bifurcations, where two or more fixed points collide annihilating each other, it is known that the dynamics can suffer the so-called delayed transition. This phenomenon emerges when the system spends a lot of time before reaching the remaining stable equilibrium, found after the bifurcation, because of the presence of a saddle-remnant in phase space. Some works have analytically tackled this phenomenon, especially in time-continuous dynamical systems, showing that the time delay, tau, scales according to an inverse square-root power law, tau similar to (mu-mu (c))(-1/2), as the bifurcation parameter mu, is driven further away from its critical value, mu (c) . In this work, we first characterize analytically this scaling law using complex variable techniques for a family of one-dimensional maps, called the normal form for the saddle-node bifurcation. We then apply our general analytic results to a single-species ecological model with harvesting given by a unimodal map, characterizing the delayed transition and the scaling law arising due to the constant of harvesting. For both analyzed systems, we show that the numerical results are in perfect agreement with the analytical solutions we are providing. The procedure presented in this work can be used to characterize the scaling laws of one-dimensional discrete dynamical systems with saddle-node bifurcations.

Author Keywords: Scaling Law; Saddle-Node Bifurcations; One-Dimensional Maps; Complex Variable

KeyWords Plus: Critical Slowing-Down; Intermittency; Cooperation; Transitions; Hypercycles; Extinctions; Models; Ghosts

Reprint Address: Duarte, J (reprint author), ISEL Engn Super Inst Lisbon, Dept Math, Rua Conselheiro Emídio Navarro 1, P-1949014 Lisbon, Portugal.

Addresses:
1. ISEL Engn Super Inst Lisbon, Dept Math, P-1959007 Lisbon, Portugal
2. Inst Super Tecn, Ctr Analise Matemat Geometria & Sistemas Dinam, Dept Math, P-1049001 Lisbon, Portugal
3. Consejo Super Invest Cient UPV, Inst Biol Mol & Celular Plantas, Valencia 46022, Spain

E-mail Address:

jduarte@deq.isel.pt; cjanuario@deq.isel.pt; nmartins@math.ist.utl.pt; josep.sardanes@upf.edu

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